

INTRODUCTION TO QUANTUM FIELD THEORY

Lectures at HDM 2017

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Lecture 1



LECTURE PLAN

- 1:** Classical Mechanics: Newton's Laws, Lagrangian Formalism, Action, Hamilton Principle, Classical Fields, Non-Relativistic Quantum Mechanics, Schroedinger's Equation, Quantum Operators, Relativistic Quantum Mechanics, Klein-Gordon Equation, Dirac Equation (1).
- 2:** Dirac Equation (2), Quantum Fields: Klein-Gordon, Dirac, Electromagnetic, Noether's Theorem, Gauge Invariance. Quantum Chromodynamics (QCD) (1).
- 3:** Quantum Chromodynamics (QCD) (2), Current Correlators, Complex (squared) Energy Plane, Analytical Solutions in QCD, Detailed Calculation of a Leading Order QCD Current Correlator.
- 4:** QCD Current Correlators at Finite Temperature, Dolan-Jackiw Formalism, Analytical Signals for Quark-Gluon Deconfinement, Relation to Lattice QCD.

Newton's Classical Mechanics: $F_i = \frac{d}{dt} p_i$, $p_i = m \frac{d r_i}{dt}$

Lagrange's Classical Mechanics:

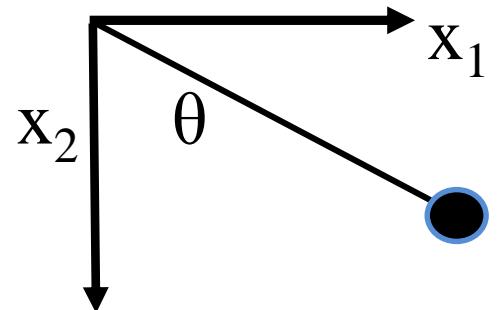
$$L \equiv T - V$$

$$\frac{d}{dt} \frac{\partial L}{d\dot{x}_i} - \frac{\partial L}{dx^i} = 0$$

Generalized Coordinates

$$x_i(t) \longrightarrow q_i(t)$$

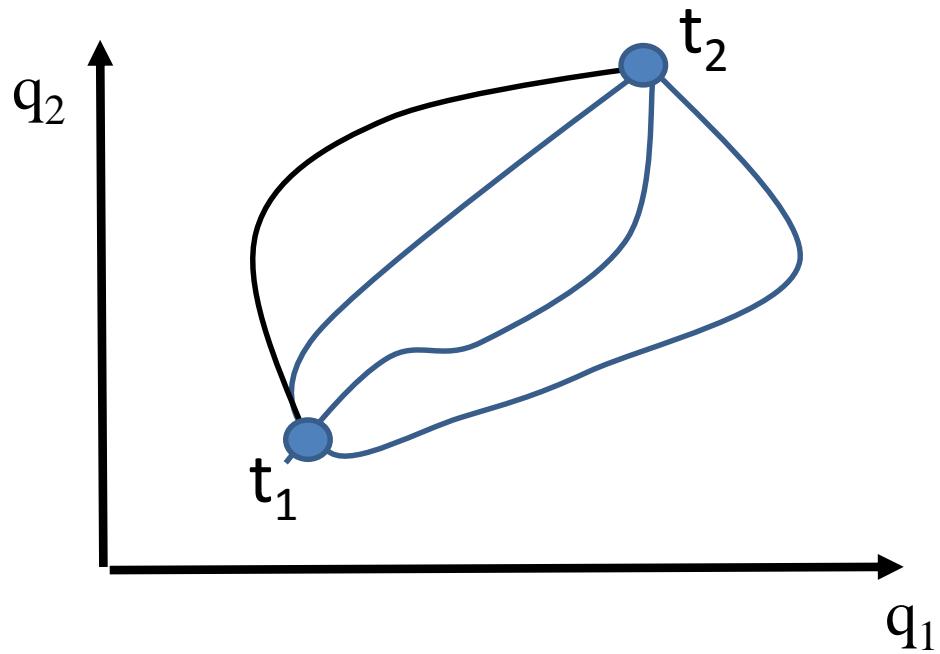
$$\frac{d}{dt} \frac{\partial L}{\partial \dot{x}_i} - \frac{\partial L}{\partial x_i} = 0$$



