

# INTRODUCTION TO QUANTUM FIELD THEORY

Lectures at HDM 2017

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C.A. Dominguez

Department of Physics & Centre for Theoretical & Mathematical Physics  
University of Cape Town



## Lecture 3



## **PARTICLE PROPAGATION:**

**particles created/annihilated in Fock space**

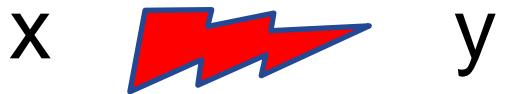
**particles propagated in Minkowski space**

# GREEN FUNCTION

- $\mathcal{D}(x, t) \psi(x, t) = f(x, t)$       e.g.  $\mathcal{D}(x, t) \equiv \nabla^2 - (1/v^2) \partial^2 / \partial t^2$
- $\mathcal{D}(x, t) G(x, t | x'', t'') = \delta^{(3)}(x - x'') \delta(t - t'')$
- $\psi(x, t) = \int d^3x'' G(x, t | x'', t'') f(x'', t'')$

# PARTICLE PROPAGATION IN QFT

- $\langle 0 | \varphi(y) \varphi^\dagger(x) | 0 \rangle$

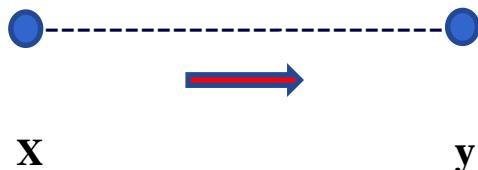


$$i \Delta(x - y) \equiv \langle 0 | \varphi(y) \varphi^\dagger(x) | 0 \rangle$$

$$\mathcal{D}(x) \Delta(x - y) \propto \delta^{(4)}(x - y)$$

# PROPAGATOR IN MOMENTUM SPACE

(SCALAR PARTICLE SPIN=0)

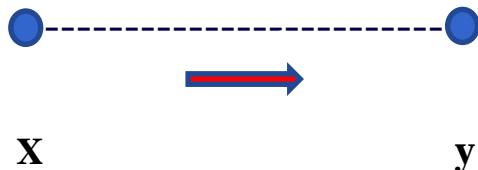


$$\Delta(x-y) = \int \frac{d^4 p}{(2\pi)^4} e^{-i p \cdot (x-y)} \Delta(p)$$

$$\Delta(p) = \frac{1}{p^2 - m_0^2 + i \varepsilon}$$

# PROPAGATOR IN MOMENTUM SPACE

(FERMION PARTICLE SPIN=1/2)



$$S_F(x - y) = \int \frac{d^4 p}{(2\pi)^4} e^{-i p \cdot (x - y)} S_F(p)$$

$$S_F(p) = \frac{1}{\gamma^\mu p_\mu - m_0} = \frac{\gamma^\mu p_\mu + m_0}{p^2 - m_0^2}$$

# PROPAGATOR IN MOMENTUM SPACE

(BOSON PARTICLE SPIN=1 e.g. photon)

$$i D_{\mu\nu}(x - y) = \langle 0 | A_\mu(x) A_\nu(y) | 0 \rangle$$

$$D_{\mu\nu}(q^2) = -\frac{1}{q^2 + i\varepsilon} T_{\mu\nu}$$

$$\mathbf{D}_{\mu\nu}(q^2) = \frac{-1}{q^2} \left[ g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right]$$

