Vectors from students' point of view.

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Abstract. First year physics lecturers reported difficulties with students' learning of concepts and skills associated with studying vectors. The results of the investigation suggested that students had insufficient knowledge and understanding of magnitude, direction, and the significance of free-body diagrams in resolving the related vectors into components [2]. [5] reported on students learning difficulties related to basic vector operations as employed in introductory physics courses. The results revealed that most students did not do well in subsequent sub-questions. For these reasons, here we report the findings obtained from the students as they respond to the sources of their challenges in dealing with sections related to vectors.

1. Introduction

Vector concepts are very important in physics, since many physical quantities are defined as vectors. Physical quantities, like force, velocity and acceleration are defined as vectors, and electric and magnetic field are defined as vector fields. The use of vectors in both forces and equilibrium concepts illustrated by free-body diagrams, are basics to all mechanics problems. In his report Knights [2], emphasized that the vector nature of forces, fields, and kinematics quantities requires that students have a good grasp of basic vector concepts if they are to be successful in mastering even introductory level physics. The fundamental concept of Newtonian mechanics is force, and forces are vectors, they must be added using vector addition to determine the net force along the axis of motion.

Although many studies have centered on students' difficulties with vector concepts in problems with mathematics and physical context [4-6], only a few of them have focused on students' difficulties in relation to the sources of their challenges in dealing with vector concepts. This is remarkable because these challenges are consistently used by students in the introductory physics university course. Therefore, the objective of this study is to analyze the difficulties that students who have just completed their introductory physics vector university courses still have with problems that involve the vector problems.

2. Experimental methods

The observations obtained in this study were taken from the responses given by first year students from University of Johannesburg (UJ). In this institution, the textbooks used in Physics modules are "Introduction of Physics" by Cutnell and Johnson [1] and "Principles and practice of Physics" by Eric Mazur [3].

2.1.1. Study Participants

The present study involved students who were enrolled in courses for first semester of introductory physics modules. Everyone had received standard lecture instructions on vectors and had been assessed in various ways in sections involving Mechanics problems. By this time, vector concepts had been explicitly covered in lecturers as well as in tutorial sessions.

2.1.2. Design of the questionnaires

Multiple choice questions, explanations of the student choices and conceptual questions were administered to 24 mainstream students and 111 four-year degree extended students using google docs. Most questions had already been discussed in different lectures, some were used for the tutorial sessions too. There were several different versions of the written problems that were explored in these theoretical contact sessions. Some involved drawing of the free-body diagrams; resolving vectors into components; addition and subtraction of two vectors; trajectory motions and others were on conceptual understanding of definition and their applications. To analyse students' responses in the multiple-choice questionnaires given, the observed responses were treated in 2 different ways. For conceptual questions, we manually found similar answers, then listed them, and finally established the proportions of each answer. In the analyses of each conceptual question we grouped the most common answers and then in the analysis of the answers we described the reasoning or challenges given by students.

3. Results and Discussion

In the following subsections, we present the results of students' answers to each of the questions. In each case, we present the most common answers and their reasoning.

3.1 Problems and explanations to the choices.

3.1.1 Graphical representation of a vector into X and Y components

Question 1: Referring to figure 1 below, the components of vectors $\vec{A}$ and $\vec{B}$ are:

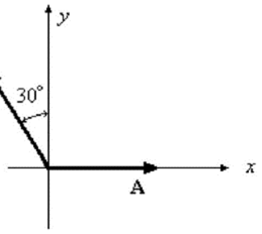


Figure A

Refer to figure above. The components of vectors $\vec{A}$ and $\vec{B}$ are

A) Ax = A sin90° Bx = B cos60° Ay = A cos90° By = B sin60°.
B) Ax = A cos90° Bx = B sin60° Ay = A sin90° By = B cos60°.
C) Ax = 0 Bx = B sin 30° Ay = 0 By = B cos30°.
D) Ax = A sin 90° Bx = B sin 30° Ay = 0 By = B cos30°.
E) Ax = A cos0° Bx = -B cos60° Ay = A cos90° By = B cos30°.

