

INTRODUCTION TO QUANTUM FIELD THEORY

Lectures at HDM 2017

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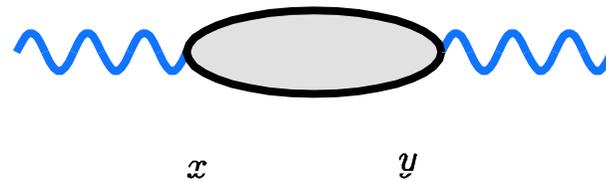
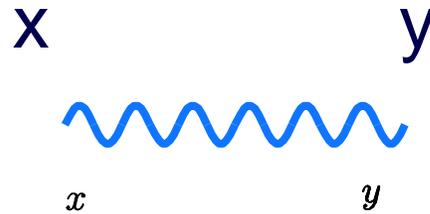


Lecture 4



CORRELATION FUNCTION IN QFT

- $\langle 0 | J_\mu(y) J_\nu^\dagger(x) | 0 \rangle$



$$J_\mu(x) |_{QCD} = \bar{\psi}(x) \gamma_\mu \psi(x) \quad J_\mu(x) |_{HAD} = \rho_\mu(x)$$

Q C D

$$\Pi(q^2) = i \int d^4x e^{iqx} \langle 0 | T (J(x) J^+(0)) | 0 \rangle$$

$$J(x) \Rightarrow A_\mu(x) |^i_j = \bar{\psi}^i(x) i \gamma_5 \gamma_\mu \psi_j(x)$$

$$V_\mu(x) |^i_j = \bar{\psi}^i(x) i \gamma_\mu \psi_j(x)$$

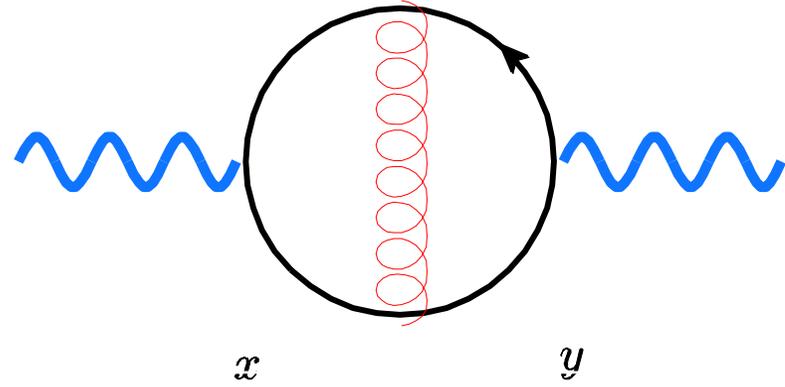
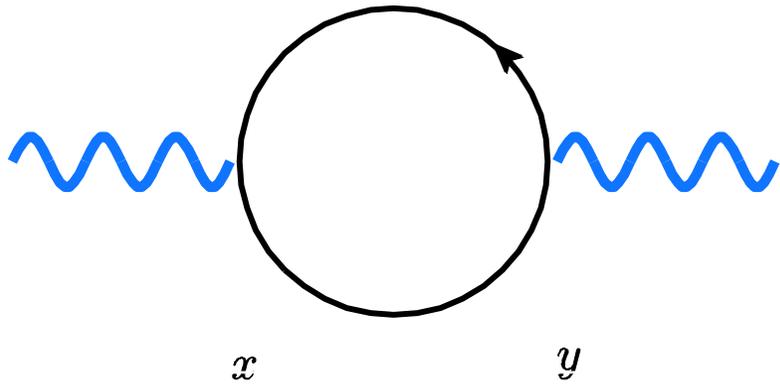
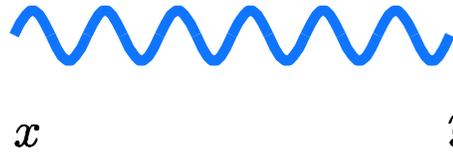
HADRONIC

$$\Pi(q^2) = i \int d^4x e^{iqx} \langle 0 | T (J(x) J^+(0)) | 0 \rangle$$

$J(x) \Rightarrow$ *Hadronic fields / EXPERIMENTAL DATA*

$$J_\mu(x) |_{QCD} = \bar{\psi}(x) \gamma_\mu \psi(x) \quad J_\mu(x) |_{HAD} = \rho_\mu(x)$$

