The energy density of a light quark jet using AdS/CFT

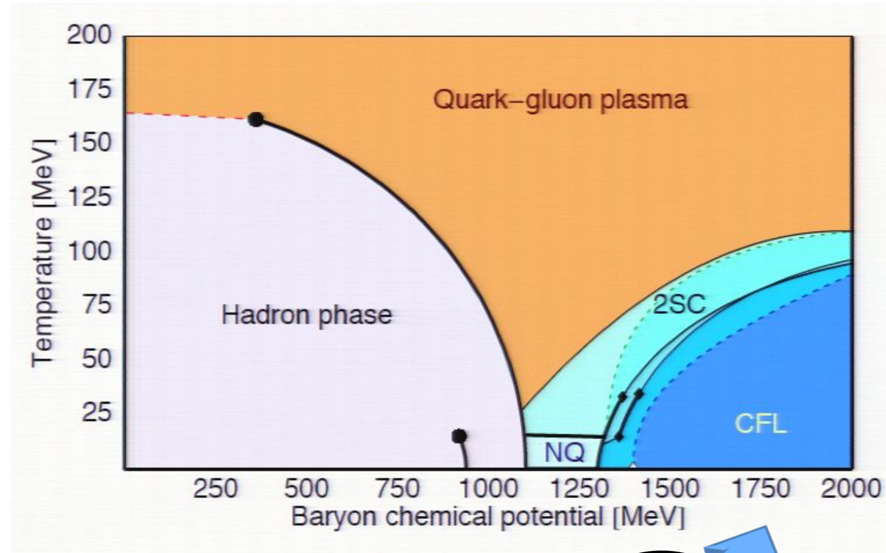
Razieh Morad

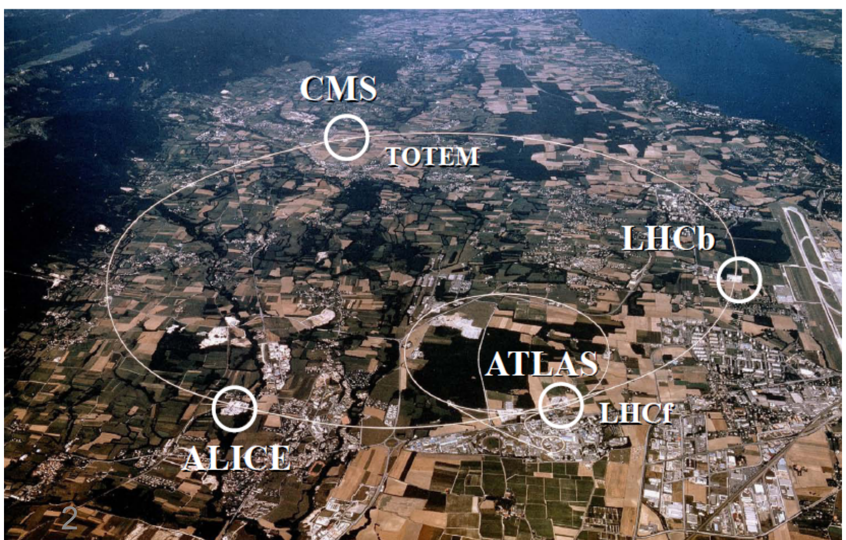
In Collaboration with Dr. W. A. Horowitz

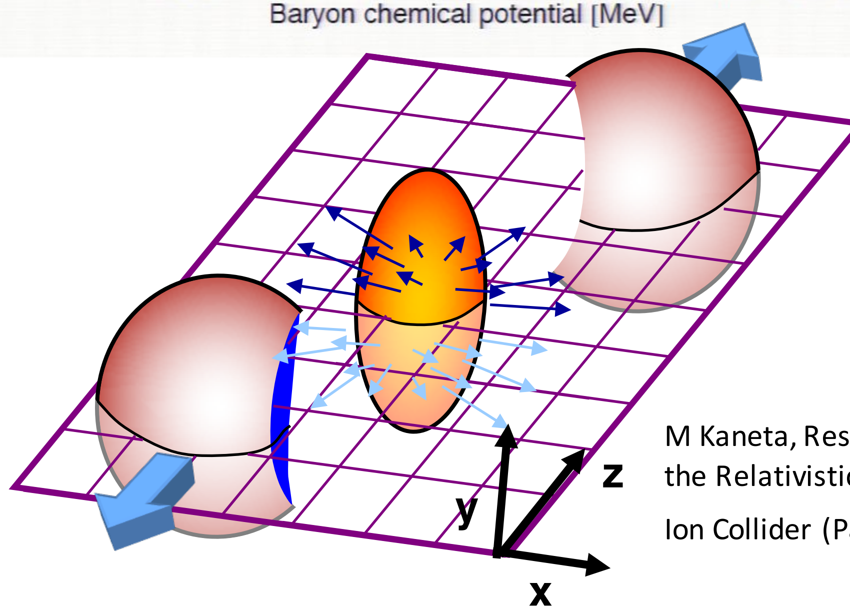
61st Annual Conference of the South African Institute of Physics
University of Cape Town
July 8th, 2016

Quark-Gluon Plasma

Quark-Gluon Plasma is formed in Heavy Ion Collision at RHIC and LHC.