We used question 1 to analyze students' difficulties in resolving vectors into their components. Students were asked to choose the correct answer and explain their choice of their answer. To answer this question students should know that these vectors have *x* and *y* components. Vector A has only *x*-component with *y*-component being zero; and vector B has both *x* and *y* components that they had to be resolved. Figure 1 shows the proportion of the answers given by the students and some of the explanations are also shown. The correct answer is **E**, of which only 36.9% students from extended group and 50% of students from mainstream got the correct answers, respectively. If we analyse the explanations of the correct answers, it reveals an understanding of the addition of vectors. These are some of the answers given by students:

1. A has only x- component and B has both x and y components with then x-component being negative.
2. The angle between vector A and the horizontal is equal to zero so the value of the y component of vector A is zero and the horizontal component is Ax =Acos0...the vector B makes an angle of 60 with the horizontal so the horizontal component of vector B is Bx=Bcos60 and the y component is By=Bsin60.
The rest of the other students did not have a good explanations of their choices.

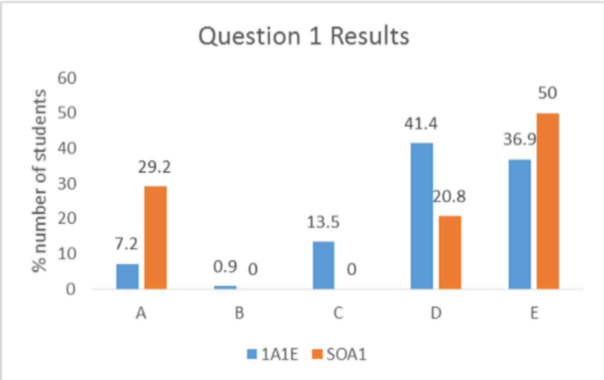


Figure 1. shows the proportion of the answers given by the students

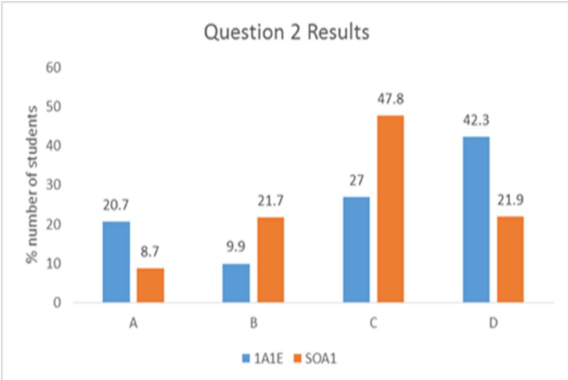
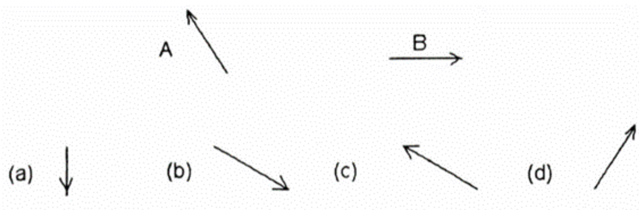


Figure 2. Shows various results given by both groups.

3.1.2 The difference between vector A and vector B

To analyze students' difficulties with the difference between two vectors, we used question 2, which asks students to choose the correct answer about the difference between vector **A** and Vector **B**. The statement of this question is: "Consider two vectors A and B as shown on the diagram below.

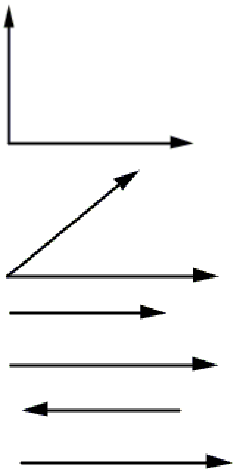


The difference between vector A and B is"…. There are different ways to answer this question, a common correct procedure is that students subtracted vector **B** from vector **A** then use the head to tail method to find the correct answer. Figure 2 shows that only 37.4% of the total number of students chose the correct answer C. Some guessed the answer with wrong explanations, while others could explain their correct choices. If we analyse the answers given, it is interesting that 32.1% of the students chose D as the correct answer. This indicates that they simply used the head to tail method and found the resultant. It is also interesting that they could explain their choice with correct physics explanation, but they did not answer the question at hand. Thus, rushing to choose the answer could have resulted in their choice.