QCD



HADRONIC



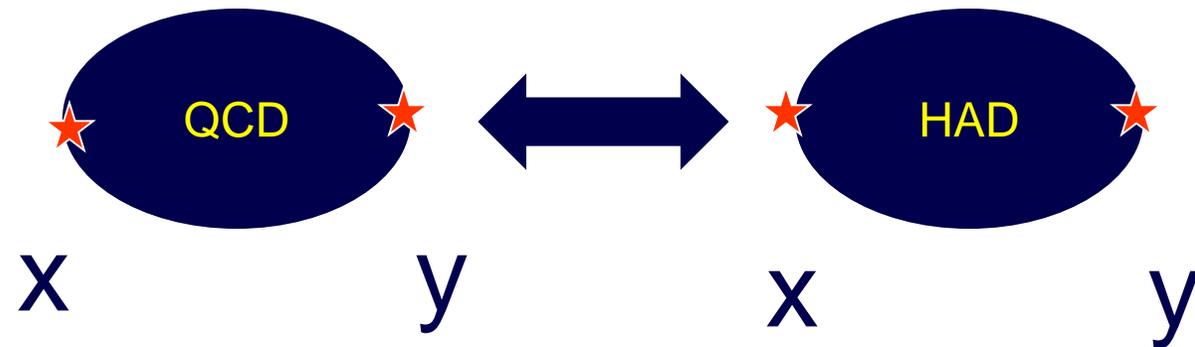
x

y



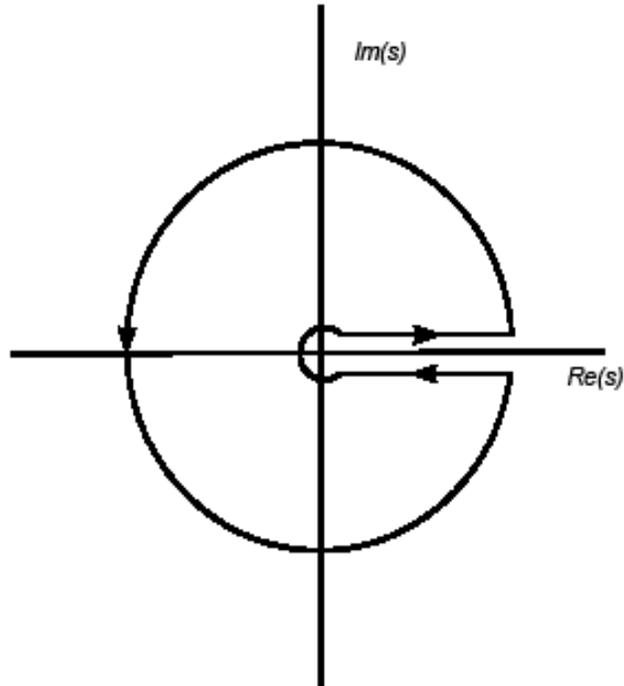
x

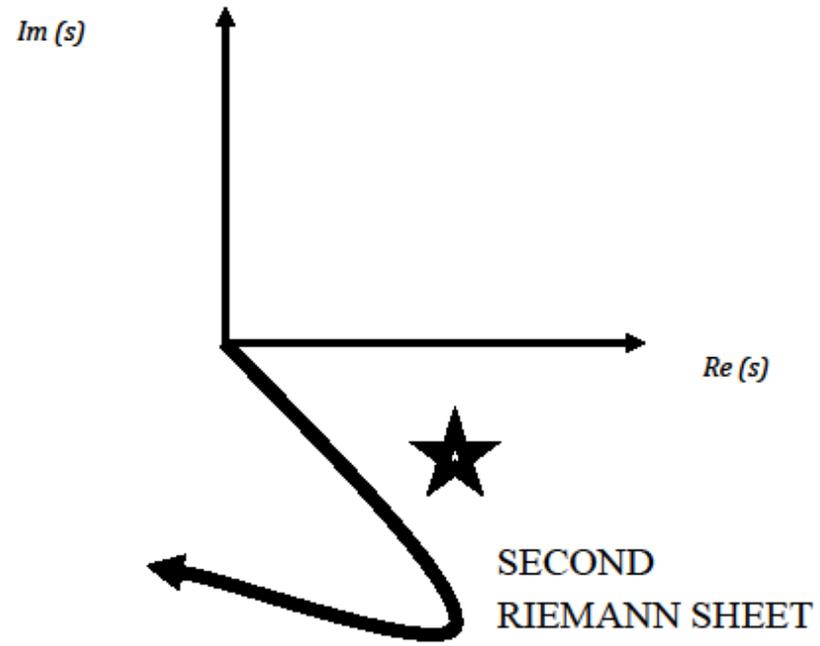
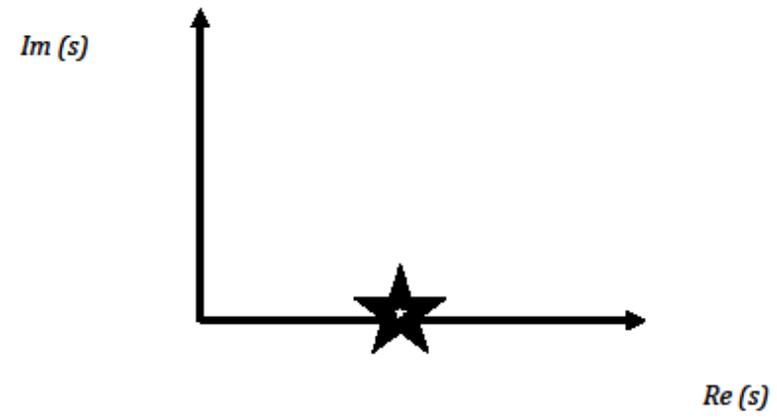
y



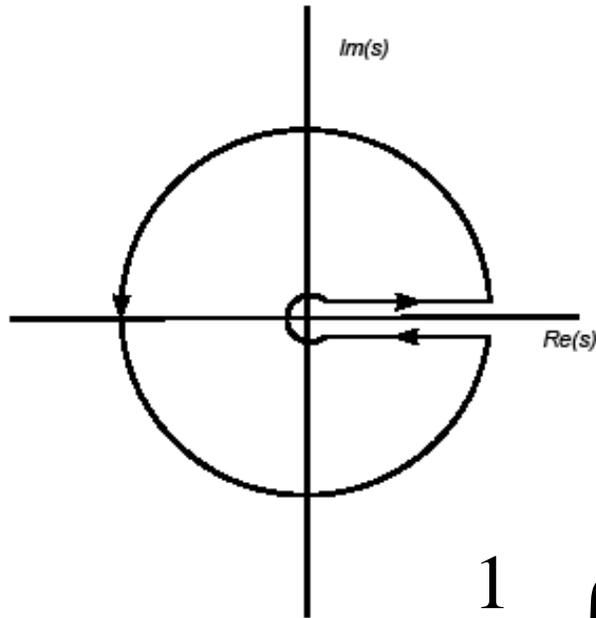
**CAUCHY'S THEOREM IN THE
COMPLEX ENERGY PLANE**

