Electron-quark scattering at next-to-leading order

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Structure of talk

- What we compute
- Why we compute it
- Results
- Things I'm still confused about
- Conclusion

What we compute

Differential cross section for massless $2 \to 2$ scattering between a quark and an electron in a t-channel photon exchange at next to leading order in the $\overline{\mathrm{MS}}$.

Couple quarks to abelian gluon field

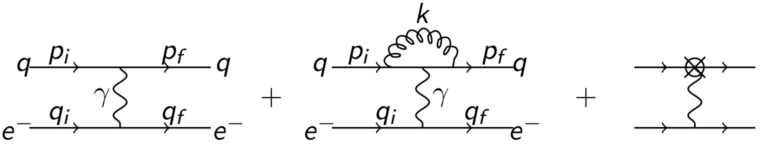
Use Dim-Reg to regulate all divergences

Why do we compute it?

- First steps in much broader research program $\rightarrow$ energy loss and running coupling in QGP.
- This precise calculation is new (to the best of my knowledge)
- Learn how IR divergences cancel at NLO

The calculation

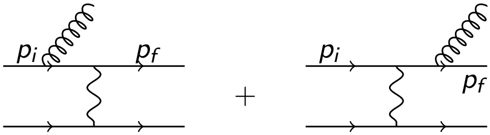
Leading order and vertex correction



$$\left(\frac{d\sigma}{d\Omega}\right)_{virtual} = \left(\frac{d\sigma}{d\Omega}\right)_0 \left[1 + \frac{\alpha_g}{2\pi}\left[-\frac{8}{\epsilon_{ir}^2} - \frac{1}{\epsilon_{ir}}\left(6 + 4\log\left(\frac{\mu^2}{t}\right)\right) + \frac{7\pi^2}{6} - 8 - 3\log\left(\frac{\mu^2}{t}\right) - \log^2\left(\frac{\mu^2}{t}\right)\right]\right].$$

The calculation

Bremsstrahlung



Soft gluon emission (Bloch-Nordsiek)

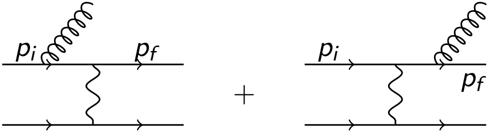
$$\left(\frac{d\sigma}{d\Omega}\right)_{sb} = \left(\frac{d\sigma}{d\Omega}\right)_0 \int g^2 \frac{2p_i \cdot p_f}{(p_i \cdot k)(p_f \cdot k)} \frac{d^{d-1}k}{(2\pi)^{d-1} 2\omega_k}$$

Integrate over all angles and from zero up to cutoff E_{cut}

$$\left(\frac{d\sigma}{d\Omega}\right)_{sb} = \left(\frac{d\sigma}{d\Omega}\right)_0 \frac{\alpha_g}{2\pi} \left[\frac{8}{\epsilon_{ir}^2} + \frac{4}{\epsilon_{ir}} \log\left(\frac{\mu^2}{\rho E_{cut}^2}\right) + \text{finite}\right], \quad \rho = \frac{t}{E_{p_i}^2}$$

The calculation

Bremsstrahlung



Hard collinear gluon emission (KLN): Integrate from zero up to cutoff angle δ

$$\left(\frac{d\sigma}{d\Omega}\right)_{hcb} = \left(\frac{d\sigma}{d\Omega}\right)_0 \frac{\alpha_g}{2\pi}\left[\frac{1}{\epsilon_{ir}}\left(8\log\left(\frac{E_{cut}}{E_{p_i}}\right)+6\right)+\text{finite}\right] \quad (1)$$

The calculation

Adding the processes

The $\frac{1}{\epsilon_{ir}^2}$ terms cancel straightforwardly.

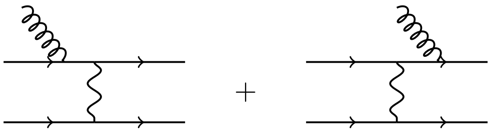
The $\frac{1}{\epsilon_{ir}}$ terms need some massaging:

$$\frac{\alpha_g}{2\pi}\frac{1}{\epsilon_{ir}}\left[-6-4\log\left(\frac{\mu^2}{t}\right)+4\log\left(\frac{\mu^2}{\rho E_{cut}^2}\right)+8\log\left(\frac{E_{cut}}{E_{p_i}}\right)+6\right]=0$$

The finite terms depend logarithmically on parameters E_{cut}, t, δ and μ.

Things I need to think about

No reason why I shouldn't include soft gluon absorption

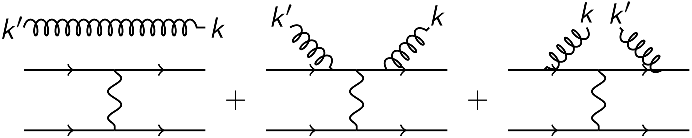


Reintroduces IR divergences

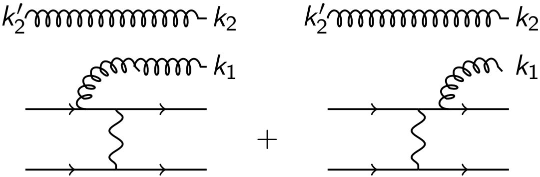
$$|\mathcal{M}_{sb}|^2 = |\mathcal{M}_0|^2 g^2 \frac{2p_i \cdot p_f}{(p_i \cdot k)(p_f \cdot k)}.$$

KLN says $\rightarrow$ find more degenerate processes contributing to $|\mathcal{M}|^2$ (same order of coupling) Lee & Naunberg, Lavelle, Zakharov

Things I need to think about



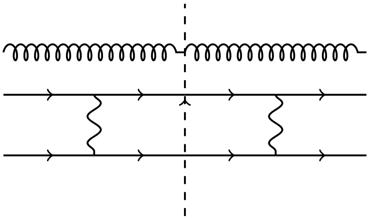
$\mathcal{M}_0(2\pi)^3 2\omega_k \delta^{(3)}(k - k')$.   Gives $-2|\mathcal{M}_{sb}|^2$



Gives $|\mathcal{M}_{sb}|^2$. Thus, $|\mathcal{M}_{sb}|^2 - 2|\mathcal{M}_{sb}|^2 + |\mathcal{M}_{sb}|^2 = 0$

Things I need to think about

So, soft IR divergences have canceled in $|\mathcal{M}|^2$, but now have completely disconnected contributions.



Indeed, seems like we can have contributions from any number of disconnected gluon lines to $|\mathcal{M}|^2$ at same order in coupling.

Conclusion

Need to explain the disconnected diagram contributions to $|\mathcal{M}|^2$.

- Can we really define asymptotic free states here?
- Take into account the softness of the photon in its wave packet in the cross section?
- Or am I doing something stupid and missing something big!!!!

Conclusion

Need to explain the disconnected diagram contributions to $|\mathcal{M}|^2$.

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Thank you