$$x_1, x_2 \longrightarrow \theta \quad x_1^2 + x_2^2 = \ell^2$$

Configuration Space

$q_i(t)$



WHICH PATH ??? HAMILTON PRINCIPLE

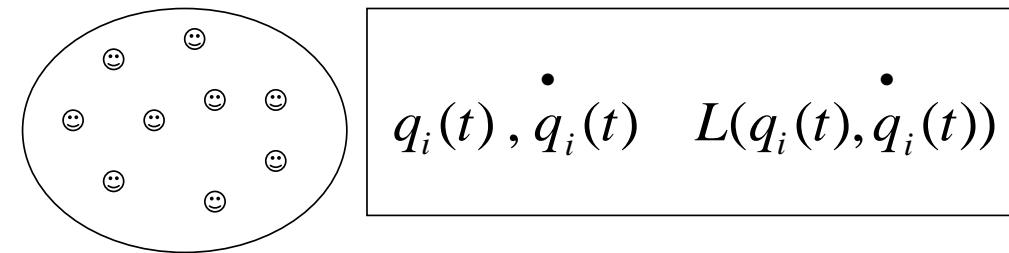
HAMILTON PRINCIPLE (LEAST ACTION)

$$S = \int_{t_1}^{t_2} L(q_i(t), \dot{q}_i(t), t)$$

$$\delta S = 0$$

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = 0 \quad !$$

FIELDS



$$q_i(t): i \rightarrow \infty$$

$$q_i(t) \rightarrow q(x, y, z, t) \equiv \varphi(x, y, z, t)$$

$$L \rightarrow \mathcal{L}(\varphi(x), \partial_\mu \varphi(x))$$

$$S = \int d^4x \mathcal{L}(\varphi, \partial_\mu \varphi)$$

$$\delta S = 0 \rightarrow (\partial_\mu \varphi \partial^\mu \varphi + m^2 \varphi) = 0$$

QUANTUM MECHANICS

**QUESTION:
NAME THE**

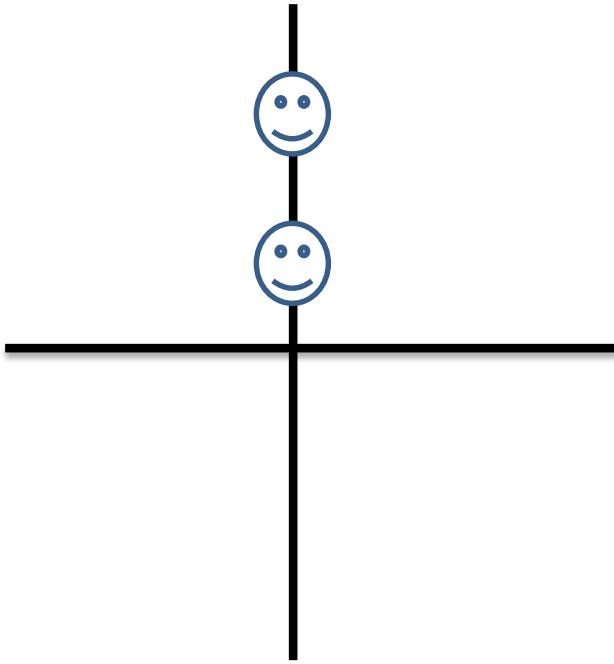
**SINGLE TOP/MOST
FUNDAMENTAL CONCEPT
BEHIND ALL MODERN
ELECTRONIC TECHNOLOGY,
CHEMISTRY, BIOLOGY
???**