QUARK-HADRON DUALITY





QUARK-HADRON DUALITY



$$\oint_C \Pi(s) ds = 0$$

$$-\frac{1}{2\pi i} \oint_{C(|s_0|)} ds \Pi(s) = \int_{s_{th}}^{s_0} ds \frac{1}{\pi} \text{Im} \Pi(s)$$

$$-\frac{1}{2\pi i} \oint_{C(s_0)} ds \Pi_{QCD}(s) = \int_{s_{th}}^{s_0} ds \frac{1}{\pi} \text{Im} \Pi(s) |_{HAD}$$

QCD @ FINITE TEMPERATURE AND DENSITY

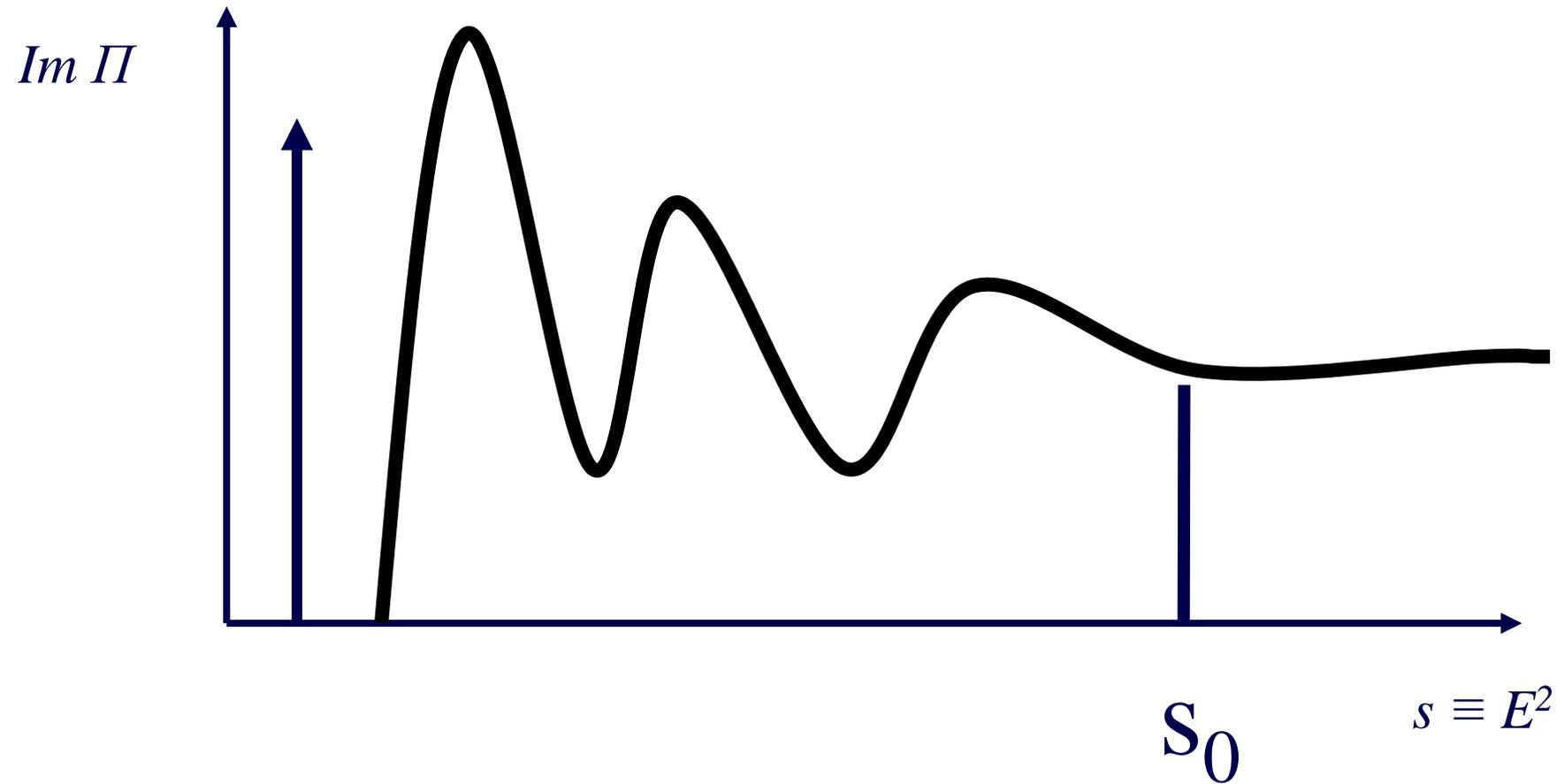
DOLAN-JACKIW (MATSUBARA)

