

# INTRODUCTION TO QUANTUM FIELD THEORY

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## Lecture 2



# QUANTUM FIELD THEORY

- Infinite number of degrees of freedom
  - FIELD (Classical)
  - $q_i(t) \ (i=1,2,\dots,N)$
  - $\lim_{N \rightarrow \infty} q_i(t) = q(x_1, x_2, x_3, t) \equiv \varphi(x)$ 
    - FIELD (Quantum)
    - $\varphi(x) \Rightarrow$  Operator
  - $[\varphi(x), \varphi(y)] \neq 0 \quad \text{or} \quad \{\varphi(x), \varphi(y)\} \neq 0$

# KLEIN-GORDON COMPLEX SCALAR FIELD

(Classical)

$$\varphi(x) \neq \varphi^*(x) \quad x = (x_0 = ct, x_i)$$

$$\mathcal{L}_{KG}^0 = \partial^\mu \varphi(x) \partial_\mu \varphi^*(x) - m_0^2 \varphi(x) \varphi^*(x)$$

$$(\partial^\mu \partial_\mu + m_0^2) \varphi(x) = 0$$

$$(\partial^\mu \partial_\mu + m_0^2) \varphi^*(x) = 0$$

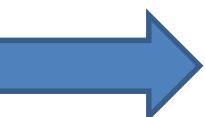
$$\varphi(x) = \sum_p [a_p e^{-ipx} + b_p^* e^{ipx}]$$

# FIELDS

CLASSICAL  QUANTUM

e.g.

## EXCITATIONS OF FIELDS

WAVES  PARTICLES

MINKOWSKI (SPACE-TIME)  FOCK (PARTICLES)

$\hat{\phi}(x)$  → operator in Fock space

$$\hat{\phi}(x)|0\rangle = 0 \quad \hat{\phi}^+(x)|0\rangle \propto |p\rangle$$

annihilation / creation of a particle of 4-momentum  $p$

creation initiated in Fock space & particle appears in Minkowski 4-d space

$$\varphi(x)=\sum_p [a_p~e^{-i\,p\,x}+~b_p^*~e^{i\,p\,x}]$$

$$\hat{\varphi}(x)=\sum_p [\hat{a}_p~e^{-i\,p\,x}+~\hat{b}_p^+~e^{i\,p\,x}]$$

$$[a_{\vec p} \; , a^+_{\vec q} ] = \delta^{(3)} \left( \vec p - \vec q \right) \qquad [b_{\vec p} \; , b^+_{\vec q} ] = \delta^{(3)} \left( \vec p - \vec q \right)$$

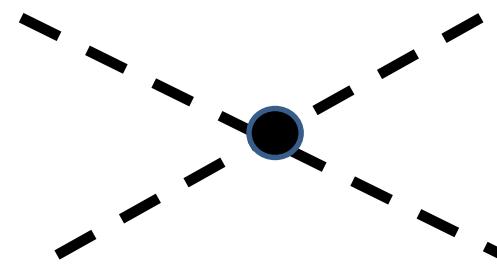
$$a_p|0\rangle\!=\!b_p|0\rangle\!=\!0\qquad a_p^+|0\rangle\propto|p\rangle\qquad b_p^+|0\rangle\propto|\bar{p}\rangle$$

**KLEIN GORDON QUANTUM FIELD:**

**PARTICLES: SPIN ZERO:  $\pi^\pm$   $K^\pm$  ...**

$$\mathcal{L}_{KG}^0 = \partial^\mu \varphi(x) \, \partial_\mu \varphi^*(x) - m_0^2 \, \varphi(x) \, \varphi^*(x)$$

$$\mathcal{L}_{KG}^1 = \lambda \, \varphi^4$$



# **QUANTUM ELECTRODYNAMICS (QED)**

**FIELDS:** Matter fields ( $e^-$ ,  $e^+$ ):  $\psi(x)$   
**massive** ( $m_0$ )

**Gauge fields ( $\gamma$ ):**  $A_\mu(x)$   
**massless**

# CURRENTS

- CLASSICAL ELECTRODYNAMICS
- $\rho(x, t)$  &  $\mathbf{J}(x, t) : \partial \rho / \partial t + \nabla \cdot \mathbf{J} = 0$
- $\mathbf{J}_\mu(\rho, \mathbf{J}) : (\partial / \partial x^\mu) J^\mu = \partial_\mu J^\mu = 0$