3.1.3 Vector operations

We used question 3 to analyze students' difficulties in the operations in vectors. The question states that: "*Which of the following operations will not change a vector?"* To answer this question correctly, students should be understanding the definition of the vector. The figure 3 below gives the distribution of how the question was answered. An overwhelming number of students from both groups managed to choose the correct answer.

Question
Two forces act on a hockey puck. For which orientation of the forces will the puck acquire an acceleration with the largest magnitude?



A)
B)
C)
D)
E) The magnitude of the acceleration will be the same in all four cases shown above.

3.1.4 Application of Newton's second law in relation to acceleration

We designed this question to analyse students' difficulties with the resolving of vectors and addition of vectors to obtain the greater net force and the largest acceleration of the motion. Figure 4 shows students responses to the question. To answer this question, students should correctly identify two vectors that will sum up to the greater net force then determine the acceleration of the puck. Almost more than 70% of students from both classes answered the question correctly.

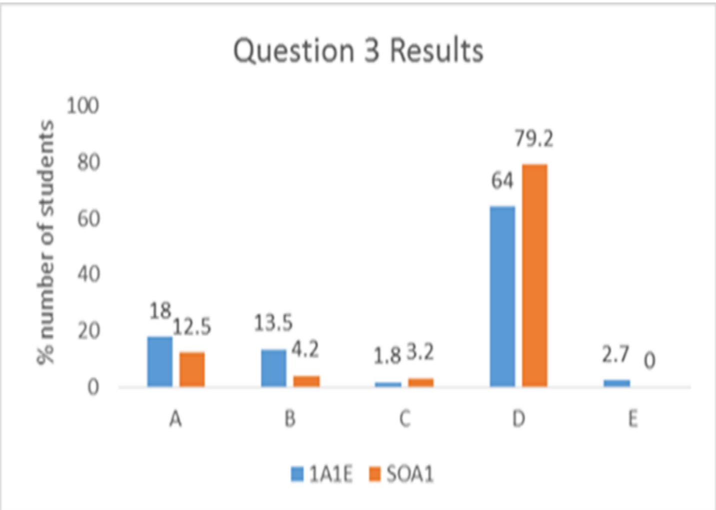


Figure 3. Shows various results given by both groups.

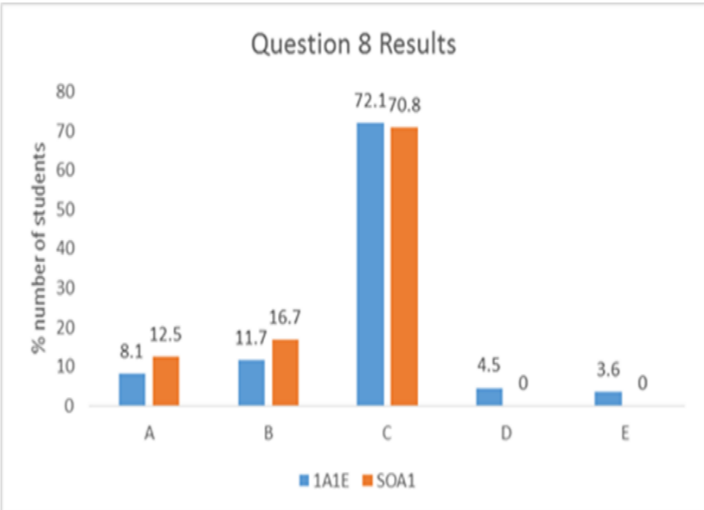


Figure 4. Shows various results given by both groups.

These are some of the supporting answers to their choices given by students:

1. The maximum value is given by the two forces acting at zero angle and minimum value is 180 degree. 2. Because both the forces acting on the puck are in the same direction and as it known that force and acceleration are directly proportional. 3. Because it will result to the addition of forces thus greater resultant. 4. Since the two forces are in the same direction the acceleration will be at its maximum.

3.2 Conceptual questions

3.2.1 Definition of a vector

We designed this question to analyze students' grasp of the definition. The statement of the question: *"Define a vector in your own words."* To answer this question correctly student should be able to mention both magnitude and direction in their definition. Most students in the 90 - 95 % range answered this question correctly, although some gave wrong definitions. Below are some of the definitions given by students:

1. "Something with both magnitude and direction."
2. "Not really sure how to define it."