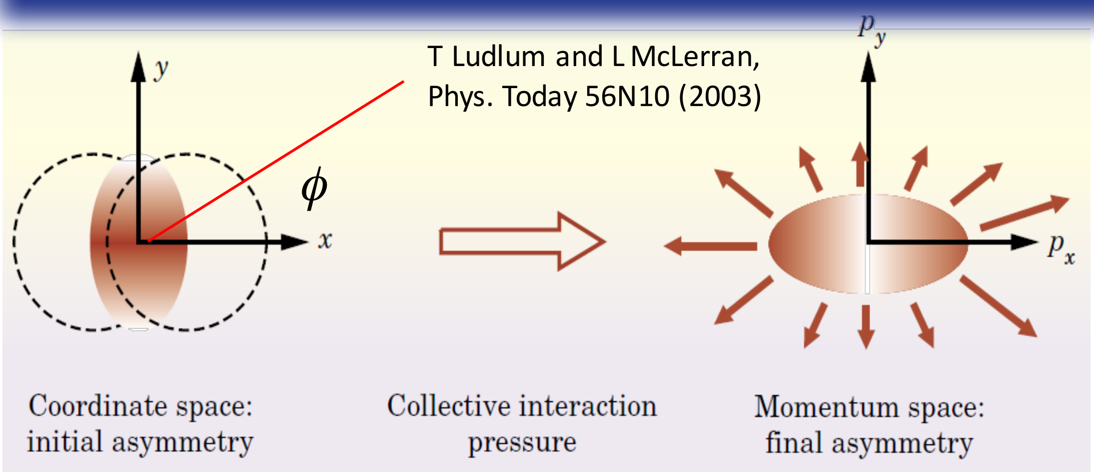






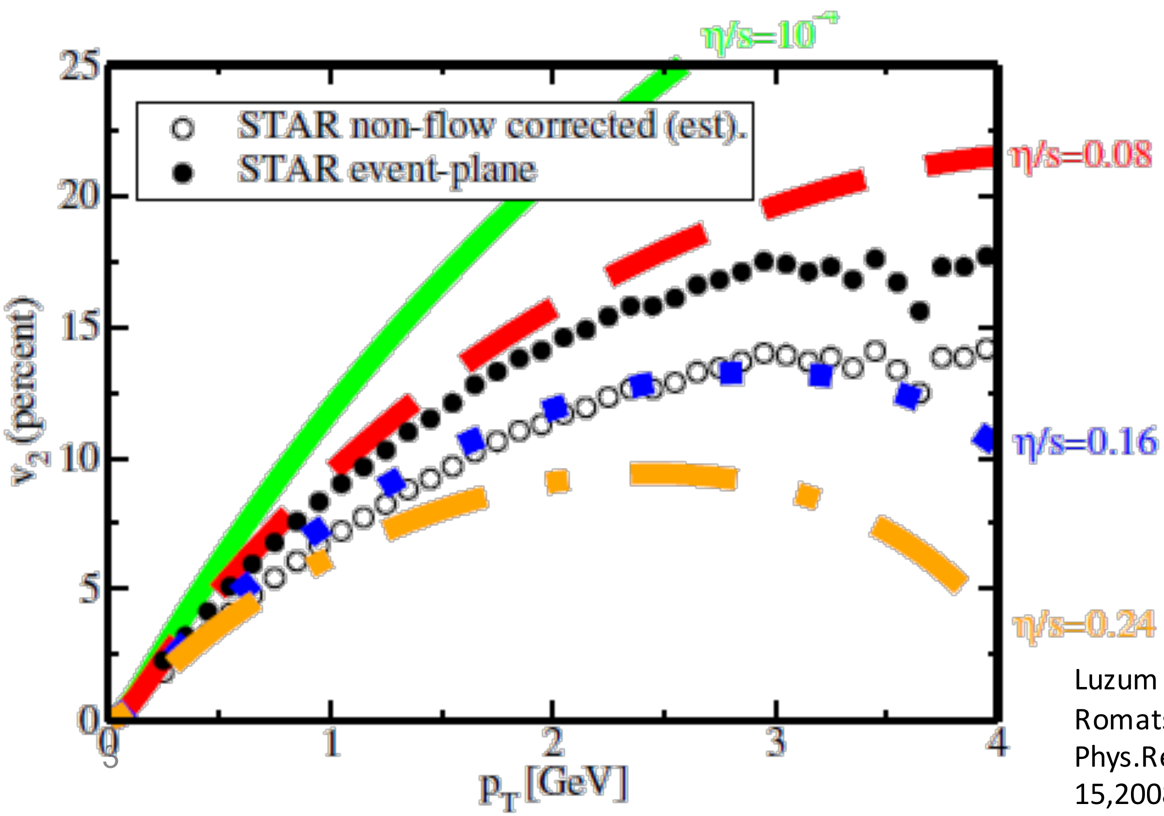
M Kaneta, Results from the Relativistic Heavy Ion Collider (Part II)

Quark-Gluon Plasma



Elliptic flow

$$\frac{dN}{dp_T}(p_T,\phi) = \frac{dN}{dp_T}(p_T)\left[1+2\,v_2(p_T)\cos(2\phi)+\ldots\right]$$



Luzum and Romatschke, Phys.Rev.C78:0349 15,2008

1) Strongly Coupled Matter
2) Rapid Thermalization

Quark-Gluon Plasma

Shear viscosity

Hydrodynamics prediction: $\frac{\eta}{s} < 0.1 - 0.2$ Teaney (2003)

Lattice: $\frac{\eta}{s} = 0.13 \pm 0.03$ Meyer (2007)

Naive pQCD: $\frac{\eta}{s} \sim 1$

N=4 SYM: $\frac{\eta}{s} = \frac{1}{4\pi} \approx 0.08$ Policastro, Son, and Starinets (2001)

AdS/CFT predicts a universal lower bound for the ratio of shear viscosity to entropy.

Kovton, Son and Starinets (2003)

Rapid thermalization: $\tau_{therm} \sim 0.35 \quad fm$

Chesler and Yaffe, PRL 106 (2011)
Janik et all (2012),(2014)