# CLASSICAL / QUANTUM ELECTRODYNAMICS

$$\begin{aligned}\mathcal{L} = & i \bar{\psi}(x) \gamma_\mu \partial^\mu \psi(x) - m_0 \bar{\psi}(x) \psi(x) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\ & + e A_\mu(x) \bar{\psi}(x) \gamma^\mu \psi(x)\end{aligned}$$

$$F_{\mu\nu}(x) \equiv \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)$$

$$j^\mu(x) = e \bar{\psi}(x) \gamma^\mu \psi(x) \quad \mathcal{L}_{\text{int}} = A_\mu(x) j^\mu(x)$$

$$(i \gamma^\mu \partial_\mu - m_0)_{ab} \psi(x)_b = 2 i e A_\mu(x) \partial^\mu \psi(x)_a$$

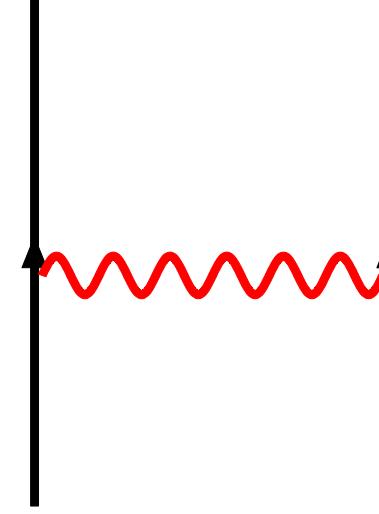
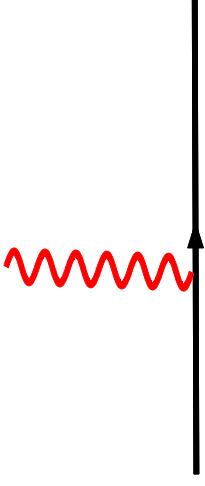
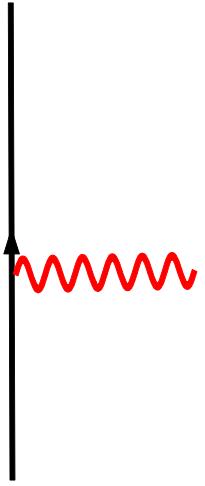
$$\partial_\nu F^{\mu\nu}(x) = i e [\bar{\psi}(x) \partial^\mu \psi(x) - \psi(x) \partial^\mu \bar{\psi}(x)]$$

$$F_{\mu\nu}(x) \equiv \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)$$

coupled non-linear equations for  $\psi(x)$  &  $A_\mu(x)$  nobody has solved (yet)

“solution”: treat  $A_\mu(x)$  as known and given, solve for  $\psi(x)$

$$\partial_\nu F^{\mu\nu}(x) = J^\mu(x) \text{ **given**}$$



## **PARTICLE PROPAGATION:**

**particles created/annihilated in Fock space**

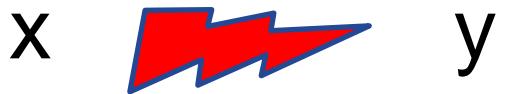
**particles propagated in Minkowski space**

# GREEN FUNCTION

- $\mathcal{D}(\mathbf{x}, t) \psi(\mathbf{x}, t) = f(\mathbf{x}, t)$       e.g.  $\mathcal{D}(\mathbf{x}, t) \equiv \nabla^2 - (1/v^2) \partial^2 / \partial t^2$
- $\mathcal{D}(\mathbf{x}, t) G(\mathbf{x}, t | \mathbf{x}'', t'') = \delta^{(3)}(\mathbf{x} - \mathbf{x}'') \delta(t - t'')$
- $\psi(\mathbf{x}, t) = \int d^3x'' G(\mathbf{x}, t | \mathbf{x}'', t'') f(\mathbf{x}'', t'')$