$$S_F(T = 0) = \frac{\not{k} + m}{k^2 - m^2 + i\epsilon}$$

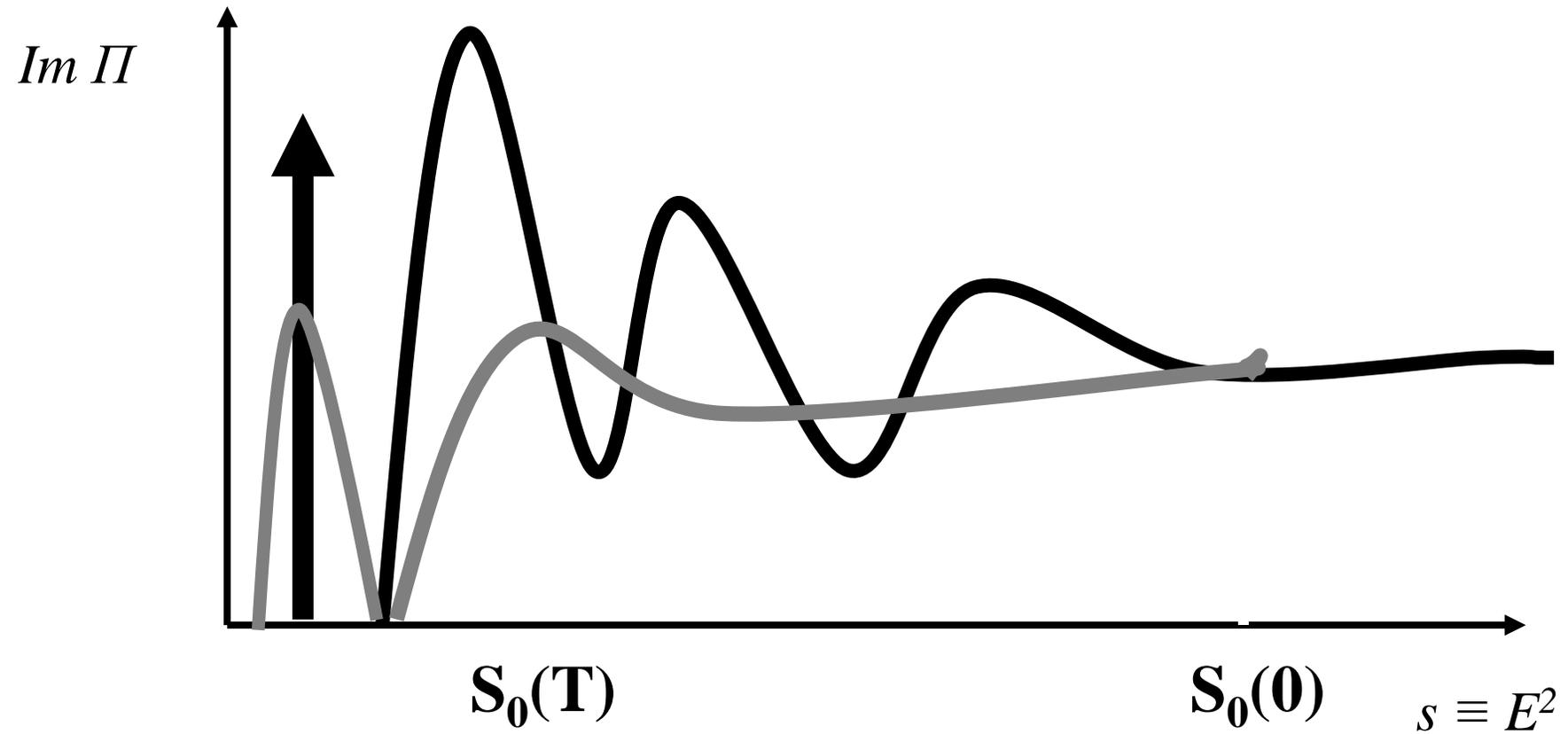
$$S_F(T) = (\not{k} + m) \left[\frac{1}{k^2 - m^2 + i\epsilon} + 2\pi i \delta(k^2 - m^2) n_F(|k_0|) \right]$$

$$n_F(z) = \frac{1}{e^{z/T} + 1}$$

Realistic Spectral Function T=0



Realistic Spectral Function (T)



THEORETICAL EXPECTATIONS

HADRONIC SECTOR

ONSET OF PQCD (CONTINUUM) $S_0(T)$

RESONANCE BROADENING : $\Gamma(T)$

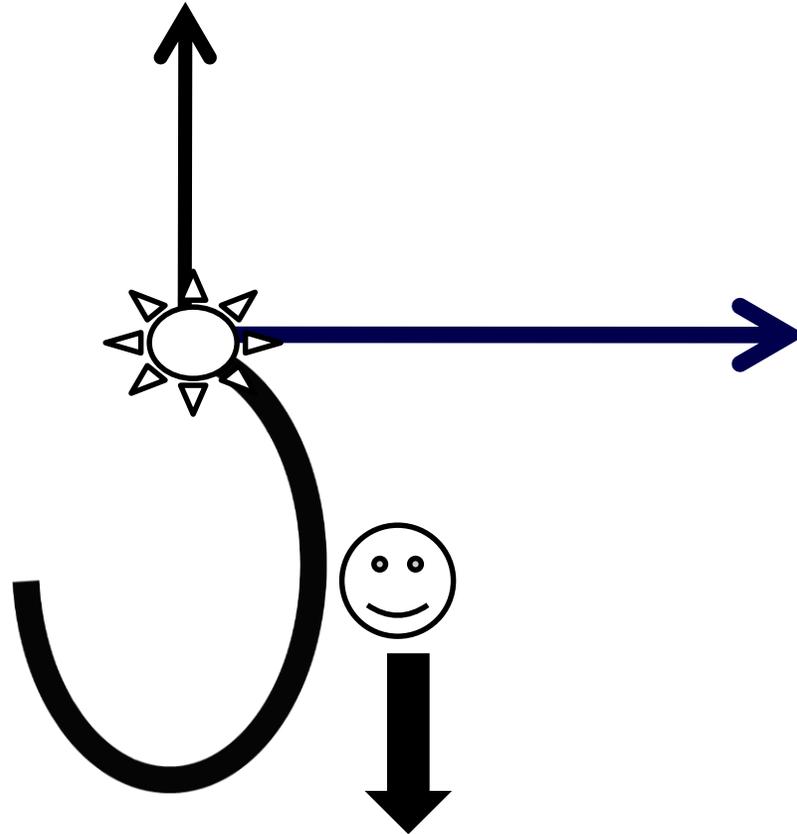
(absorption of hadrons in the thermal medium)