AdS/CFT Correspondence

Maldacena *Conjecture*

Classical gravity on AdS_{d+1}

Strongly coupled d - dimensional CFT which lives on boundary of AdS_{d+1}

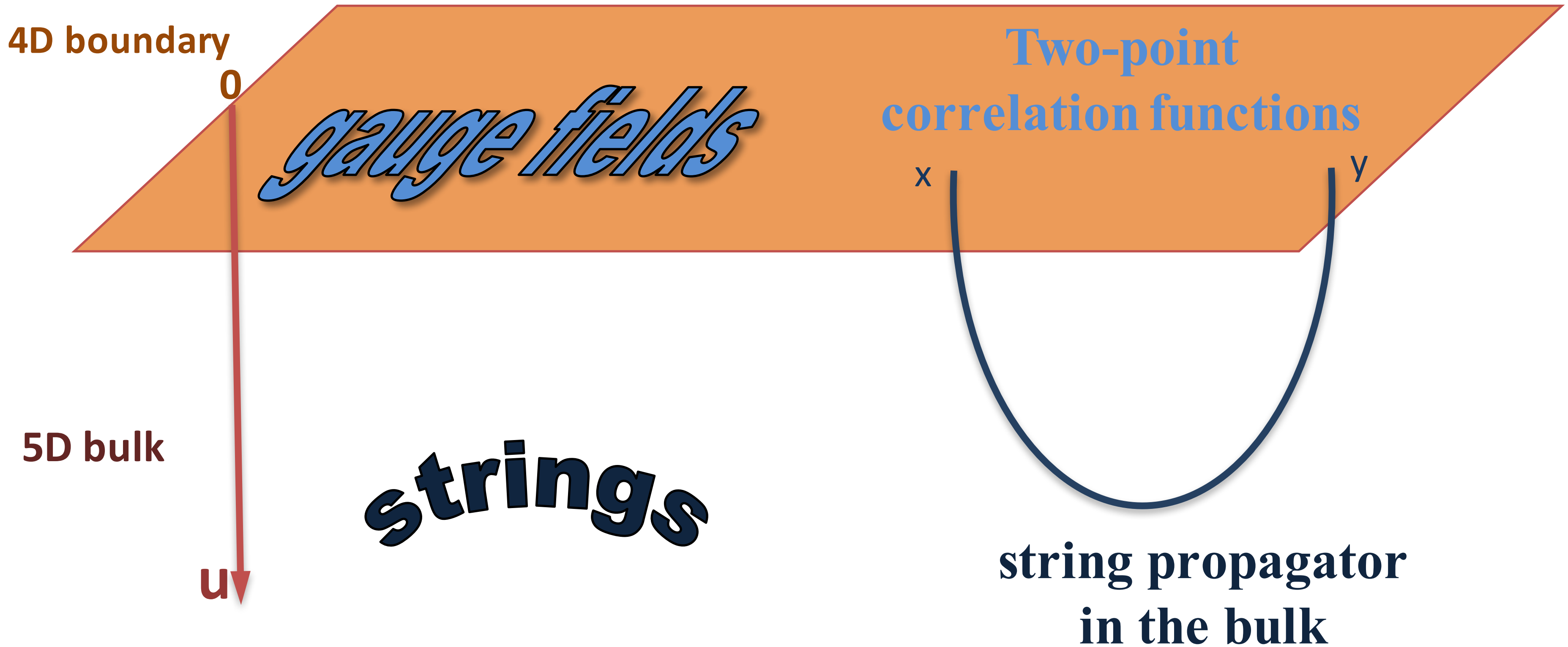
Maldacena 98

Duality unproven, but many consistency checks performed.

AdS/CFT Correspondence

Anti-de-Sitter space (AdS_5)

$$ds^2 = \frac{dx^\mu d\, x_\mu + du^2}{u^2}$$



u plays a role of inverse energy scale in 4D theory

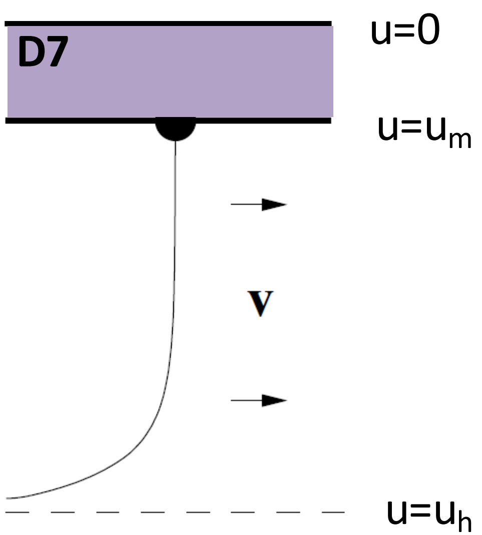
Light-Quark in string Setup

$\mathcal{N}=4$ Super-Yang-Mills theory in 4d in large N_C and strong coupling limit λ ⟷ A Classical supergravity on the 10d $AdS_5 \times S^5$

Studying the theory at finite temperature ⟷ Adding black hole to the geometry: AdS-schwarzchild metric

Fundamental quarks in theory ⟷ Open strings moving in the 10d geometry

Fundamental quark is dual to a string in the bulk with an endpoint attached to a D7-brane ending at u_m.



For a massive quark at rest: $$m_Q = T_0 L^2 \left(\frac{1}{u_h} - \frac{1}{u_m}\right)$$

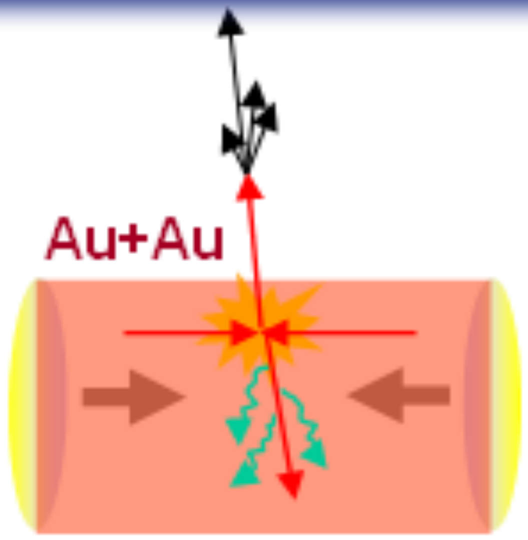
Light Quarks ⟷ **Falling Strings**

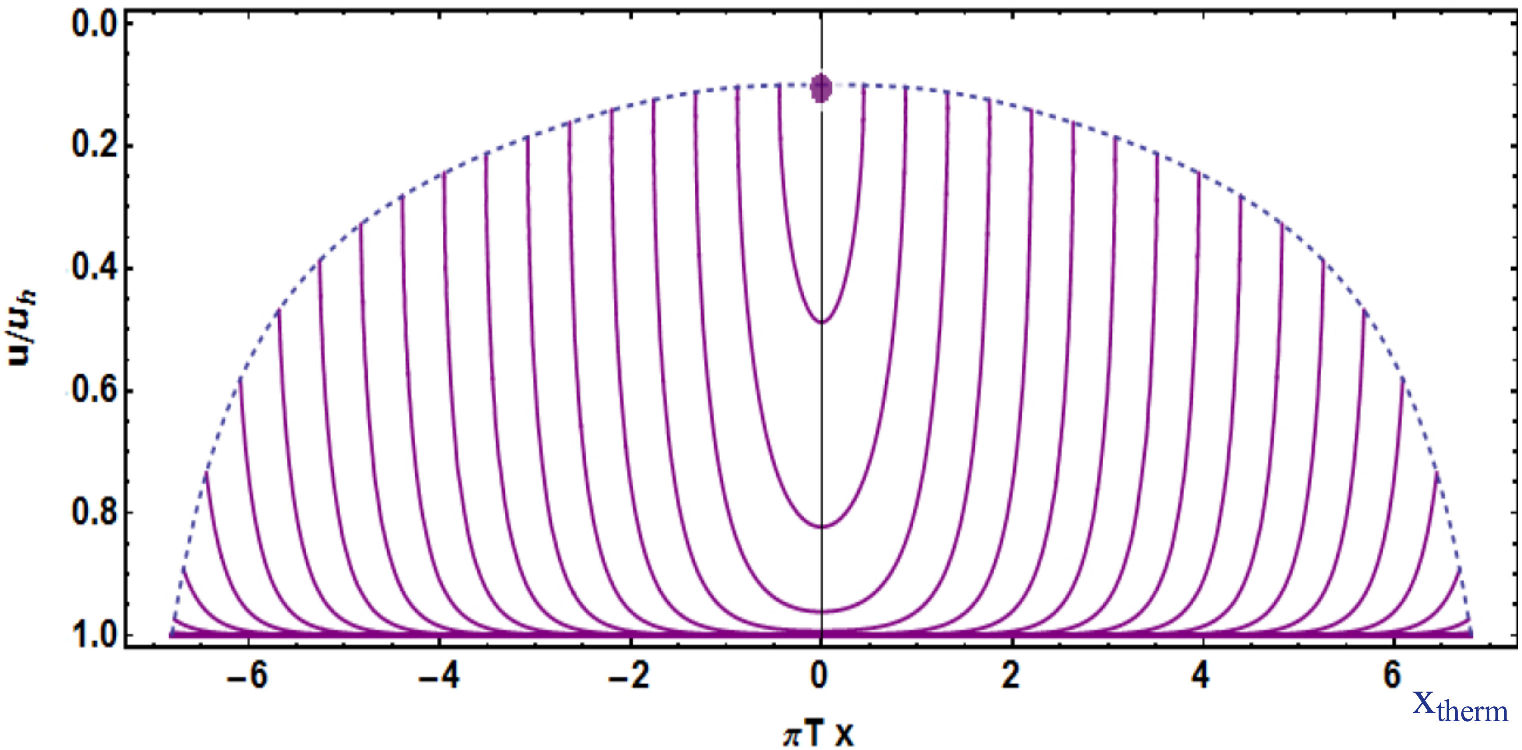
Falling String

$$S_P = -\frac{T_0}{2}\int d^2\sigma\,\sqrt{-\eta}\,\eta^{ab}\,\partial_a X^\mu \partial_b X^\nu\, G_{\mu\nu}$$