# PARTICLE PROPAGATION IN QFT

- $\langle 0 | \varphi(y) \varphi^\dagger(x) | 0 \rangle$



$$i \Delta(x - y) \equiv \langle 0 | \varphi(y) \varphi^\dagger(x) | 0 \rangle$$

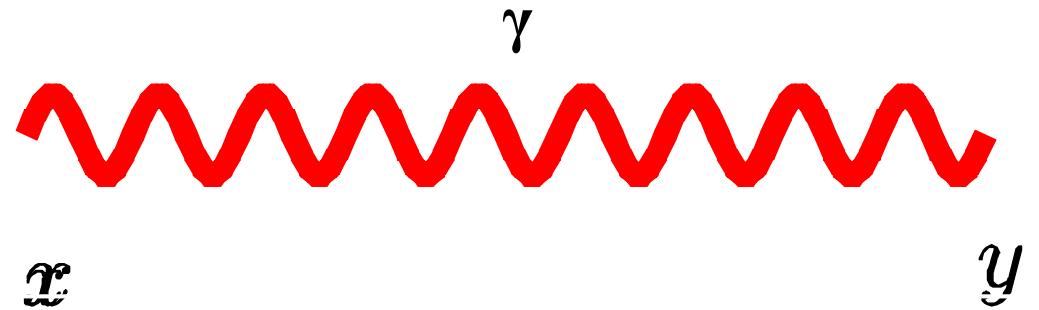
$$\mathcal{D}(x) \Delta(x - y) \propto \delta^{(4)}(x - y)$$

# PROPAGATOR IN MOMENTUM SPACE

$$\Delta(x-y) = \int \frac{d^4 p}{(2\pi)^4} e^{-i p \cdot (x-y)} \Delta(p)$$

$$\Delta(p) = \frac{1}{p^2 - m_0^2 + i\epsilon}$$

# PHOTON PROPAGATOR



$$i D_{\mu\nu}(x - y) = \langle 0 | A_\mu(x) A_\nu(y) | 0 \rangle$$

$$D_{\mu\nu}(q^2) = -\frac{1}{q^2 + i\varepsilon} T_{\mu\nu}$$

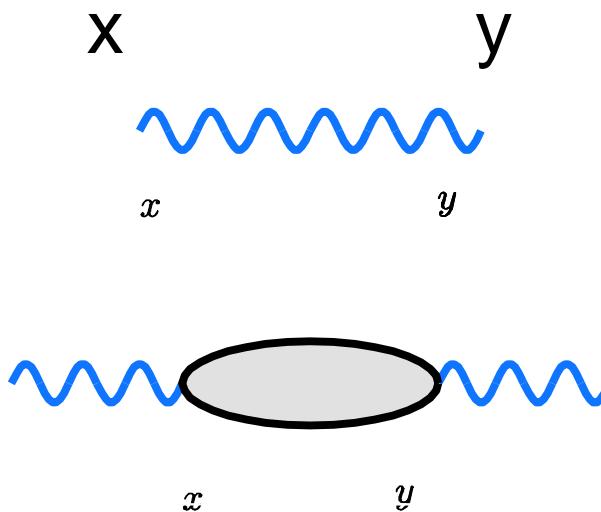
# CURRENTS

- Local bilinear functions of fields
- Definite quantum numbers
- e.g. Electromagnetic current:

$$J_\mu(x) |_{QCD} = e \bar{\psi}(x) \gamma_\mu \psi(x)$$

# CORRELATION FUNCTION IN QFT

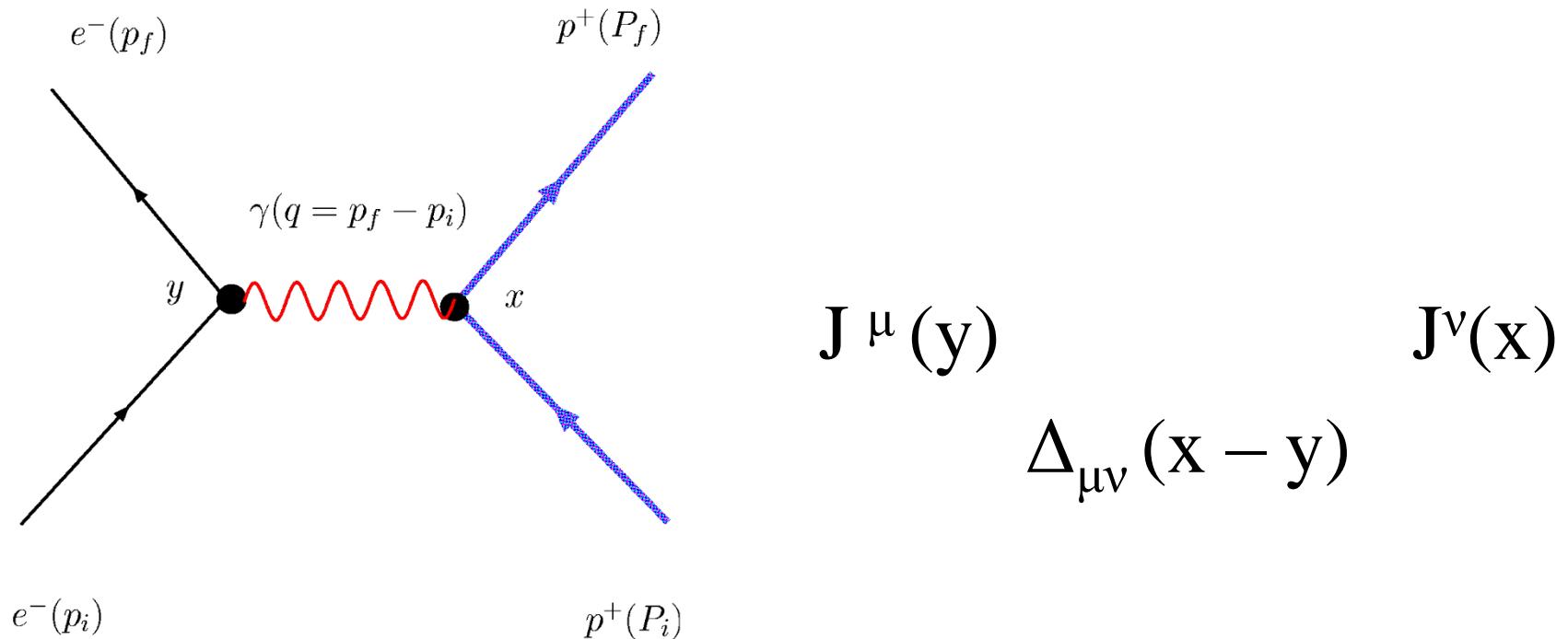
- $\langle 0 | J_\mu(y) J_\nu^\dagger(x) | 0 \rangle$