VANISHING OF HADRONIC COUPLINGS

BROADENING OF RADII

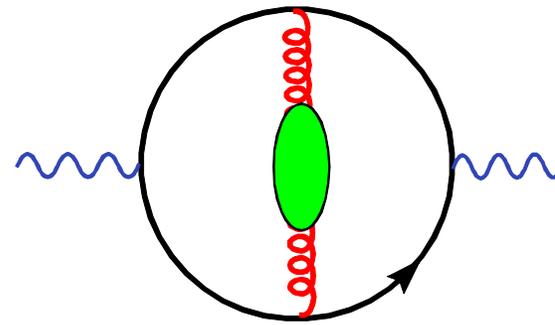
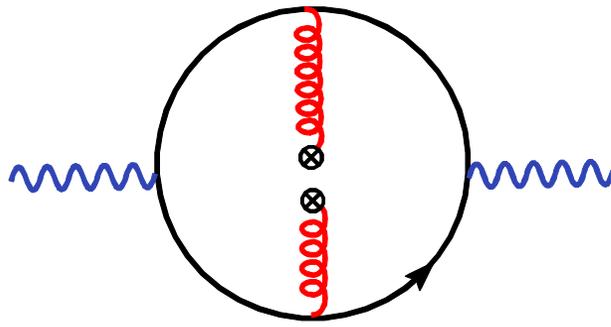
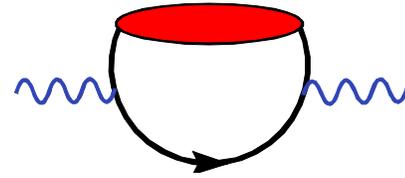
ORDER PARAMETERS

$S_0(T)$ & COUPLING(T) & WIDTH(T) [



DECONFINEMENT *SIGNALS*

- **RESONANCE BROADENING WITH (increasing) T**
- **ELECTROMAGNETIC & HADRONIC RADII OF HADRONS: SWELLING WITH (increasing) T**
- **HADRONIC COUPLINGS: VANISHING WITH (increasing) T**
- **HADRONIC (weak) DECAY CONSTANTS: VANISHING WITH (increasing) T**