IC: $t(0,\sigma) = t_c\,,\;\; x(0,\sigma) = 0\,,\;\; u(0,\sigma) = u_c$

BC: $X'^\mu(\tau, \sigma^*) = 0$

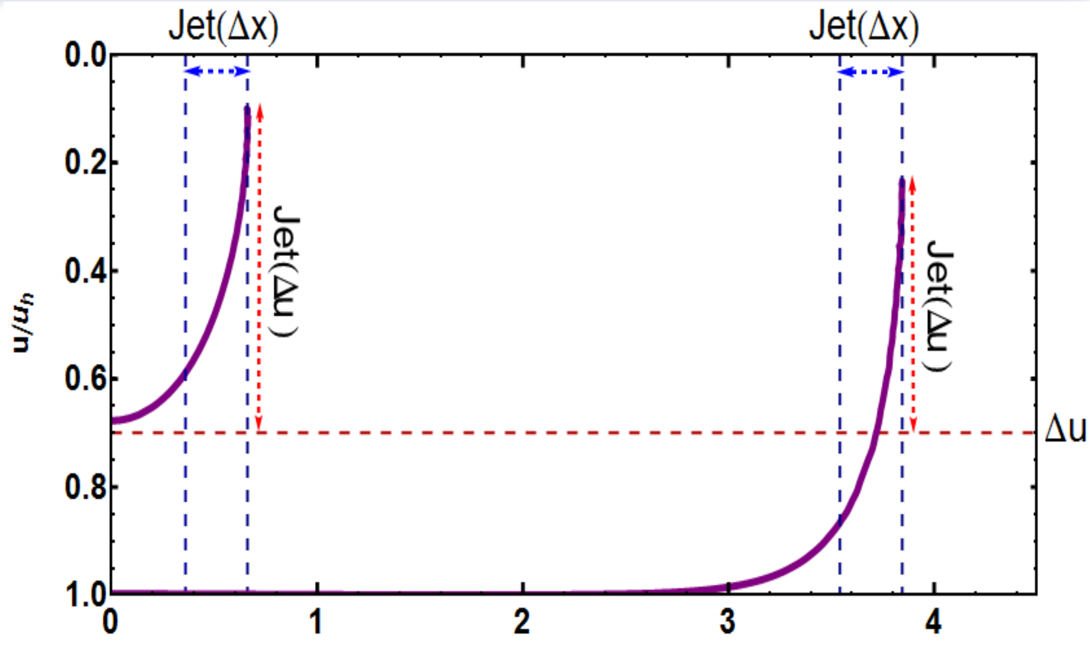




Jet Energy Lost

Prescription of jet in AdS/CFT

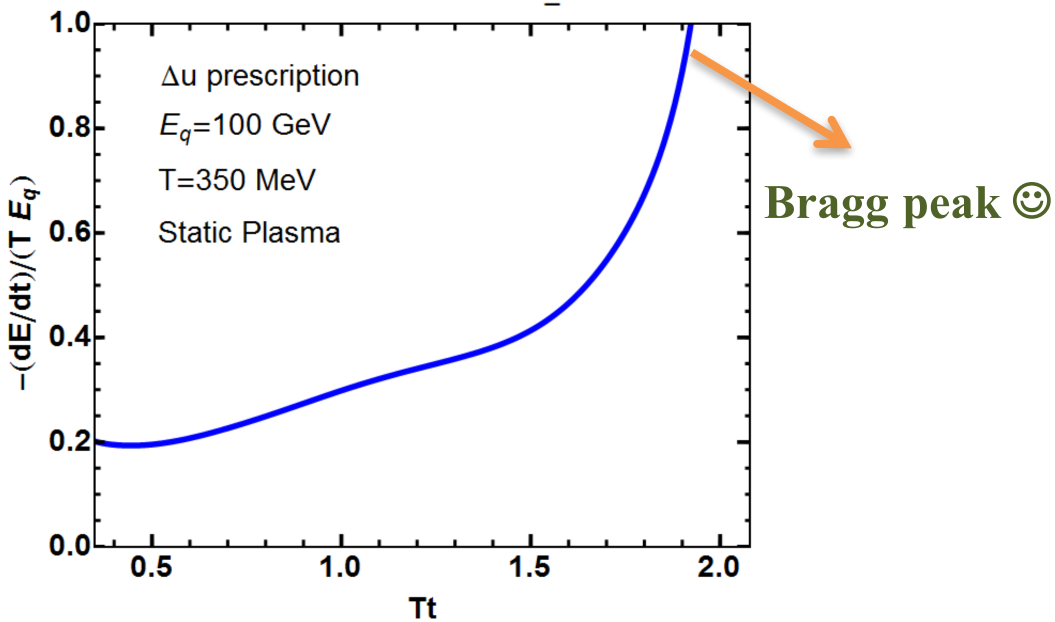
New Jet Prescription based on separation of hard and soft sectors:



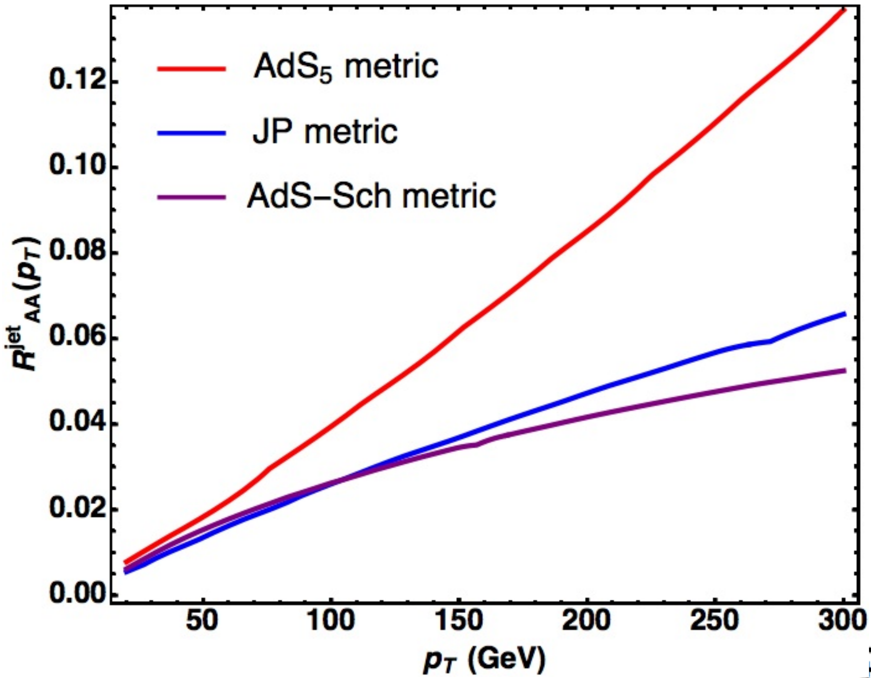
Energy Loss:

$$\Pi^{a}_{\mu}(\tau,\sigma) \equiv \frac{1}{\sqrt{-\eta}} \frac{\delta S_{\mathrm{P}}}{\delta(\partial_a X^{\mu}(\tau,\sigma))}$$

$$\frac{dp_{\mu}}{dt} = -\sqrt{-\eta}\left(\Pi^{\sigma}_{\mu} - \Pi^{t}_{\mu}\,\dot{\sigma}_{\kappa}\right)$$

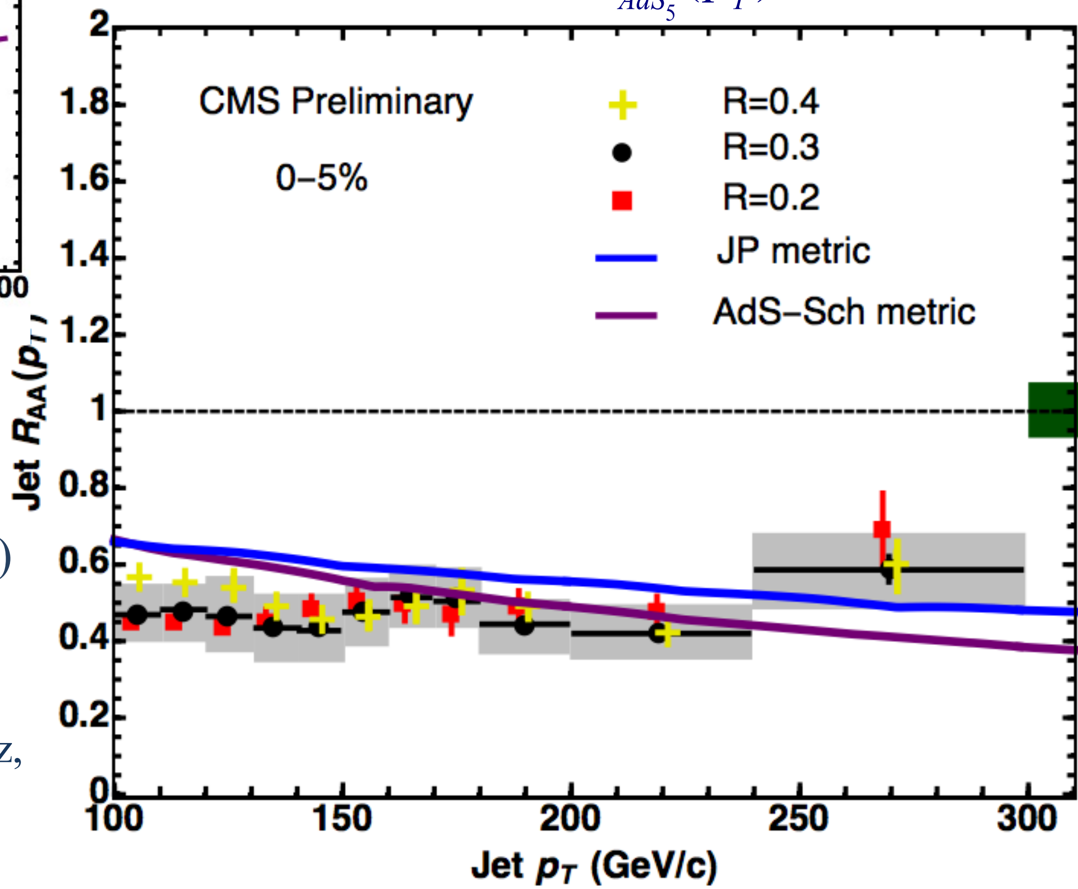


Jet Nuclear Modification Factor



We define a renormalized R_{AA} in AdS/CFT:

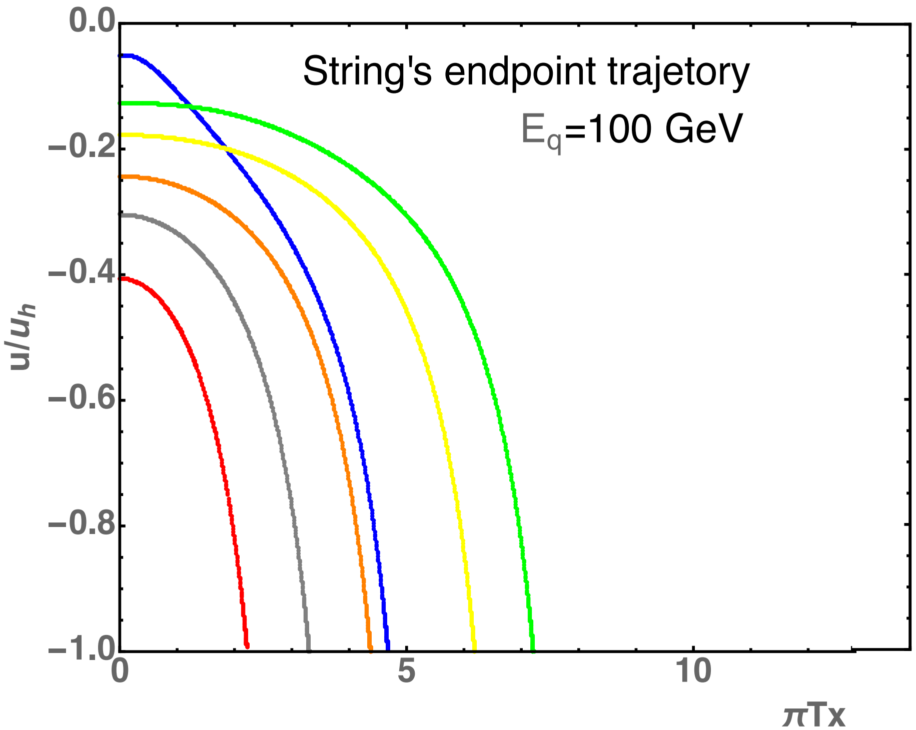
$$R^{jet}_{AA}(p_T)_{AdS/CFT} \equiv \frac{R^{jet}_{medium}(p_T)}{R^{jet}_{AdS_5}(p_T)}$$



$$\Delta E^{sub,\,ren}(p_i\,,L,T) \equiv \Delta E^{medium}(p_i\,,L,T) - \Delta E^{vacuum}(p_i\,,L,T)$$

