Answers to the Referees report: List of corrections for the conference proceeding

Effects of atmospheric turbulence on entangled photon field generated by a partially coherent pump beam

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Dear Editor,

We would like to thank you and the reviewers for reviewing our conference proceeding entitled, "Effects of atmospheric turbulence on entangled photon field generated by a partially coherent pump beam". In particular, we greatly thank you for the useful comments by the referees on how the proceeding can be improved. We have considered the conclusive input of the referees and editor, and amended the proceeding accordingly. In particular we made the changes listed in the answers to the referee, which are listed below.

We strongly hope that in this amended version of our improved manuscript, we have addressed the concerns of the referees.

Kind Regards
Stuti Joshi (on behalf of the authors)

Reviewer #0:

The reviewer suggested that the quantities with the theoretical sections need to be defined at their first occurrence.

The authors would like to thank the review for this observation. Upon checking the proceeding, the authors have identified all the quantities that was not defined at the first occurrence. The manuscript was corrected accordingly. The changes are highlighted in yellow.

Theoretical Background

A generic situation to study the effect of atmospheric turbulence on the coincidence counts of the two-photon fields is represented in figure (1). The signal-idler photons produced by SPDC are detected in coincidence by detectors D_1 and D_2 respectively. The two-photon field can be expressed as [8],

$$|\psi\rangle = \int\int \mathrm{d}q_s \mathrm{d}q_i \Gamma(q_s, q_i) \hat{\mathrm{b}}_s^{\dagger}(q_s) \hat{\mathrm{b}}_i^{\dagger}(q_i) |0,0\rangle, \tag{1}$$

where, $\mathrm{b}_s^{\dagger}$ and $\mathrm{b}_i^{\dagger}$ are the creation operators for signal (s) and idler (i) with the corresponding transverse wave-vectors q_s and q_i respectively. The vacuum state is denoted by $|0,0\rangle$ and $\Gamma(q_s, q_i)$ describes the phase-matching and perfect energy conservation in the SPDC process,

$$\Gamma(q_s, q_i) = A\int \mathrm{d}q_p \mathrm{U}(q_p)\delta(q_p - q_s - q_i)\tilde{\zeta}(q_s, q_i), \tag{2}$$

where $\mathrm{U}(q_p)$ is the pump field and and $\zeta(q_s,q_i)$ is defined as,

$$\tilde{\varsigma}(q_s,q_i)=\mathrm{sinc}\left(\frac{\Delta qL}{2}\right)\exp\left(-\mathrm{i}\frac{\Delta qL}{2}\right), \tag{3}$$

and A is the integral constant, L is the crystal length. The positive electric field component of the signal and idler photon at the detection plane after propagation through an arbitrary optical system is given by [8],

$$E_\alpha^+=\int \mathrm{d}q_\alpha H_\alpha(x,q_\alpha)\exp(-\mathrm{i}\omega_\alpha t)\hat{\mathrm{b}}_\alpha^-, \qquad \alpha=s,i, \tag{4}$$

where, $H_\alpha(x,q_\alpha)$ is the response of the signal (idler system), ω_α.the frequency and t is the time photons take to reach the detector. The detection probability of signal photon at x_1 and idler photon at x_2 is given by [8],

$$\mathrm{C}(x_1,x_2)=\langle\psi|\hat{E}_s^+(x_1)\hat{E}_i^+(x_2)\hat{E}_i^-(x_2)\hat{E}_s^-(x_1)|\psi\rangle. \tag{5}$$

Substituting equations (1)-(4) into equation (5) and considering the crystal is illuminated by a partially coherent pump beam and the two photon field is propagated through a turbulent atmosphere, we have,

$$\mathrm{C}(x_1,x_2)=A\int\!\!\int\!\!\int\!\!\int \mathrm{W}(x_1',x_1'';x_2',x_2'')\langle h_s(x_1,x_1')h_s^*(x_1,x_1'')h_i(x_2,x_2')h_i^*(x_2,x_2'')\rangle,$$
$$\mathrm{d}x_1'\mathrm{d}x_1''\mathrm{d}x_1'\mathrm{d}x_2'' \tag{6}$$

where $h_s(x,x')$ is the spatial Fourier transform of $H_s(x,q_s)$ and similarly for the idler system. The Cross-spectral density (CSD) of the photon field is expressed as,

$$\mathrm{W}(x_1',x_1'';x_2',x_2'')=\int\!\!\int\langle U_p(x)U_p^*(x_0)\rangle\eta(x+x_1',x+x_2')\eta^*(x_0+x_1'',x_0+x_2'')\mathrm{d}x\mathrm{d}x_0, \tag{7}$$