QUANTUM MECHANICS

QUANTUM FIELD THEORY

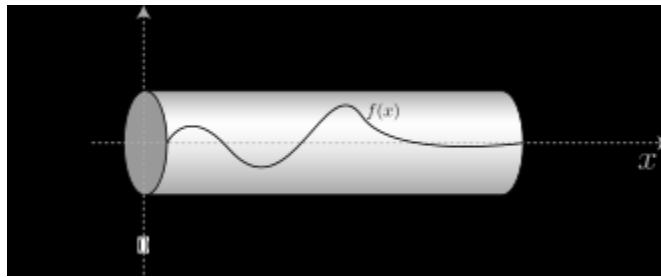
(ELECTRODYNAMICS, STRONG & WEAK INTERACTIONS)

- **QM: $\sqrt{-1}$!!!**



- **NO MODERN TECHNOLOGY**
- **NO UNDERSTANDING OF PHYSICS/CHEMISTRY**

$$\frac{\partial \Psi(x,t)}{\partial t} - k \frac{\partial^2 \Psi(x,t)}{\partial x^2} = 0$$



FOURIER EQUATION: DISSIPATION

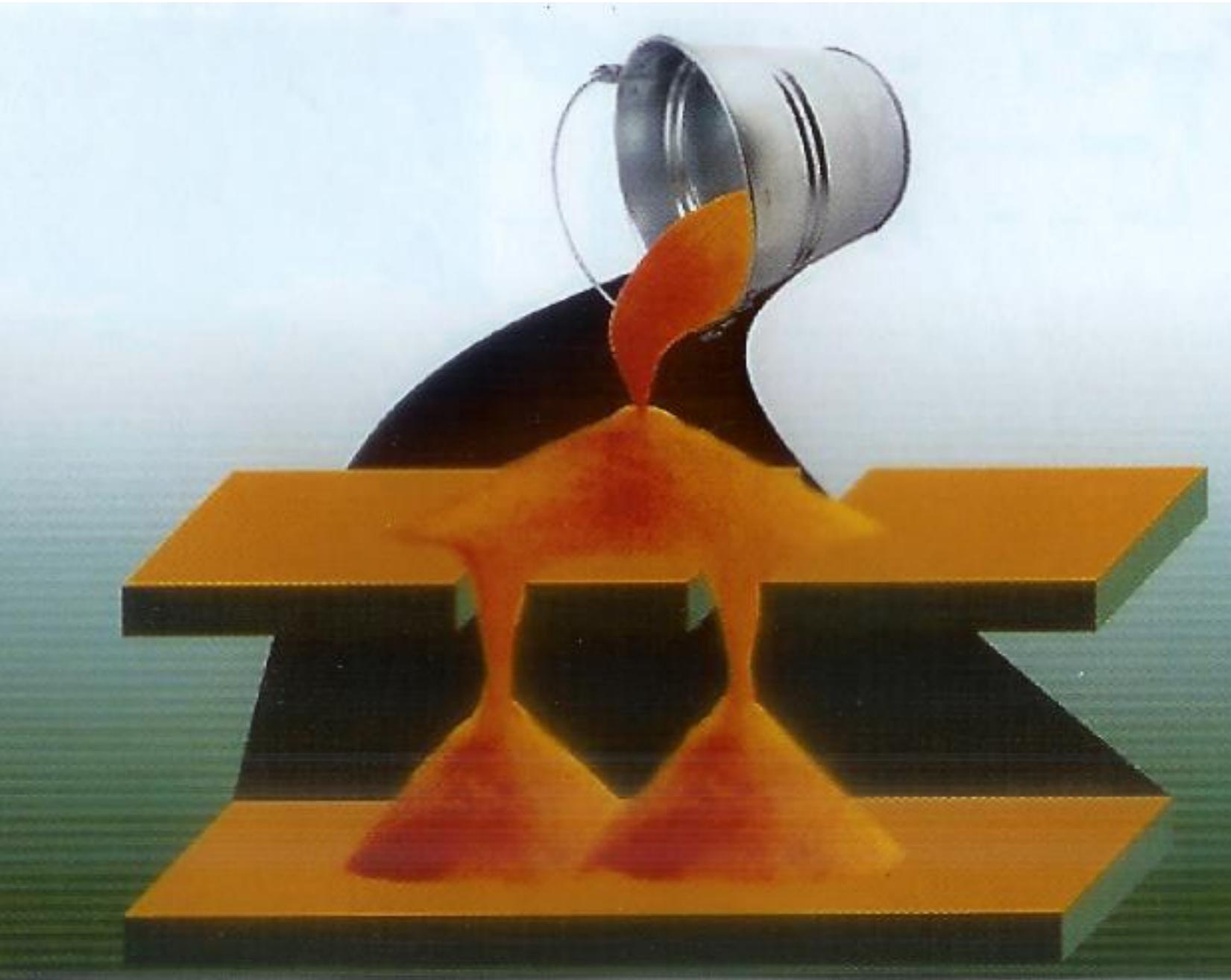
D'ALEMBERT EQUATION: WAVE PROPAGATION

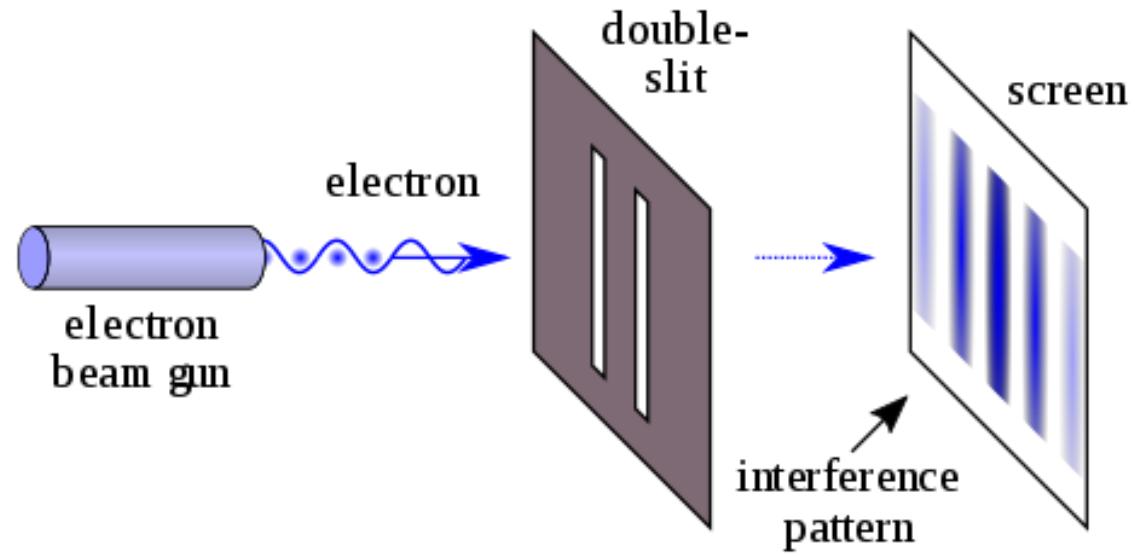
$$\frac{1}{v^2} \frac{\partial^2 \Psi(x,t)}{\partial t^2} + \frac{\partial^2 \Psi(x,t)}{\partial x^2} = 0$$

QUANTUM MECHANICS: FOUNDING FATHERS IN MENTAL CHAOS

Planck, Einstein, Schroedinger, Bohr, Heisenberg, etc. etc.