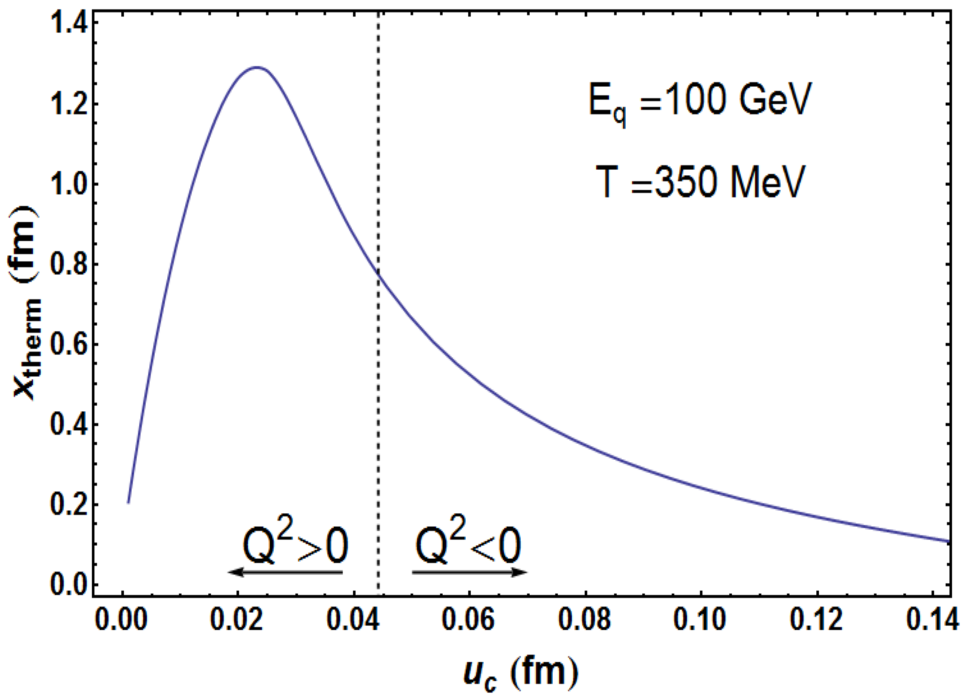
R. Morad and W. A. Horowitz, JHEP 11 (2014) 017

Light-Quark Dynamics



Light quark dynamics highly depends on the initial conditions of the string.

Virtuality of quark: $Q^2 \equiv E_q^2 - P_q^2$



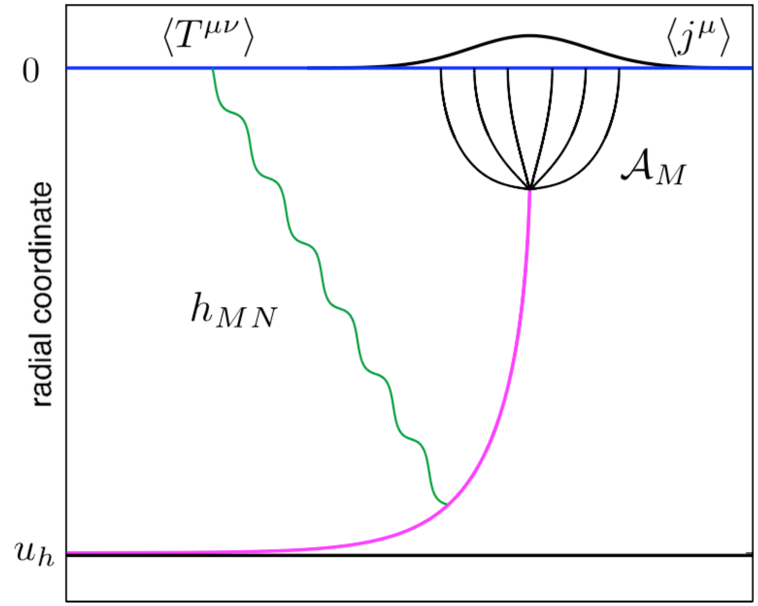
Further progress in describing experimental results will require significant advances in the understanding of string initial conditions.

The only way, is calculating the energy-momentum tensor of the string on the boundary and compare with the QCD results.

SYM Stress-Tensor

- Presence of string source with the following energy-momentum profile in the bulk perturb the metric:

$$t^{MN} = -\frac{T_0}{\sqrt{-G}} \sqrt{-g} g^{ab} \partial_a X^M \partial_b X^N \delta^3(\boldsymbol{r} - \boldsymbol{r}_s)$$



- Metric perturbation h_{MN}: $G_{MN} = G^{(0)}_{MN} + h_{MN}$

- Linearized Einstein equation for h_{MN}:

$$- D^2 h_{MN} + 2D^P D_{(M} h_{N)P} - D_M D_N h + \tfrac{8}{L^2} h_{MN} + \left(D^2 h - D^P D^Q h_{PQ} - \tfrac{4}{L^2} h\right) G^{(0)}_{MN} = 2\kappa_5^2 \, t_{MN} \, ,$$

- On-shell gravitational action: $S_G = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-G} \left(\mathcal{R} + \frac{12}{L^2}\right) + S_{GH}$

- SYM energy-momentum tensor: $T^{\mu\nu}(x) = \frac{2}{\sqrt{-g}} \frac{\delta S_G}{\delta g_{\mu\nu}(x)}$

h_{MN} has 15 degrees of freedom ⟷ **?** **$T_{\mu\nu}$ has 5 degrees of freedom**

Gauge-Invariants

It is possible to construct gauge invariant quantities out of linear combinations of h_{MN} and its derivatives.

