Testing the scattering distribution of a photon in a turbid medium using Monte Carlo simulations

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Abstract. The scattering distribution of a photon in a homogeneous turbid medium is too complex to be represented by an analytical expression, and therefore requires a numerical solution. Photon propagation may be treated as a stochastic process. In this study, a Monte Carlo simulation is used to reproduce the behaviour of photons in turbid media. The fraction of photons transmitted and reflected depends on the optical properties of a turbid medium as well as the angle of the incoming photon. We determine the transmission coefficients through several model atmospheres for three representative solar zenith angles. The fraction of the transmitted photons is further reduced depending on the angle of the incoming photons. The results presented in this study shows that the optical depth of the medium and the incoming angle of photons have a significant impact on the scattering distribution of photons in a turbid medium and on the amount of photons reaching the ground.

1. Introduction

Several studies done in the past show that photon propagation through scattering media can be solved by analytical expression only if there are few scattering events [1]. A photon experiences different kinds of scattering as it enters a homogeneous turbid medium. The propagation of light in such a medium may be analysed using either the wave model or photon model. This study uses the photon model.

The radiative transfer equation is often used to describe photon propagation through scattering media. However, the exact analytical solutions in turbid media are too complex and impractical [2]. Therefore, numerical methods based on the statistical Monte Carlo technique (which is one among other methods) are used to solve the radiative transfer equation in complex geometries [3]. The scattering distribution of a photon is influenced by the optical properties of a turbid medium, which is characterized by the absorption coefficient, scattering coefficient, as well as the scattering phase function. The absorption and scattering coefficients are determined by the probability of photon absorption and photon scattering per unit path length, respectively [4]. The scattering phase function describes the amount of scattered photons into a unit solid angle in a given direction.

Information about the optical properties of the turbid media such as the absorption coefficient, scattering coefficient as well as the asymmetry parameter of the medium are required in order to describe the scattering distribution of a photon through a homogeneous turbid medium. The new direction of a photon after each scattering event in a medium rich in aerosols can be randomly generated from a Henyey-Greenstein phase function [5, 6]. This function is appropriate for Mie scattering, where aerosols size is approximately equal to the incoming light wavelength. The Rayleigh scattering phase function is used instead when particles have a small size compared to the wavelength of the incoming light.

This study uses a python code to model the scattering distribution of a photon in a turbid medium. The Monte Carlo program tracks scattered photons as they propagate through a turbid medium based on the scattering angles as well as the azimuthal angles in order to determine the scattering distribution.

2. Monte Carlo simulations

The Monte Carlo code developed to model the scattering distribution of a photon in a turbid medium is as per several studies done in the past for photon propagation through scattering media [1-8], except that in this study the focus is more on modeling the scattering distribution of photon propagation in a scattering and non-absorbing medium in which photons are scattered multiple times before reaching the receiver.

2.1. Set photons in motion

In this study, the first step was to launch photons (i.e. photon packet) through the turbid medium from the origin defined by the coordinates [*x*, *y*, *z*] and their path is followed until they are completely absorbed or exit the medium. The photon propagation direction was initially set along the *z*-direction pointing inside the medium defined by the coordinates system: [*x*, *y*, *z*] = [0, 0, 1] in units of airmass.

2.2. Photon step size

The photon travels along a straight line before it encounters the scattering center and is scattered in a random direction. When the photon propagates in a turbid medium, the path length ΔS, also known as the step size, is equal to the inverse of the extinction coefficient. This is the distance that a photon travels between two consecutive scattering events in the medium. It is calculated using random numbers between 0 and 1 generated by a random number generator as follows [5, 7]:

$$\Delta S = \frac{-\ln \xi}{\mu_t} \tag{1}$$

where μ_t is the total extinction coefficient obtained from scattering and absorption coefficients of a turbid medium, and ξ is the random number generated by the program which is distributed between zero and one.

The aerosol optical depth which characterizes the size and the amount of particles in a turbid medium is defined as the integral of the total extinction coefficient, μ_t over the photon path, ds:

$$\tau(s) = \int_0^s \mu_t ds \tag{2}$$

2.3. The new direction of a photon and scattering function

A single scattering model which is based on the Beer-Lambert law works very well on clear atmospheres, but is inappropriate for highly attenuating turbid media [9]. For our turbid medium we made the simplified assumption that scattering is dominated by aerosols, [10], and hence the Rayleigh scattering is not considered in this study. The redistribution of a photon in different directions is determined through the stochastic treatment of the multiple scattering events and corresponding phase function.