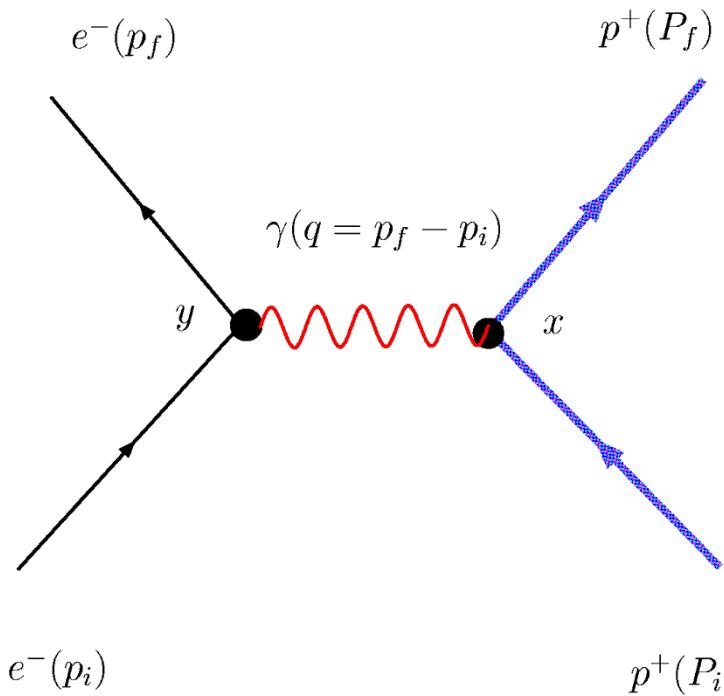
# CURRENTS

- Local bilinear functions of fields
- Definite quantum numbers
- e.g. Electromagnetic current:

$$J_\mu(x) |_{QCD} = e \bar{\psi}(x) \gamma_\mu \psi(x)$$

# CURRENTS

- QUANTUM ELECTRODYNAMICS
  - Matter Current  $J_\mu(x, t)$



$$J^\mu(y)$$

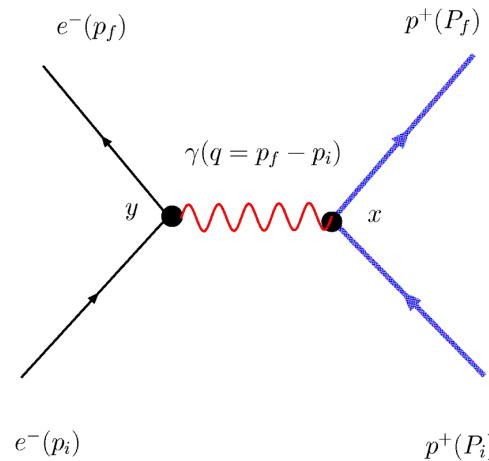
$$J^\nu(x)$$

$$V(q) = \frac{|e|}{q^2}$$

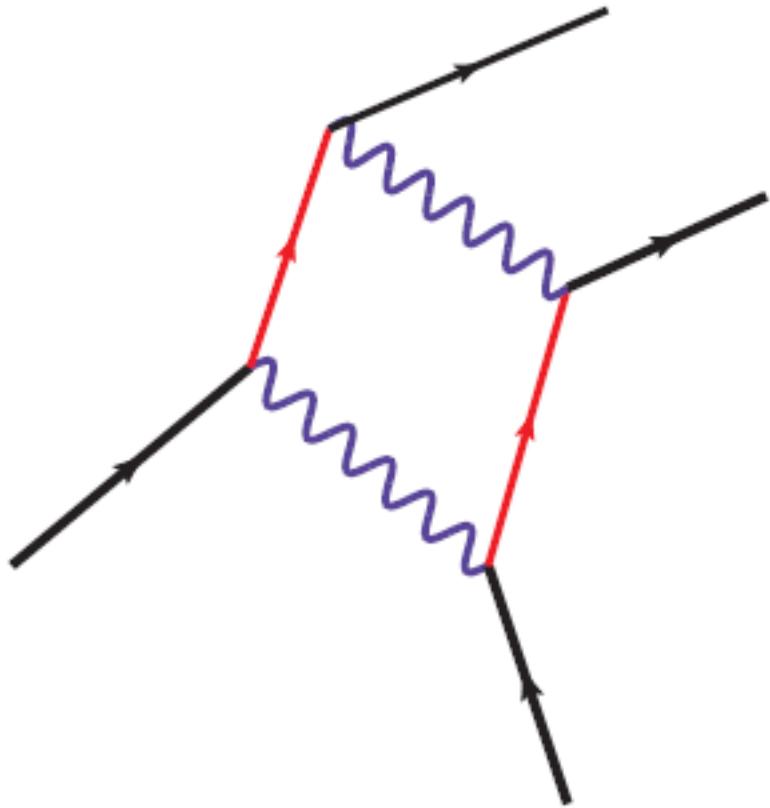
$$\Delta_{\mu\nu}(x - y)$$

$$V(r) = \frac{1}{4\pi} \int d^3q \ e^{-iq \cdot r} V(q)$$

$$V(q) = \frac{|e|}{q^2} \quad \longrightarrow \quad V(r) = \frac{|e|}{r} \text{ COULOMB !}$$



Coulomb's Law : One photon exchange

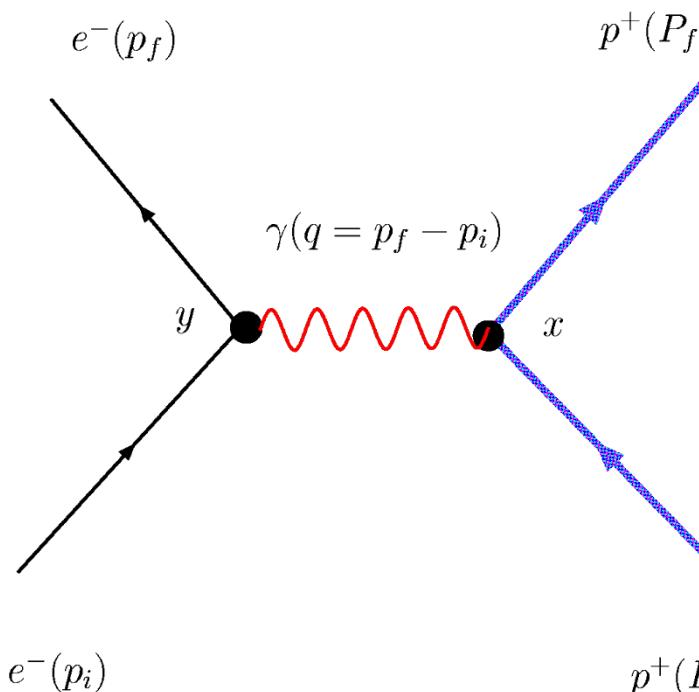


# FEYNMAN DIAGRAMS

$$q^2 = (p_f - p_i)^2 = (q_0^2 - \vec{q}^2) \neq 0 !!!$$

Insisting on energy-momentum conservation  $\longrightarrow$  *virtual particles*

## Feynman's (practical) invention



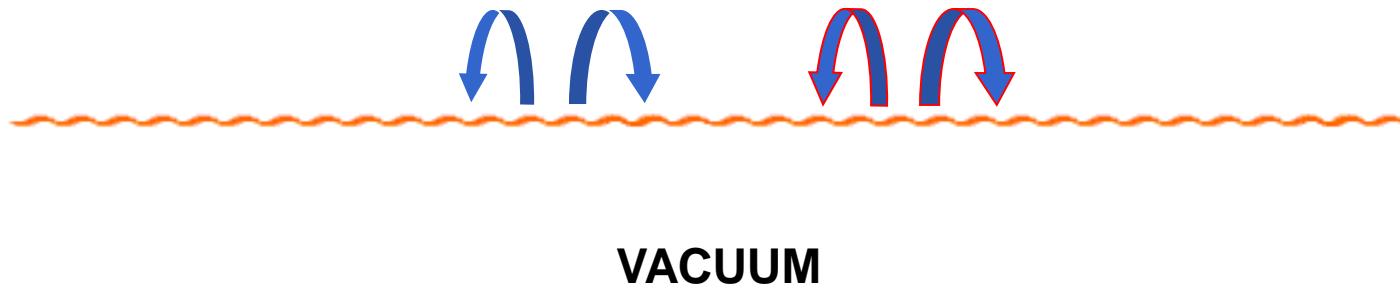
# THE QUANTUM VACUUM

- First Law of Thermodynamics (conservation of energy-momentum)
- Heisenberg's Uncertainty Principle:

$$\Delta p_x \Delta x \approx \hbar \quad \text{etc.}$$

$$\Delta E \Delta t \approx \hbar$$

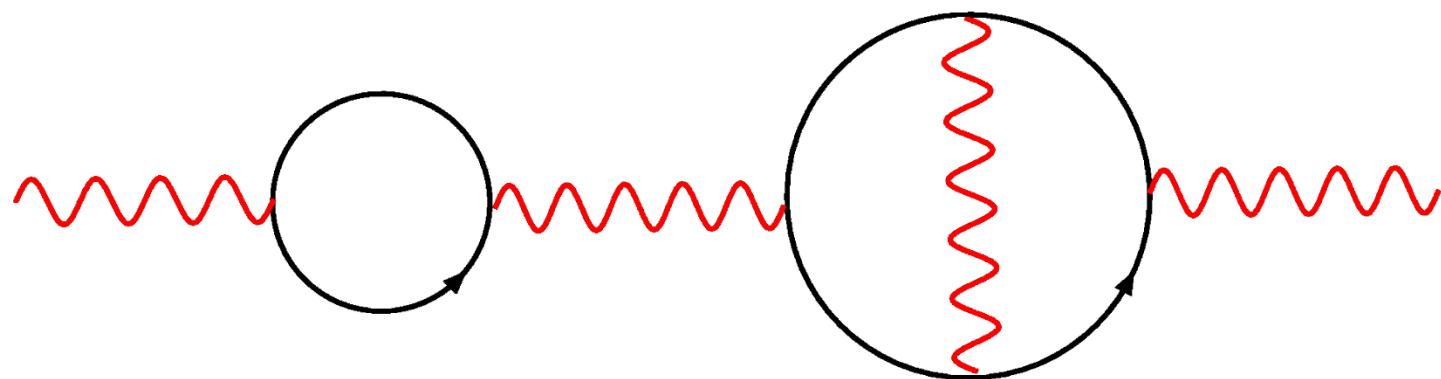
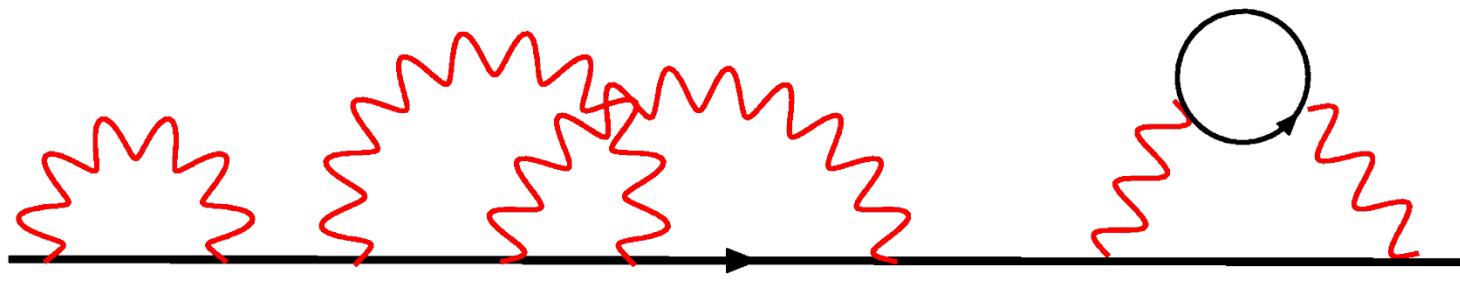
# Particle-Antiparticle production out of nothing