- ✓ **There are just 5 of them.**
- ✓ **Their equation of motions are completely decoupled.**

The scalar gauge invariant Z, can give us the energy density of the SYM stress tensor on the boundary:

In AdS_5 background:

$$Z'' + AZ' + BZ = S \qquad A = -\frac{5}{u} \qquad B = \omega^2 - q^2 + \frac{9}{u^2}$$

$$S = 8t'_{00} + \frac{4}{3}u(q^2\delta^{ij} - 3q^i q^j)t_{ij} + 8i\omega t_{05} + \frac{8}{3u}(q^2u^2 - 6)t_{00} - \frac{8}{3}q^2 u\, t_{55} - 8iq^i t_{i5}$$

Asymptotic behavior of Z: $$Z(u) = Z_{(2)}\,u^2 + Z_{(3)}\,u^3 + \cdots \quad ; \quad u \to 0$$

Energy density: $$\mathcal{E} = -\frac{L^3}{8\kappa_5^2} Z_{(3)}$$

P. Chesler et al, hep-th/1001.3880

Boundary Energy Density in AdS_5

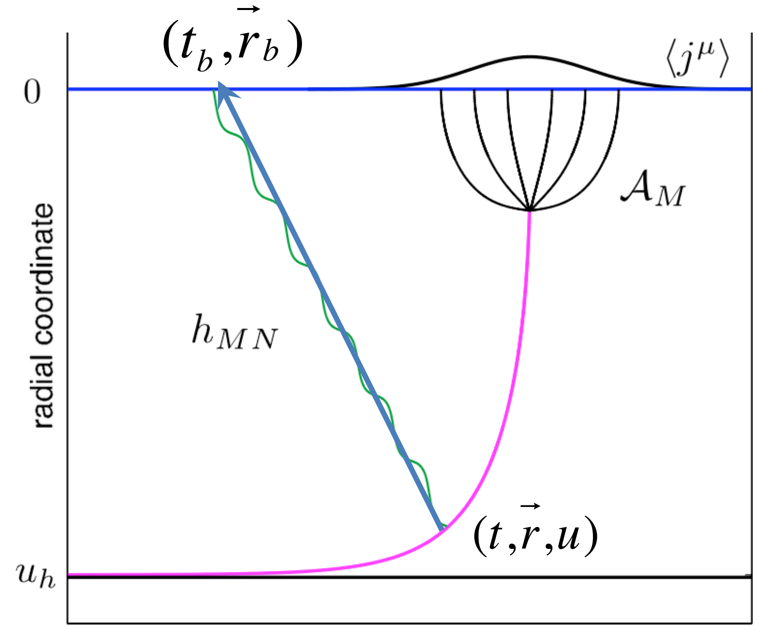
$$\mathcal{E}(t_b, \mathbf{r_b}) = \mathcal{E}_{\mathcal{A}}(t_b, \mathbf{r_b}) + \mathcal{E}_{\mathcal{B}}(t_b, \mathbf{x_b})$$

$$\mathcal{E}_{\mathcal{A}}(t_b, \mathbf{r_b}) = \frac{2L^3}{\pi} \int d^4r \, \frac{du}{u^2} \, \Theta(t_b - t) \, \delta''(W) \, [u(2t_{00} - t_{55}) - (t_b - t)t_{05} + (x_b - x)^i t_{i5}]$$

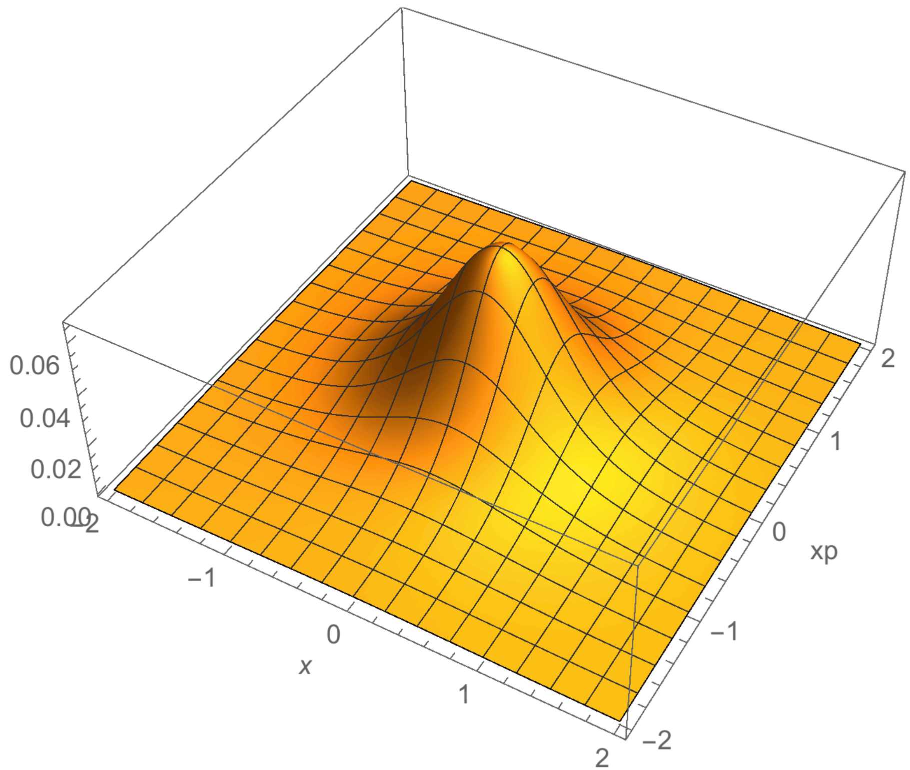
$$\mathcal{E}_{\mathcal{B}}(t_b, \mathbf{r_b}) = \frac{2L^3}{3\pi} \int d^4r \, \frac{du}{u} \, \Theta(t_b - t) \, \delta'''(W) \, [|\mathbf{r_b} - \mathbf{r}|^2 \, (2t_{00} - 2t_{55} + t_{ii}) - 3(x_b - x)^i \, (x_b - x)^j \, t_{ij}]$$

$$W = -(t - t_b)^2 + (\vec{r} - r_b)^2 + u^2$$

At time t, the bulk excitation localized at (t, r, u) emits a gravitational wave h_{MN} which propagates through AdS_5 at the respective speed of light up to the measurement point (t_b, r_b) on the boundary.