The Henyey-Greenstein phase function is used to describe the scattering phase function of the photon moving in the turbid media and this is used to calculate the photon scattering angle in different directions. This means that the scattering angle is derived from that phase function and it depends on the asymmetry parameter, g which is normally used to take care of the asymmetry in the scattering process [11]:

$$P(\theta) = \frac{1}{4\pi} \frac{1 - g^2}{\left[1 + g^2 - 2g\cos\theta\right]^{3/2}} \tag{3}$$

where *P(θ)* is a probability density function, the parameter *g* is the asymmetry factor for a homogeneous turbid medium, and *θ* is the scattering angle of the photon which here is generated stochastically using the expression [11]:

$$\cos\theta = \frac{1}{2g}\left\{1 + g^2 - \left(\frac{1 - g^2}{1 + g\xi}\right)^2\right\} \tag{4}$$

In addition to the scattering angle is a random azimuthal angle, *Φ* which is assumed to be uniformly distributed between 0 and 2*π* is also needed to describe statistically the direction of a photon:

$$\Phi = 2\pi\,\xi \tag{5}$$

3. Results and discussion

The propagation of a photon in a turbid medium is treated as a random walk using stochastic means. The histogram for the step size of photons in a turbid medium is illustrated in figure 1 with photons undergoing exponential attenuation. The total interaction coefficients values assumed to be of the medium under study were estimated from the aerosol optical depth provided by sun-photometer [12].

The total interaction coefficient of the medium used in this study is varied from 0.2 to 1. Photons can be scattered or absorbed as they propagate through the turbid medium. As shown in figure 1, photon movement through a turbid medium depends largely on the total interaction coefficient, which is inversely related to the step size, see equation (1). It is rather straight forward to see that photon step size depends solely on the total extinction coefficient encountered in various turbid media. As expected, by increasing the size of the total extinction coefficient from 0.2 to 1.0, photons travel a shorter distance.