where $\eta(x,x')$ is the Fourier transform of $\tilde{\varsigma}(q_s,q_i)$ and

$$h_\alpha(x,x')=\left(-\mathrm{i}\frac{k_\alpha}{2\pi z}\right)^{1/2}\exp\left[\mathrm{i}\frac{k_\alpha}{2z}(x-x')^2+\phi_\alpha(x,x')\right], \quad \alpha=s,i, \tag{8}$$

where k_α is the wavenumber, z is the distance between nonlinear crystal and detectors and $\phi_\alpha(x,x')$ is the phase turbulence due to scattering for a Kolmogorov atmosphere model and is given by [7],

$$\langle\exp[\phi_\alpha^*(x_1,x_1')+\phi_\alpha(x_2,x_2')]\rangle=\exp\left[-\frac{(x_1-x_2)^2+(x_1-x_2)(x_1'-x_2')+(x_1'-x_2')^2}{\rho_\alpha^2}\right], \tag{9}$$

where, $\rho_\alpha=(0.55\mathrm{C}_\mathrm{n}^2k_\alpha^2z)^{-3/5}$ $(\alpha=s,i)$.. C_n^2, describes the turbulence level. Within the paraxial approximations, we have assumed $k_s\approx k_i\approx k_p/2$ and $\Delta q=|q_s-q_i|^2/(2k_p)$, k_p is the wavenumber of the pump. Using the approximation $\mathrm{sinc}(\Delta qL/2)\approx\exp[-\gamma L\sqrt{\Delta q}^2/2]$, the CSD is given by,

$$\mathrm{W}(x_1',x_1'';x_2',x_2'')=\frac{4\pi k_p}{L\sqrt{\gamma^2+1}}\left\langle U_p\left(-\frac{x_1'+x_2'}{2}\right)U_p^*\left(-\frac{x_1''+x_2''}{2}\right)\right\rangle$$
$$\times\exp\left[-\frac{(x_1'+x_2')^2k_p}{4L(\gamma+\mathrm{i})}-\frac{(x_1''+x_2'')^2k_p}{4L(\gamma-\mathrm{i})}\right], \tag{10}$$

For the special case of partially coherent pump field of Gaussian-Shell model type the correlation of the field is represented as [9],

$$\left\langle U(x_1')U^*(x_2')\right\rangle = S_0 \exp\left[-\frac{x_1'^2+x_2'^2}{4\sigma^2}\right]\exp\left[-\frac{\left(x_2'-x_1'\right)^2}{2\delta^2}\right], \tag{11}$$

where, S_0 is a constant, □ is the beam width and □ is the spatial coherence length of the pump beam. Substituting equations (7)-(11) into equation (6) we get,

$$\mathrm{C}(x_1,x_2) = \left(\frac{4\pi k_p}{L(\gamma^2+1)}\right)\mathrm{S}_0\left(\frac{k_p}{4\pi z}\right)^2 \iiiint \exp\left[-\frac{(x_1'+x_2')^2+(x_1''+x_2'')^2}{16\sigma^2}\right]\exp\left[-\frac{\left(x_1''+x_2''-x_2'-x_1'\right)^2}{2\delta^2}\right]$$

$$\times\exp\left[-\frac{(x_1'+x_2')^2 k_p}{4L(\gamma+\mathrm{i})}-\frac{(x_1''+x_2'')^2 k_p}{4L(\gamma-\mathrm{i})}\right]\exp\left[-\frac{\mathrm{i}k_p}{4z}\left((x_1-x_1'')^2-(x_1-x_1')^2+(x_2-x_2'')^2-(x_2-x_2')^2\right)\right]$$

$$\times\exp\left[-\frac{\left(x_1'-x_1''\right)^2+\left(x_2'-x_2''\right)^2}{\rho^2}\right]\mathrm{d}x_1'\mathrm{d}x_2'\mathrm{d}x_1''\mathrm{d}x_2''\,. \tag{12}$$

Reviewer #1:

The reviewer suggested the following grammatical changes. All these changes were made in the respective positions.

Abstract:
Line 1: fields and not field
Line3: insert "the" between "...of and the"
Line4: Change "find" to "found"

Introduction
Line9 (from heading): insert "a" between"....by and partially
Line12: fields and not field

Theoretical background:
Line2(from heading) delete "and"
Equation9: there should be 2 coefficient for the second term within the "exp" bracket

Result and discussion

Line 2: under "figure 4" insert "the" between ".....almost and same"
Line 10: under figure4" generated instead of generate

Conclusion:
Line3: under heading concluded instead of conclude