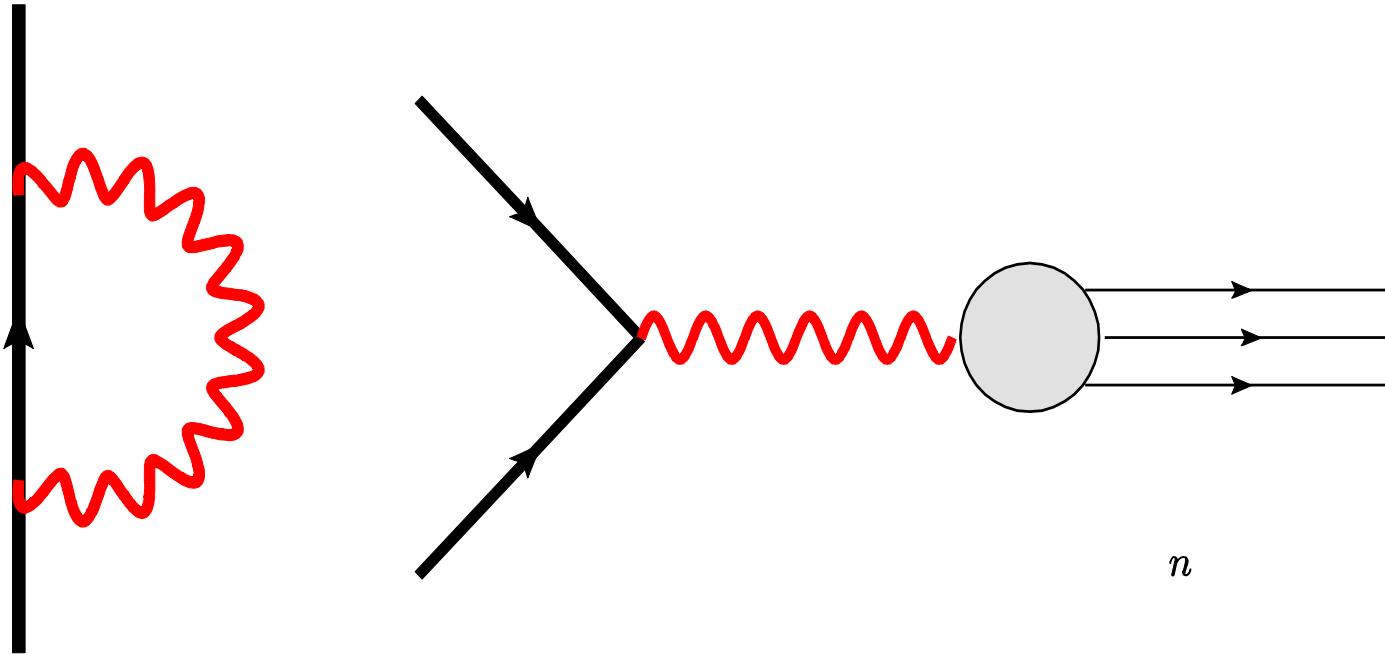
$$J_\mu(x)|_{QCD} = \bar{\psi}(x) \gamma_\mu \psi(x) \quad J_\mu(x)|_{HAD} = \rho_\mu(x)$$