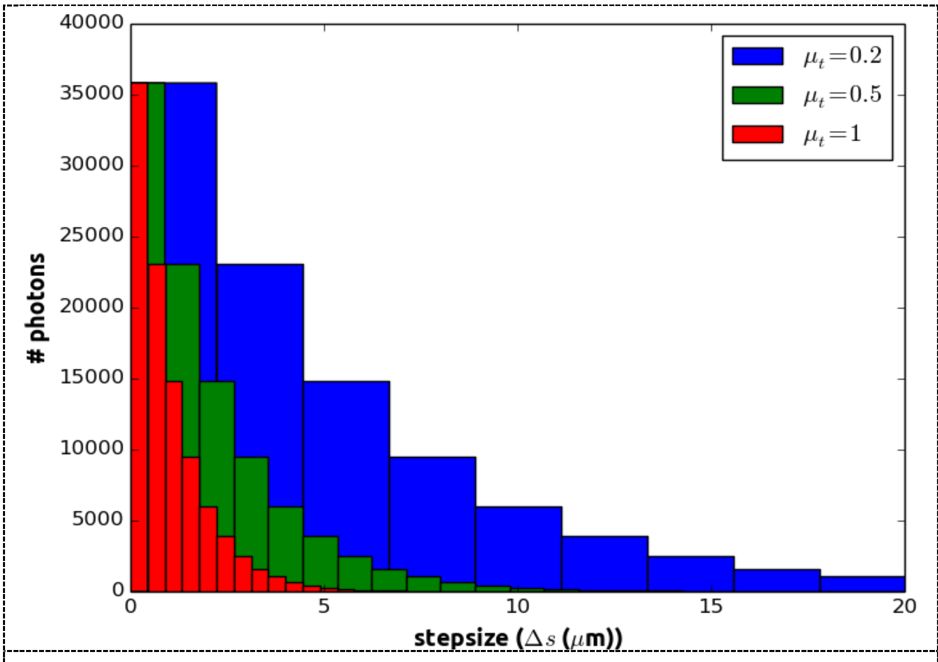

Figure 1. Histogram of the photon step size in a turbid medium

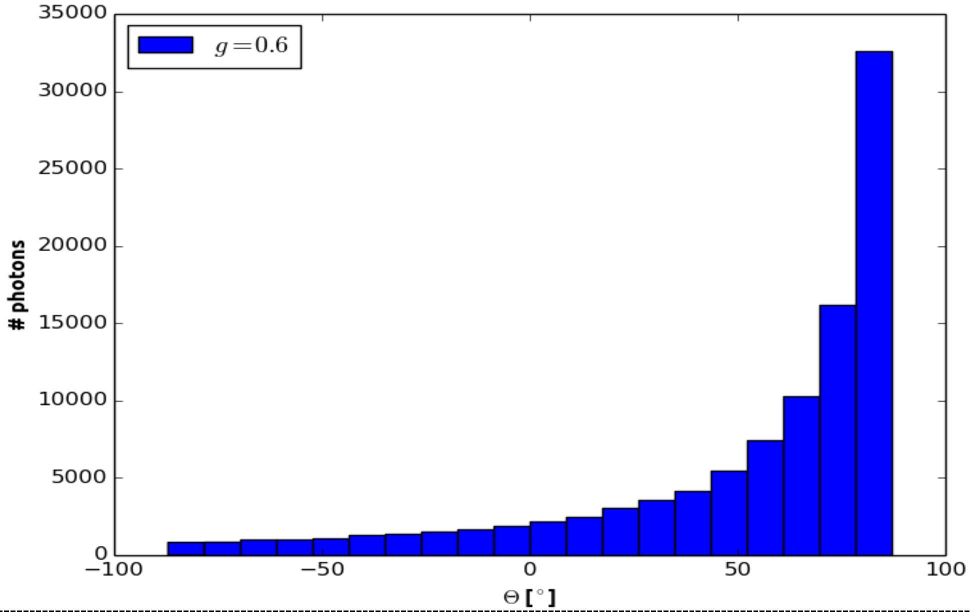

Figure 2. Scattering angles generated throughout the Monte Carlo simulation with the asymmetry parameter, g = 0.6

The above figure shows the histogram of the deflection angles, θ for g = 0.6. In this study, we used g = 0.6 as it appeared a suitable value to characterize aerosol scattering in the forward direction [13]. The focus was on modelling the aerosol scattering of photons. With anisotropy g equals to 0.6 indicates the forward directed scattering. Obviously from the formula, one can clearly see that more photons are scattered in the forward direction as the asymmetry parameter tends towards the positive value.

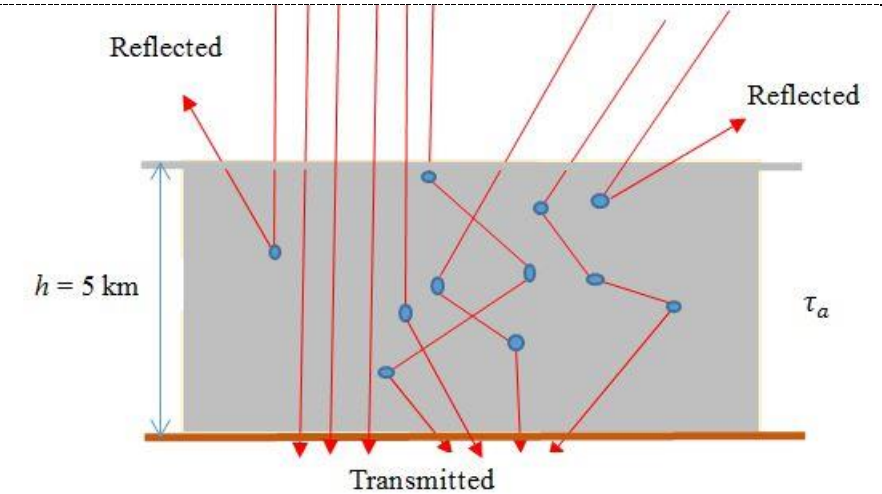

Figure 3. Schematic diagram of a photon movement through a plane parallel aerosol layer, h is the height above the ground level

Figure 3 shows examples of the photon propagation in a homogeneous turbid medium being scattered on its way to the ground level. Given the bias towards forward scatter, most photons reach the ground after passing through a highly scattering medium. The scatter is determined by considering the optical properties of the atmospheric aerosol. Figures 4 and 5 illustrate the transmittance and reflectance of photons versus optical depth for three representative solar zenith angles. The aerosol optical depth as well as the incident angle of the photon have a significantly impact on the number of photons reaching the ground. Some photons are scattered back to the top of the atmosphere and their fraction to the total number of launched photons gives the reflection values of the turbid medium while the transmission values are given by the ratio of the transmitted ones to the total number of launched photons from the top of the atmosphere.