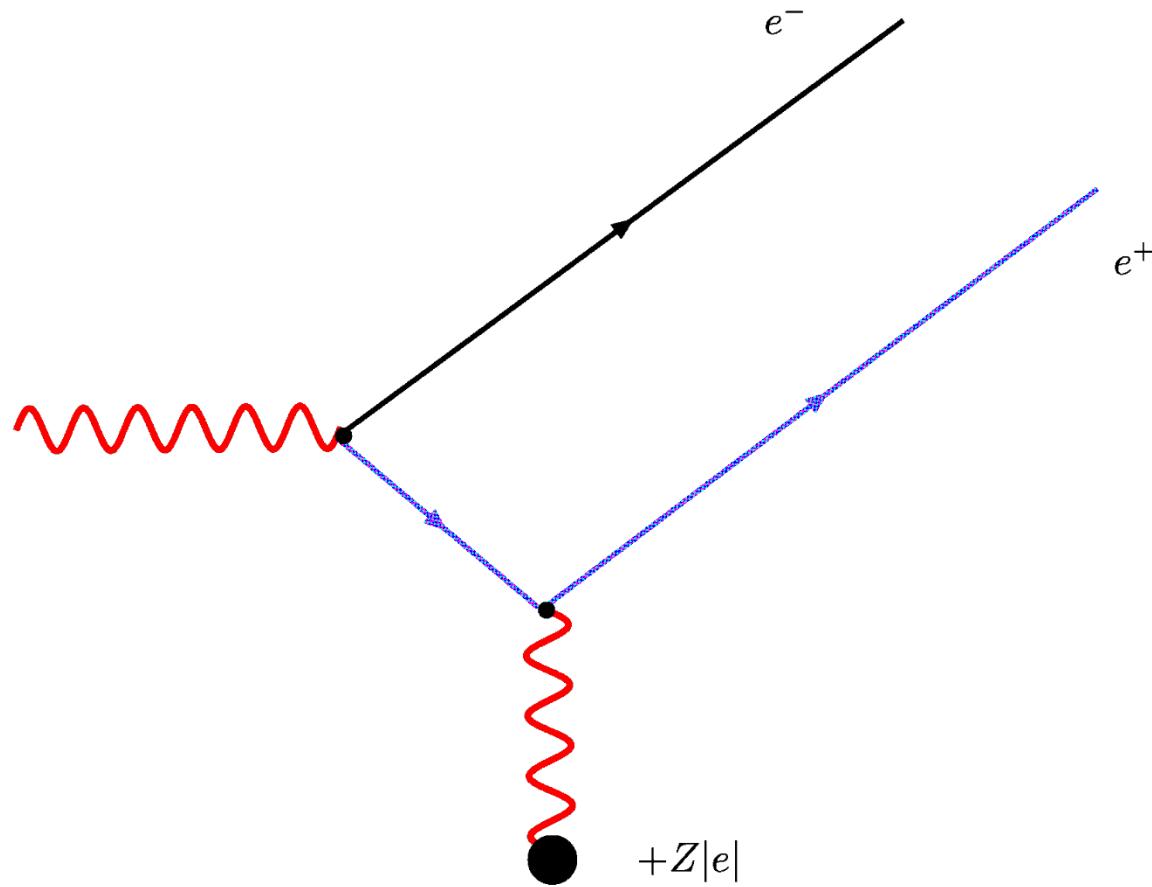
An e- e+ pair produced and reabsorbed in  $\sim 10^{-21}$  s