# CURRENTS

- QUANTUM ELECTRODYNAMICS
  - Matter Current  $J_\mu(x, t)$



## e<sup>+</sup> e<sup>-</sup> annihilation into hadrons

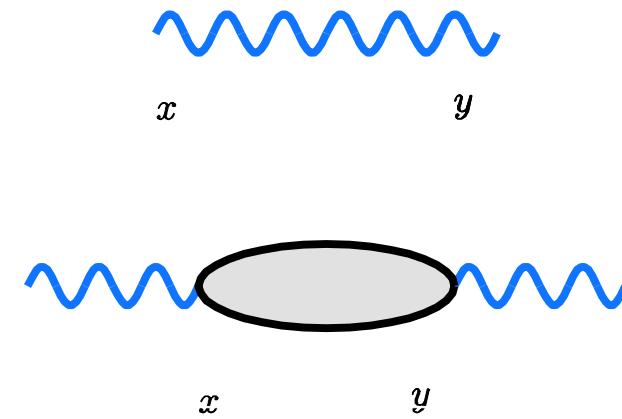


$$\Sigma_n |\langle 0 | V_\mu | n \rangle|^2$$

$$\sigma(e^+e^- \rightarrow \gamma^* \rightarrow h) \propto \frac{1}{q^2} \int d^4x \exp(iq \cdot x) \langle 0 | V_\mu(x) V_\nu(0) | 0 \rangle$$

$$\langle 0 | V_\mu(x) V_\nu(0) | 0 \rangle \Rightarrow \sum_n \langle 0 | V_\mu(x) | n \rangle \langle n | V_\nu(0) | 0 \rangle$$

$$\sum |n\rangle\langle n| = 1$$



# QUANTUM CHROMODYNAMICS

Fritzsch & Gell-Mann 1972

$$\begin{aligned}\mathcal{L} = & i \bar{\psi}_a(x) \gamma_\mu \partial^\mu \psi_a(x) - m_0 \bar{\psi}_a(x) \psi_a(x) - \frac{1}{4} F^i{}_{\mu\nu} F_i{}^{\mu\nu} \\ & - g G_{i\mu}(x) \bar{\psi}_a(x) \gamma^\mu \lambda_{ab}^i \psi_b(x)\end{aligned}$$

$i = 1, 2, \dots, 8$  (*gluons*)     $a = 1, 2, N_c = 3$  (*colours*)

$$F^i{}_{\mu\nu} \equiv \partial_\mu G_\nu^i - \partial_\nu G_\mu^i - g f_{ijk} G_\mu^j G_\nu^k$$

# **QCD**

## **MATTER PARTICLES**

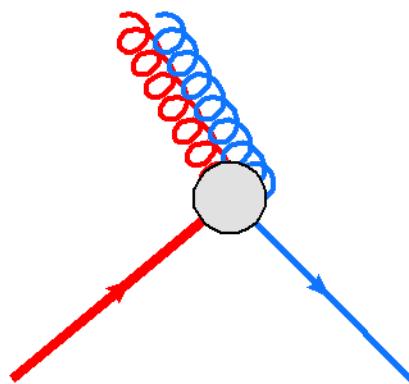
$$\begin{pmatrix} u \\ d \end{pmatrix}_{1,2,3}$$

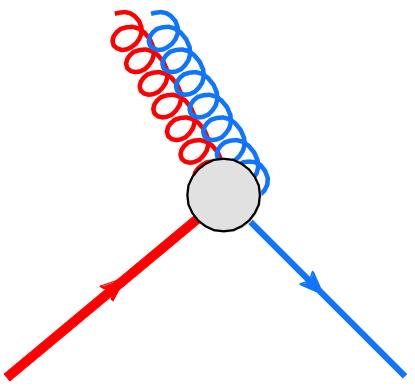
$$\begin{pmatrix} c \\ s \end{pmatrix}_{1,2,3}$$

$$\begin{pmatrix} t \\ b \end{pmatrix}_{1,2,3}$$

# QCD GAUGE PARTICLES (analog of photon)

$g_1, g_2, \dots g_8$







$$\Box \cdot A_\mu(x) = J_\mu(x)$$

$$\Box \cdot \equiv \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2$$