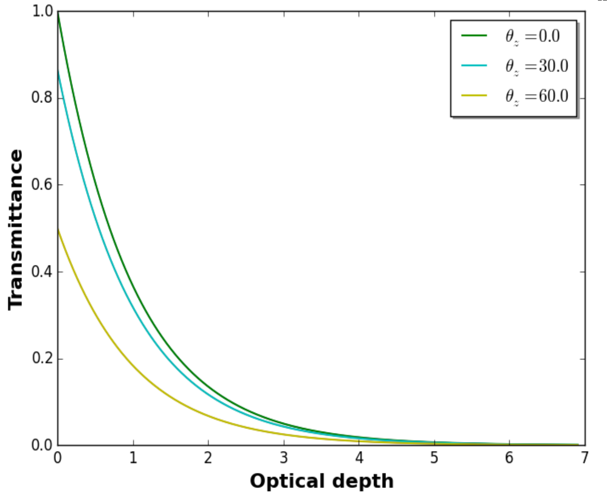

Figure 4. Transmittance of a turbid medium as function of the optical depth for three different solar zenith angles.

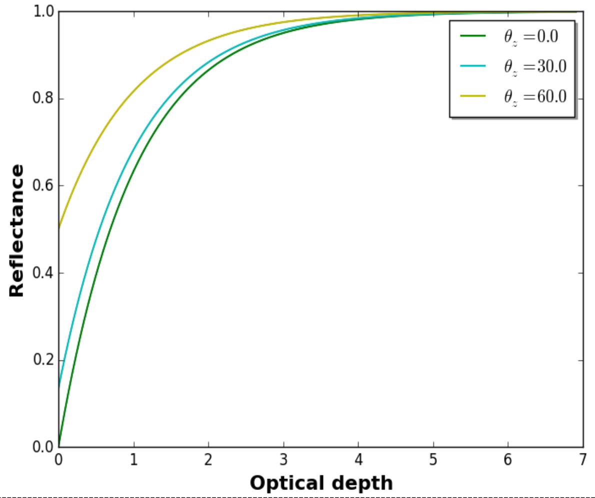

Figure 5. Reflectance as function of optical depth for three different solar zenith angles.

The three solar zenith angles, 0°, 30°, and 60° used in this study were found to be representative of different sun's position and daily variation, i.e, 0° is when the sun is overhead (airmass is one), 30° is toward the afternoon, and 60° is when the sun is approaching sunset. The aerosol optical depth of 1.5, 2.0, and 3.0 were chosen in order to examine the turbid medium.

Table 1: Transmission and reflection of a plane parallel aerosol layer for three different solar zenith angles at different aerosol optical depth for a wavelength of 440 nm [12].

θ_z (degrees)	τ_a	Transmission (T)	Reflection (R)
0	1.5	0.795	0.204
	2.0	0.737	0.262
	3.0	0.635	0.364
30	1.5	0.752	0.247
	2.0	0.692	0.307
	3.0	0.589	0.410
60	1.5	0.594	0.406
	2.0	0.529	0.471
	3.0	0.454	0.546

The results presented in the table above from stochastic simulations show the relation between the fraction of transmitted and reflected photons with an aerosol optical depth (AOD), for a wavelength of 440 nm, corresponding to blue light. Scattering at other wavelengths will be explored in a later publication. As seen in our results, the transmission and reflection depend on the optical properties of the aerosol layer as well as on the incident angle. The direct transmittance decreases exponentially with increasing AOD.

4. Conclusion

The Monte Carlo algorithm for simulating photon transport in turbid medium has been implemented in a 2-D code, where a scattering distribution of photons within a medium rich in aerosols has been tested in this study by changing the total interaction coefficients of the medium. A Monte Carlo Model was used to calculate the upward and downward solar photon flux characteristics based on different aerosols extinction coefficients. The results indicate that aerosol loading has a significant impact on the solar photons. Photon tracing through Monte Carlo simulation showed the true complexity of the scattering distribution of photons in turbid media. However, the Monte Carlo method was found to be suitable for multiple scattering events as well as for non-isotropic scattering. The redistribution of a photon in different directions in a turbid medium was determined.

5. Further study

The Monte Carlo method will be used to quantify the amount of solar radiation that is received by the Earth's surface under turbid atmosphere for the South African conditions. Future work also involves developing a statistical model to simulate the effect of high vapor concentration in the atmosphere on solar radiance, and the effect of heavy smoke-induced aerosols during winter dry seasons.

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