QUARK CONDENSATE

$$\langle 0 | \bar{q} q | 0 \rangle$$

GLUON CONDENSATE

$$\langle 0 | \alpha_s G_{\mu\nu}^a G_{\mu\nu}^a | 0 \rangle$$

$$\mathbf{T} \neq \mathbf{0}$$

QCD SECTOR

$$s_0(\mathbf{T})$$

$$\langle \mathbf{0} | \alpha_s \mathbf{G}^2 | \mathbf{0} \rangle (\mathbf{T})$$

$$\langle \mathbf{0} | \bar{\psi} \psi | \mathbf{0} \rangle (\mathbf{T})$$

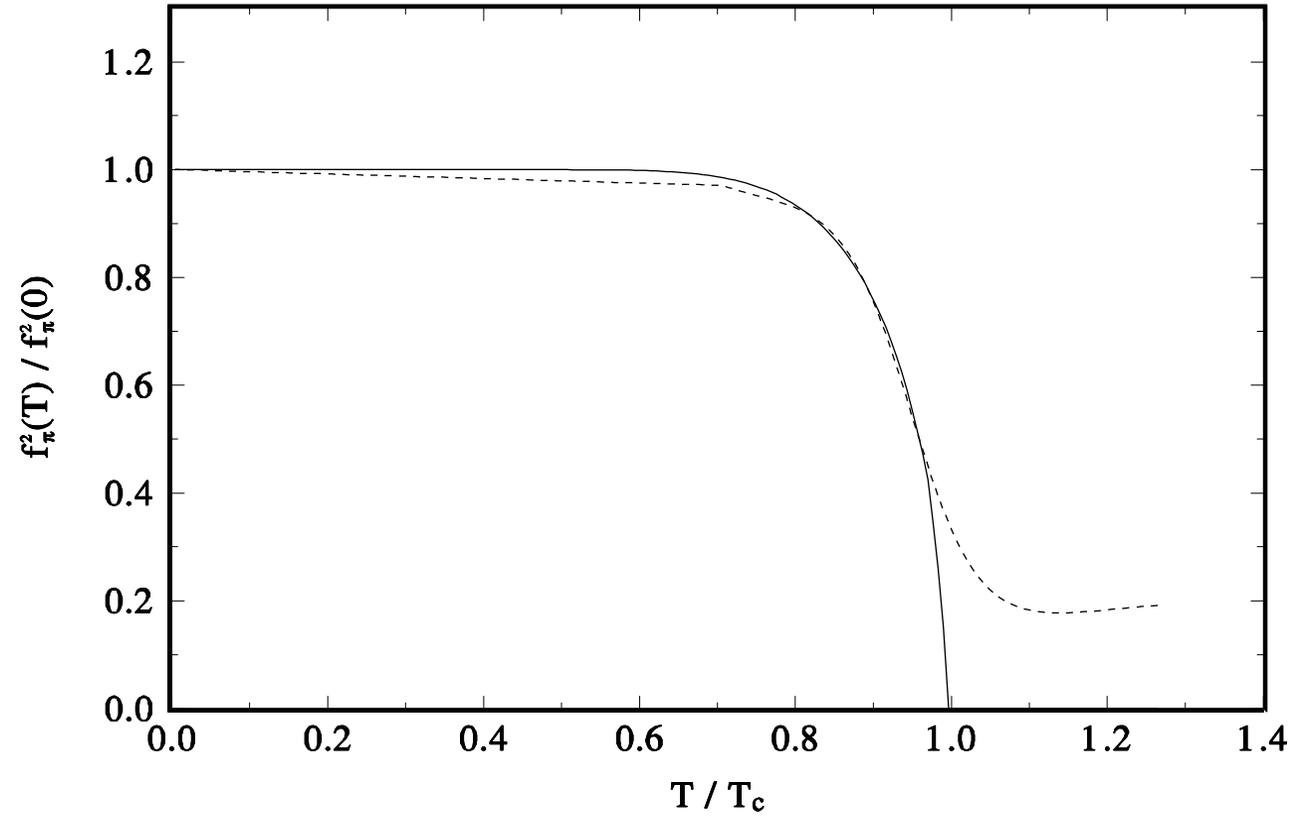
QUARK CONDENSATE

$$\lim_{T \rightarrow T_c} \langle 0 | \bar{q} q | 0 \rangle_T \rightarrow 0$$

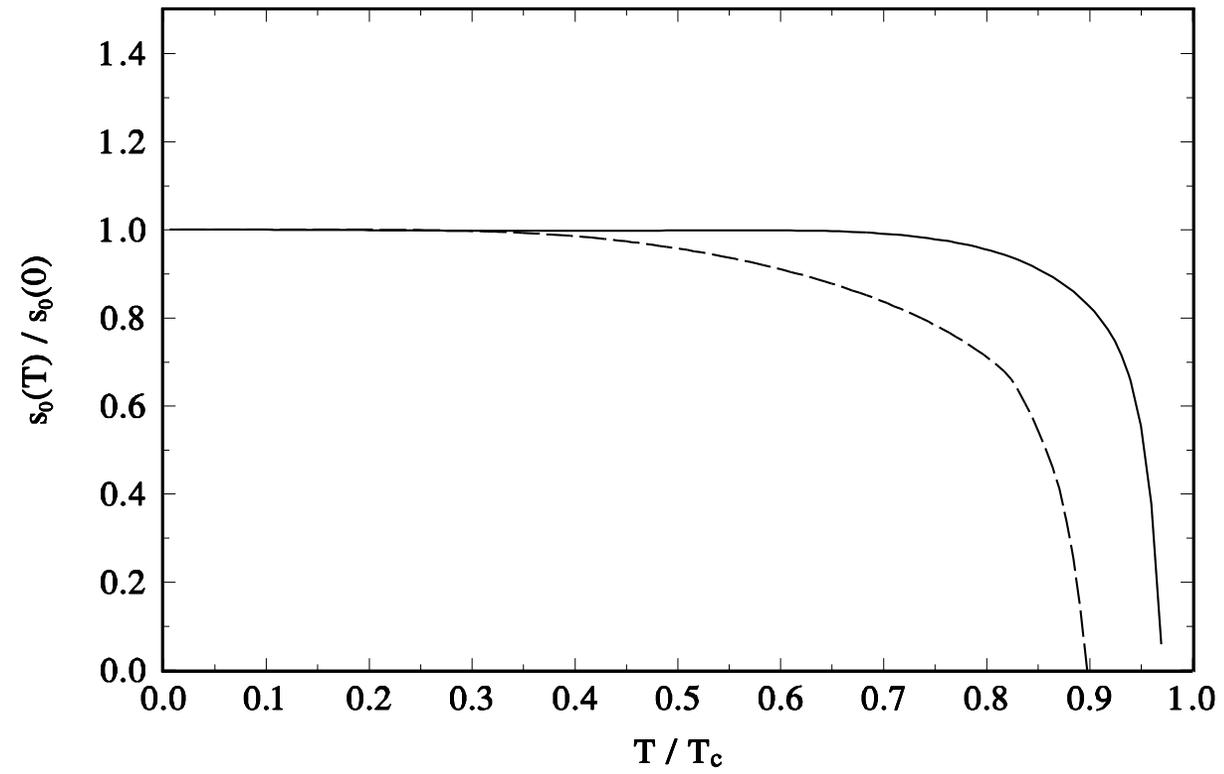
GLUON CONDENSATE

$$\lim_{T \rightarrow T_c} \langle 0 | \alpha_s G^2 | 0 \rangle_T \rightarrow 0$$

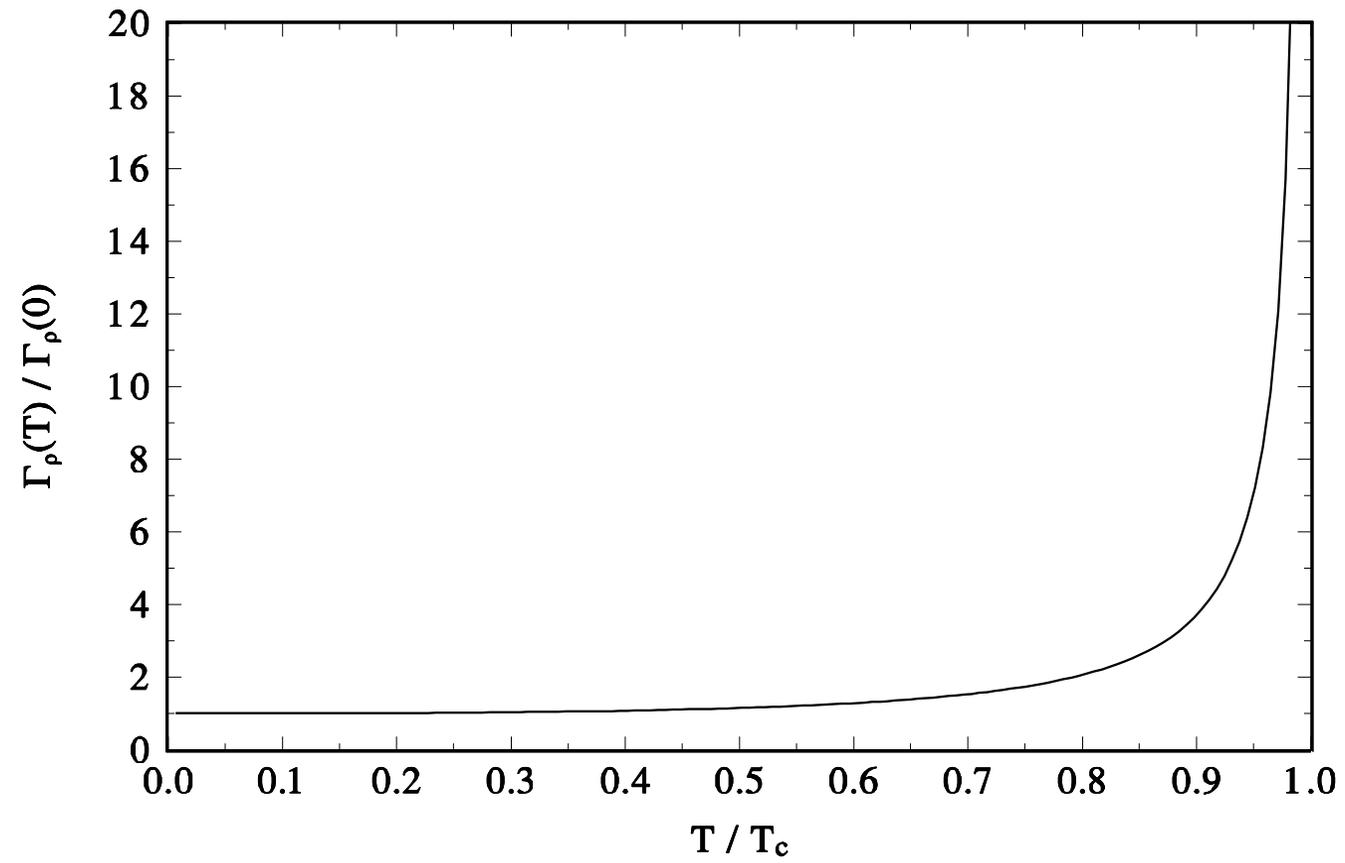
$$f_{\pi}^2 \propto |\langle 0 | \bar{q} q | 0 \rangle|$$