A proton-antiproton pair in  $\sim 10^{-24}$  s

# VACUUM CORRECTIONS



# ANTIMATTER PRODUCTION



# QUANTUM CHROMODYNAMICS

Fritzsch & Gell-Mann 1972

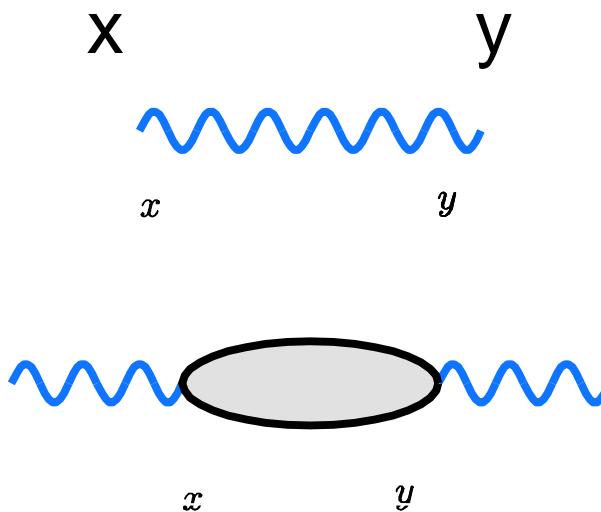
$$\begin{aligned}\mathcal{L} = & i \bar{\psi}_a(x) \gamma_\mu \partial^\mu \psi_a(x) - m_0 \bar{\psi}_a(x) \psi_a(x) - \frac{1}{4} F^i{}_{\mu\nu} F_i{}^{\mu\nu} \\ & - g G_{i\mu}(x) \bar{\psi}_a(x) \gamma^\mu \lambda_{ab}^i \psi_b(x)\end{aligned}$$

$i = 1, 2, \dots, 8$  (*gluons*)     $a = 1, 2, N_c = 3$  (*colours*)

$$F^i{}_{\mu\nu} \equiv \partial_\mu G_\nu^i - \partial_\nu G_\mu^i - g f_{ijk} G_\mu^j G_\nu^k$$

# CORRELATION FUNCTION IN QFT

- $\langle 0 | J_\mu(y) J_\nu^\dagger(x) | 0 \rangle$



$$J_\mu(x)|_{QCD} = \bar{\psi}(x) \gamma_\mu \psi(x) \quad J_\mu(x)|_{HAD} = \rho_\mu(x)$$

**Q C D**

$$\Pi(q^2) = i \int d^4x \ e^{iqx} <0|T(J(x)J^+(0))|0>$$

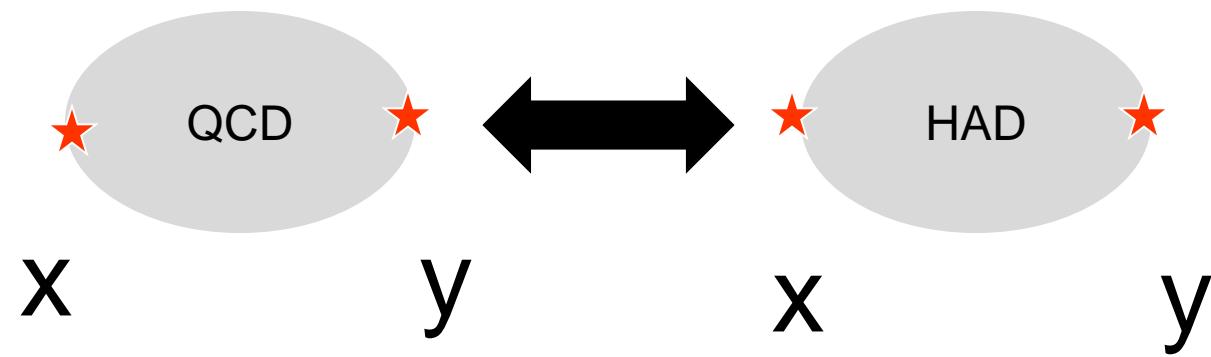
$$J(x) \Rightarrow A_\mu(x) |_j^i = \bar{\psi}^i(x) i \gamma_5 \gamma_\mu \psi_j(x)$$

$$V_\mu(x) |_j^i = \bar{\psi}^i(x) i \gamma_\mu \psi_j(x)$$

# HADRONIC

$$\Pi(q^2) = i \int d^4x e^{iqx} <0|T(J(x)J^+(0))|0>$$

*J(x) ⇒ Hadronic fields / EXPERIMENTAL DATA*



# CAUCHY'S THEOREM IN THE COMPLEX ENERGY PLANE