3.2.2 Addition of two or more vectors

We designed this question specifically to analyze students understanding and challenges with regarding addition/subtraction of vectors. The statement of the question is this: "Can you add two or more vectors? Explain." To answer this question correctly students should be able to classify the vectors into their respective component and add them algebraically. All students answered this question correctly. Some of the answers given are as follows:

1. "Yes. Vectors behaves like scalars. As you can add scalars, the same way, vectors will be added but with vectors, they should be a specified sign, indicating the direction of the vector."
2. "Yes, vectors can be added to obtain resultant vectors which are useful in static and dynamic physical problems. Multiple vectors can, almost, always be reduced to a single vector."

3.2.3 *Understanding of free-body diagram and extracting information from the free-body diagram*

We designed these two questions specifically to analyze students understanding and application or extraction of mathematical formulae from the free-body diagram. The statement of the first question is this:

"What is a free-body diagram?" To answer this question correctly students should be able to mention representation of all the vectors acting on an object using their own understanding. All students showed understanding of what is a free-body diagram. These are some of chosen answers:

The statement of the second question is this: "Can you draw a free-body diagram from statement given for various problem in physics? Explain." We expect that students should answer this correctly with confidence since it is related to the first question. All students indicated yes as an answer, this gave a positive impression. Some of the explanations given are:

1. "Yes. Drawing of a center of mass symbol, the identify forces, determine which ones are larger and smaller.
2. Separate forces that have components resolved into their respective components.
3. Draw the arrows representing each force on the center of mass symbol and indicate directions of accelerations.
4. Yes, I can, by analyzing the problem and separating the objects in question with their forces.
5. Yes, first draw a dot, and then draw all the forces acting on the object.

3.2.4 *Challenges students have with the drawing of the free-body diagram*

We designed this question to analyze students' challenges with the actual drawing of the free-body diagram and to formulate equations from the information in the diagram. The question: "*What challenges do you have with the drawing of the free-body diagram?"* Regarding the answer given in the previous question we expect student not to have challenges in drawing and interpreting the diagram. Figure 5 shows different challenges that students have with the actual drawing of the free-body diagram. We see from this figure that 75 % of the students have challenges in drawing the free-body diagram, however, students were not asked to provide explanations of their answer, and evidence provided by student performance for Question 1 and 2 strongly suggest the reasons why many students could not get the correct answers in both questions.

A : None B: Identifying forces C: Reference frames D: Free Body Diagrams E: Resolving vectors into components

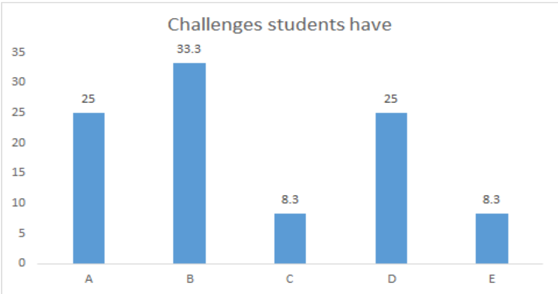


Figure 5. Shows various results given by both groups.

4. Conclusion

The responses from the students show that there is a general improvement in the understanding of this concept involving vectors. Although some fundamental challenges which informs a bit of confusion in the concept were observed from some students, but the overall outlook has showed improvements. This improvement is ascribed to time spent in this section, which involved difference teaching, learning and assessment approaches of this important concept.

5. Acknowledgements

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References

[1]Cutnell, J. D. (2013). Introduction to Physics (International Student Version). *John Wiley & Sons, Inc*.
[2]Knight, R. (1995). Vector knowledge of begginin physics students. *The Physysics Teacher 33*, 74-78.
[3]Mazur, E. (2015). Principles & Practice of Physics. *Pearson Education Limited*.
[4]Nguyen, N. &. (2003). Initial understanding of vector concepts among students in introductory phyisics courses. *AM. J. Phys. 71*, 630-638.
[5]Ortiz, L. G., Heron, P. R., Shaffer, P. S., & McDemott, L. C. (2001). Identifying student reasoning difficulties with the mathematical formalism of rotational mechanics. *AAPT Announcer, 31 (4)*, 103.
[6]Shaffer, P. S. (2005). A research-based approach to improving student undestanding of the vector nature of kinematical concepts. *AM. J. Phys. 73*, 921-931.