- * **SCHROEDINGER'S WAVE EQUATION**
 - $i \hbar \partial\Psi/ \partial t = - (\hbar^2 / 2 m) \nabla^2 \Psi + V \Psi$
 - e^-
- * **Ψ : WAVE or PARTICLE???**
- **MENTAL FLAGELATION**



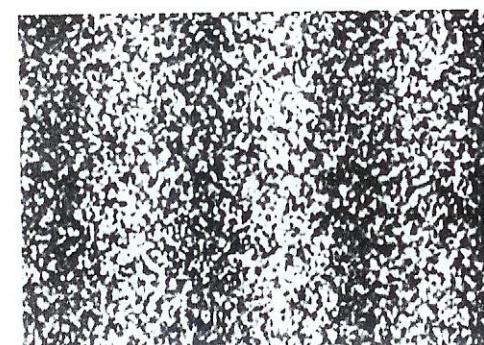




(b) After 100 electrons



(c) After 3000 electrons



70,000

NON-RELATIVISTIC QUANTUM MECHANICS

(after the facts)

$$\mathbf{p} \rightarrow -i\hbar\nabla$$

$$p_x \rightarrow -i\hbar \frac{\partial}{\partial x} \text{ etc.}$$

$$E \rightarrow i\hbar \frac{\partial}{\partial t}, \quad H = T + V = \frac{p^2}{2m} + V \rightarrow -\frac{\hbar^2}{2m} \nabla^2 + V$$

$$H_{OP} \Psi(\mathbf{x}, t) = E_{OP} \Psi(\mathbf{x}, t)$$

$$\left[-\frac{\hbar^2}{2m} \nabla^2 + V \right] \Psi(\mathbf{x}, t) = i\hbar \frac{\partial}{\partial t} \Psi(\mathbf{x}, t) !!!$$

RELATIVISTIC QUANTUM MECHANICS

SPECIAL RELATIVITY: $E^2 = m_0^2 c^4 + \mathbf{p}^2 c^2$

C=1

$$E^2 = m_0^2 + \mathbf{p}^2$$

$$\mathbf{p} \rightarrow -i\hbar\nabla \quad p_x \rightarrow -i\hbar \frac{\partial}{\partial x} \text{ etc.}$$

$\hbar=1$

$$E = \pm \sqrt{(m_0^2 + p^2)} \rightarrow \pm \sqrt{(m_0^2 - \nabla^2)} \text{ ???}$$

KLEIN – GORDON EQUATION ($c=1, \hbar=1$)

$$E = \pm \sqrt{(m_0^2 + p^2)} \rightarrow \pm \sqrt{(m_0^2 - \nabla^2)} \quad ???$$

$$H_{OP} \Psi(x) = E_{OP} \Psi(x) \quad (x \equiv t, x)$$

$$\{ \pm \sqrt{(m_0^2 - \nabla^2)} \} \varphi(x) = i \hbar \frac{\partial}{\partial t} \varphi(x) \quad ???$$

SOLUTION: apply the operators “twice”!!!

$$\{ \pm \sqrt{(m_0^2 - \nabla^2)} \} \varphi(x) = i \hbar \frac{\partial}{\partial t} \varphi(x)$$

$$(m_0^2 - \nabla^2) \varphi(x) = - \frac{\partial^2}{\partial t^2} \varphi(x)$$

$$[\frac{\partial^2}{\partial t^2} - \nabla^2] \varphi(x) + m_0^2 \varphi(x) = 0 \quad \frac{\partial^2}{\partial t^2} - \nabla^2 \equiv \square$$