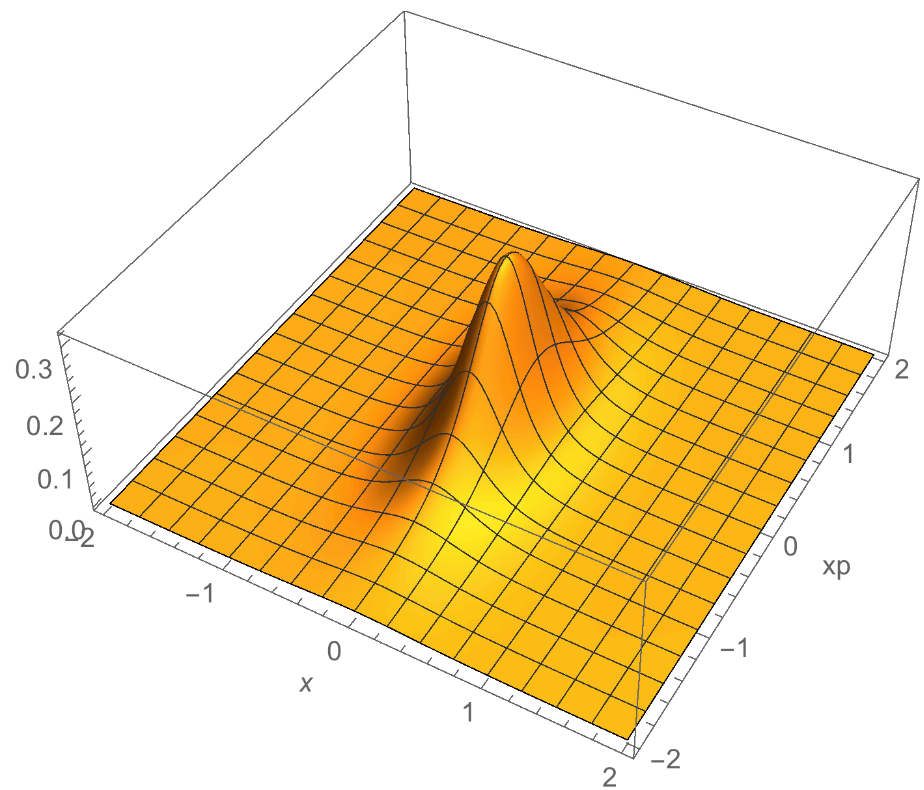


Boundary Energy Density

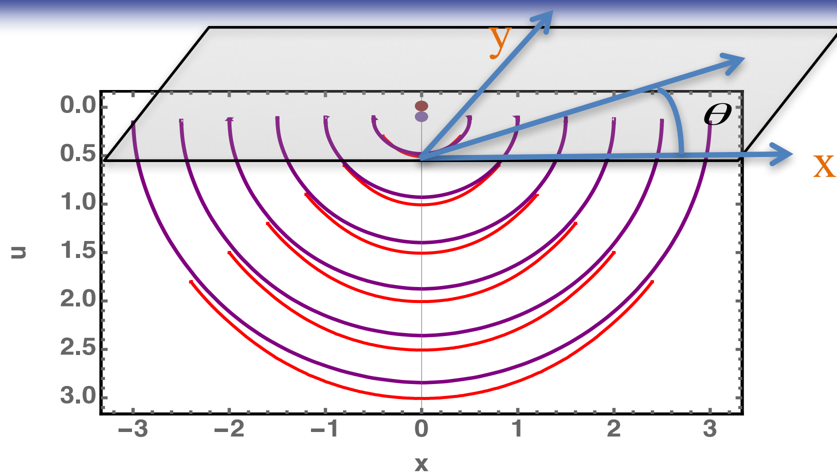


Heavy quark at rest with finite mass in AdS_5

Heavy quark with finite mass and velocity v=0.9 in x direction in AdS_5

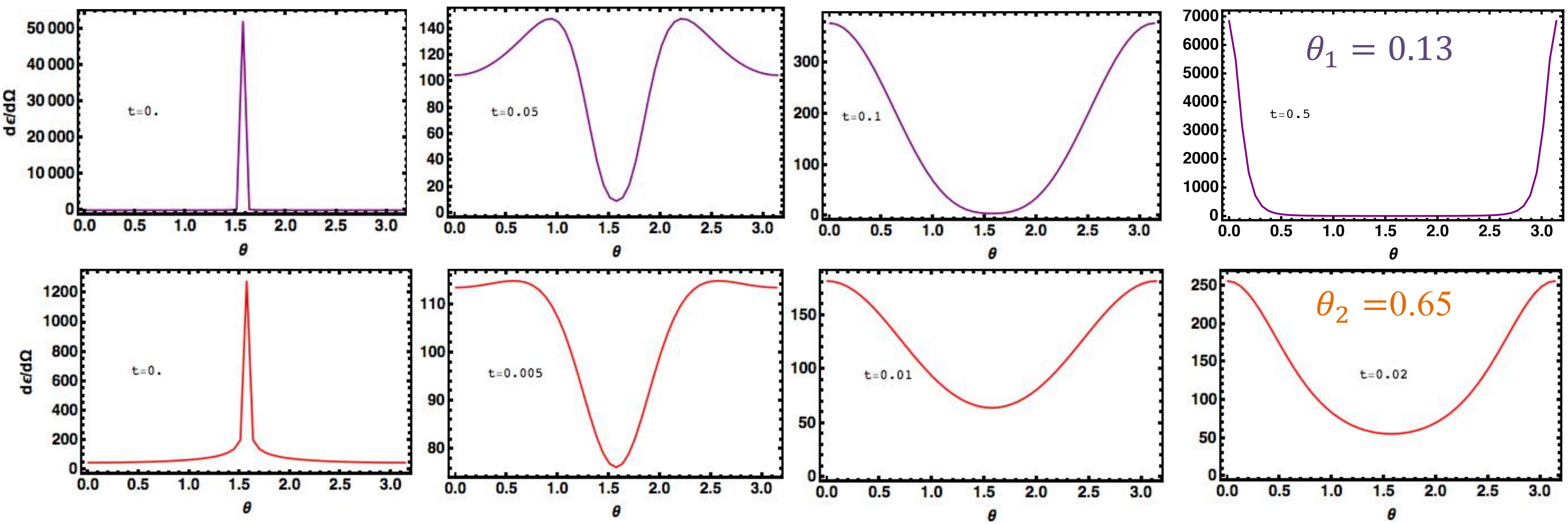


Boundary Jet Energy Density in AdS_5



String 1 created at u_c=0.1 :
E_q=100 GeV, Q^2=176 GeV^2

String 2 created at u_c=0.01 :
E_q=100 GeV, Q^2=6000 GeV^2



One can define the opening angle of jet: $\theta_j = ArcTan[\frac{\sqrt{Q^2}}{E_q}]$

Current plan:

In AdS-Sch background:

$$Z'' + AZ' + BZ = S$$

P. Chesler et al, hep-th/1001.3880

$$A \equiv -\frac{24+4q^2u^2+6f+q^2u^2f-30f^2}{uf\left(u^2q^2+6-6f\right)}, \quad B \equiv \frac{\omega^2}{f^2}+\frac{q^2u^2(14-5f-q^2u^2)+18(4-f-3f^2)}{u^2f\left(q^2u^2+6-6f\right)}$$

$$\frac{S}{\kappa_5^2} \equiv \frac{8}{f}t'_{00}+\frac{4\left(q^2u^2+6-6f\right)}{3uq^2f}(q^2\delta^{ij}-3q^iq^j)t_{ij}-\frac{8q^2u}{3}t_{55}-8iq^it_{i5}$$

$$+\frac{8i\omega}{f}t_{05}+\frac{8u\left[q^2\left(q^2u^2+6\right)-f\left(12q^2-9f''\right)\right]}{3f^2\left(q^2u^2-6f+6\right)}t_{00}$$

Construct Green's functions G(u, u') out of homogeneous solutions by considering the appropriate boundary conditions at the boundary and at the horizon and convolve with source as

$$Z(u)=\int_0^{u_h}du'G(u,u')\,S(u') \longrightarrow \varepsilon(\vec{q},\omega)=-\frac{L^3}{8\kappa_5^2}Z^{(3)}$$

$$\downarrow$$

$$E(\vec{r},t)=\int\frac{d^4q}{(2\pi)^4}\varepsilon(\vec{q},\omega)e^{iQx}$$

Thank you