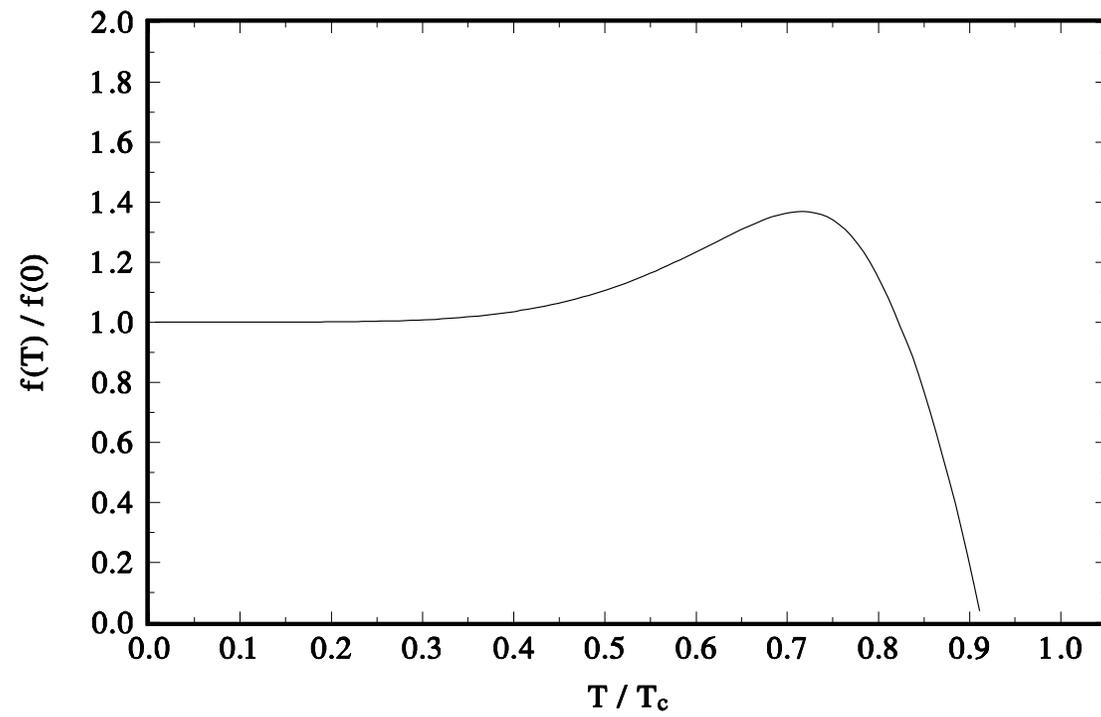
$S_0(\mathbf{T})|_{\rho}$ v: —
A: - - -



$\Gamma_\rho(\mathbf{T})$



$f_{\rho}(T)$



APPENDIX

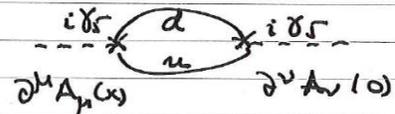
EXPLICIT CALCULATION OF A QCD CURRENT CORRELATOR

THE TWO-POINT FUNCTION $\chi_5^{(d)}$ (q^2) in PQCD

$$\chi_5(q^2) \triangleq i \int d^4x e^{iq \cdot x} \langle 0 | T(\partial^\mu A_\mu(x) \partial^\nu A_\nu^\dagger(0)) | 0 \rangle \quad (1)$$

where $\partial^\mu A_\mu(x) \Big|_i^d = (\bar{m}_i + \bar{m}_j) : \bar{q}_i(x) i\gamma_5 q_j(x) \quad (2)$

with $\bar{m}_{i,j}$ the running QCD quark masses (u,d).