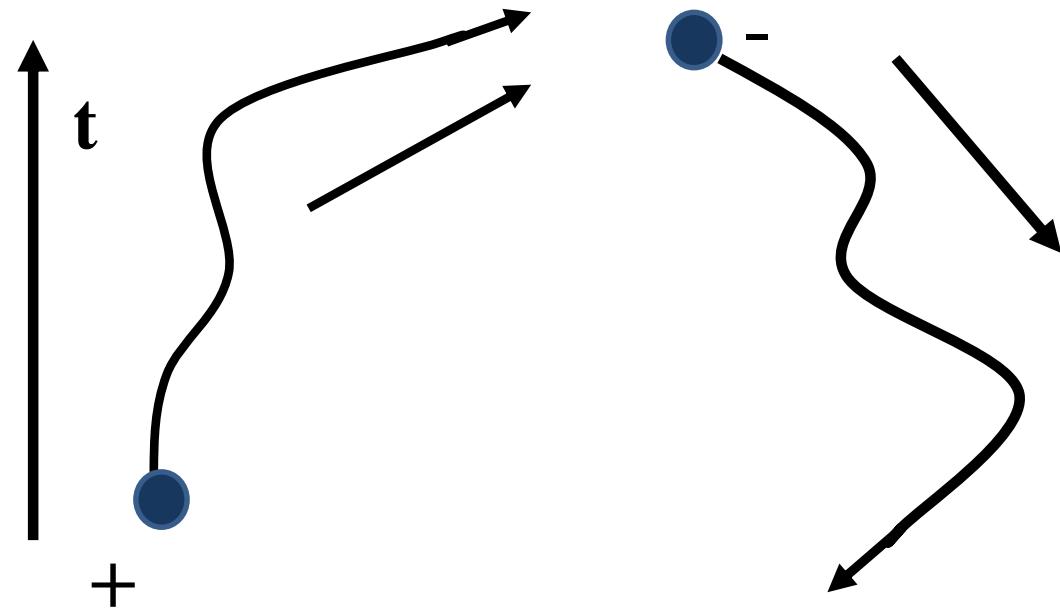
D'Alambertian

$$[\square + m_0^2 \left(\frac{c^2}{\hbar^2} \right)] \varphi(x) = 0$$

$$[\square + m_0^2 \left(\frac{c^2}{\hbar^2} \right)] \varphi(\mathbf{x}) = 0 \quad E > 0 \quad \& \quad E < 0 ???!!!$$

Klein-Gordon (1926) \longrightarrow Pauli-Weisskopf (1934)

(i) Charged particles (\pm) (ii) Motion in space



$$\varphi(\mathbf{x})^{(\pm)}|_{\pm p} = C_p e^{\pm i \mathbf{p} \cdot \mathbf{x}} e^{\mp i E t}$$

$$\textbf{Spin} = \frac{1}{2} (\textbf{e}^-)$$

Paul Dirac (1934): quest for a linear QM equation

$$\sqrt{(a^2 + b^2)} \neq \alpha a + \beta b \quad ???$$

Answer: No (if α, β are ordinary numbers)

How about α, β obeying a non-ordinary algebra???

CLIFFORD ALGEBRA

$$\gamma_\mu = (\gamma_0, \gamma_i), \quad \mu=0,1,2,3 \quad i=1,2,3 \quad (\text{time \& space components})$$

$$\{\gamma_\mu, \gamma_\nu\} \equiv \gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu = 2 g_{\mu\nu} \quad g_{\mu\nu} = \text{diag}(1, -1, -1, -1)$$

$$(\gamma^0)^2 = 1 \quad (\gamma^i)^2 = -1 \quad (i=1,2,3) \quad \text{Tr } (\gamma^0) = \text{Tr } (\gamma^i) = 0$$

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

$$\gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}, \quad \gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}.$$

DIRAC EQUATION

$$(i \gamma^\mu \partial_\mu - m_0)_{ab} \psi(x)_b = 0 \quad (a, b=1,2,3,4)$$

a 4 x 4 matrix equation $m_0 \equiv m_0 \delta_{ab}$ ($\delta_{ab} = 1$, $a=b$, $\delta_{ab} = 0$, $a \neq b$)

Describes two spin $1/2$ particles of charge $q = +|e|$ and $-|e|$ (electron & positron), with spin-up and spin down

Spin = $1/2$, & magnetic moment $\mu = \mu_B \equiv |e| \hbar / (2 m_e)$.

Free-particle solution

$$(i \gamma^\mu \partial_\mu - m_0)_{ab} \psi(x)_b = 0 \quad \psi(x)_b = \omega_b(p) \exp(-i p \cdot x)$$