Substituting Eq. (2) into Eq. (1)

$$\chi_5(q^2) = i (\bar{m}_u + \bar{m}_d)^2 \int d^4x e^{iq \cdot x} \langle 0 | T([\bar{d}(x) i\gamma_5 u(x)] \times [\bar{d}(x) i\gamma_5 u(x)]^\dagger) | 0 \rangle \quad (3)$$

$$\mathcal{T} \triangleq T(\partial^\mu A_\mu(x) \partial^\nu A_\nu^\dagger(0)) =$$

$$= \bar{d}_i^a(x) (i\gamma_5)_{ij} u_j^a(x) \bar{u}_k^b(0) (i\gamma_5)_{kl}^\dagger d_l^b(0) \quad (4)$$

use $(\gamma_5)^\dagger = \gamma_5$

$$\mathcal{T} = i^2 \overset{\textcircled{1}}{\bar{d}_i^a(x)} (\gamma_5)_{ij} \overset{\textcircled{2}}{u_j^a(x)} \overset{\textcircled{3}}{\bar{u}_k^b(0)} (\gamma_5)_{kl} \overset{\textcircled{4}}{d_l^b(0)}$$

$\textcircled{1}\textcircled{2}\textcircled{3}\textcircled{4} \Rightarrow \textcircled{2}\textcircled{3}\textcircled{4}\textcircled{1} = -\textcircled{1}\textcircled{2}\textcircled{3}\textcircled{4}$
Wick sign = -1

$$\mathcal{T} = (-) i^2 S_F^{(d)}(-x) \Big|_{li} \delta^{ab} i S_F^u(x) \Big|_{jk} \delta^{ab} (\gamma_5)_{ij} (\gamma_5)_{kl}$$

$$\mathcal{T} = (-) i^2 N_c (\gamma_5)_{ij} S_F^{(u)}(x) \Big|_{jk} (\gamma_5)_{kl} S_F^{(d)}(-x) \Big|_{li} \quad (5)$$

②

$$\tau = (-)i^2 N_c \text{Tr} [\gamma_5 S_F^{(u)}(x) \gamma_5 S_F^{(d)}(-x)]$$

$$= (-)i^2 N_c \text{Tr} [S_F^{(d)}(-x) \gamma_5 S_F^{(u)}(x) \gamma_5] \quad (6)$$

In momentum k -space this trace becomes

$$\tau = (-)i^2 N_c \text{Tr} [(k_2 + m_d) \gamma_5 (k_1 + m_u) \gamma_5]$$

$$\times \frac{1}{k_2^2 - m_d^2} \times \frac{1}{k_1^2 - m_u^2} \quad (7)$$

$$\text{where } S_F(x) = \int \frac{d^4 k}{(2\pi)^4} S_F(k) e^{-ik \cdot x} \quad (8)$$

$$\tau = (-)i^2 N_c \text{Tr} \left\{ k_2 \gamma_5 k_1 \gamma_5 + k_2 \gamma_5 m_u \gamma_5 + m_d \gamma_5 k_1 \gamma_5 + m_d \gamma_5 m_u \gamma_5 \right\} \quad (9)$$

$$\tau = (-)i^2 N_c \text{Tr} [-k_2 k_1 + m_u m_d] \quad (10)$$

$$\tau = (-)i^2 N_c 4 (m_u m_d - k_1 \cdot k_2) \quad (11)$$

Then $N_5(q^2)$ becomes

$$N_5(q^2) = i \int d^4 x e^{iq \cdot x} (-)i^2 N_c (m_u + m_d)^2 \times \\ \times \int \frac{d^4 k_1}{(2\pi)^4} \int \frac{d^4 k_2}{(2\pi)^4} \frac{4 (m_u m_d - k_1 \cdot k_2)}{(k_1^2 - m_u^2)(k_2^2 - m_d^2)} e^{i(k_2 - k_1) \cdot x} \quad (12)$$

Integrating in $d^4 x$ using

$$\int d^4 x e^{iq \cdot x} e^{ik_2 \cdot x} e^{-ik_1 \cdot x} = (2\pi)^4 \delta^{(4)}(q + k_2 - k_1) \quad (12)$$

(3)

and using the delta function to integrate over k_2 & calling $k_1 \equiv k$, gives

$$\psi_S(q^2) = -4 i^3 N_c (m_u + m_d)^2 \int \frac{d^4 k}{(2\pi)^4} \frac{[m_u m_d - k \cdot (k - q)]}{[(k - q)^2 - m_d^2] (k^2 - m_u^2)} \quad (13)$$

The term $(m_u + m_d)$ is of higher order & can be neglected (justification given later on).

$$\psi_S(q^2) = -4 i N_c (m_u + m_d)^2 \int \frac{d^4 k}{(2\pi)^4} \frac{(k^2 - k \cdot q)}{(k - q)^2 k^2} \quad (14)$$

The integral of the first term (k^2) vanishes, so

$$\psi_S(q^2) = 4 i N_c (m_u + m_d)^2 q^\mu \int \frac{d^4 k}{(2\pi)^4} \frac{k_\mu}{(k - q)^2 k^2} \quad (15)$$
$$\frac{-i}{(4\pi)^2} \frac{1}{2} q_\mu \ln(-q^2/\mu^2)$$

$$\psi_S(q^2) = (m_u + m_d)^2 \left\{ \frac{3}{8\pi^2} - q^2 \ln(-q^2/\mu^2) \right\} \quad (16)$$

There is an overall (-) sign inserted above, which must be there, but does not seem to follow from this derivation.

Justification of neglecting $O(m_q^4)$

The full expression of $\psi_S(q^2)$ is

$$\psi_S(q^2) = (m_u + m_d)^2 \left\{ -\frac{3}{8\pi^2} q^2 \ln(-q^2/\mu^2) + (m_u + m_d) \frac{\langle \bar{\psi} \psi \rangle}{q^2} - \frac{1}{8\pi^2} \langle \frac{dS}{d\pi} G^2 \rangle + \mathcal{O}\left(\frac{dS}{q^4}\right) \right\} \quad (17)$$

