DEPARTMENT OF PHYSICS
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NITheP
National Institute for Theoretical Physics

Are we gauging the pressure correctly?

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(thanks to André Peshier)

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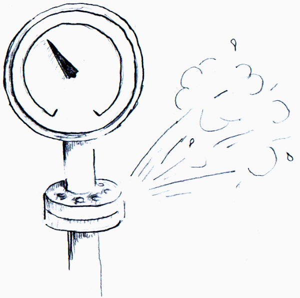
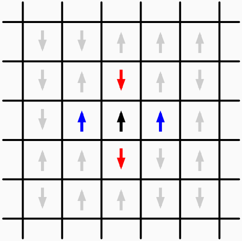


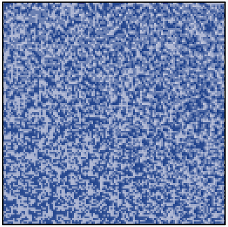


Ising model for a ferromagnet

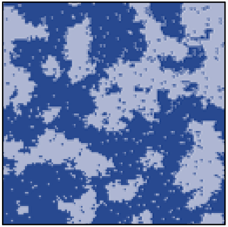
$$\mathcal{H} = -\sum_{\langle i,j \rangle} s_i s_j \qquad \text{spins: } s_i = \pm 1$$

short-range … GLOBAL SYMMETRY

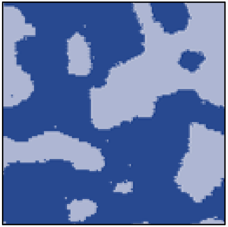
$\beta = .1$

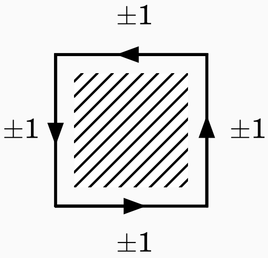


$\beta = .5$



$\beta = .9$





link variables ....

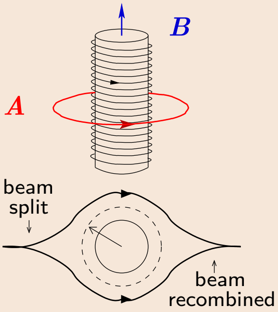
$$\mathcal{H} = -\sum_{\square} U(\square) \qquad \text{spins: } U_{ij} = \pm 1$$

... GAUGE SYMMETRY

which symmetries?

'spins'	$\mathbb{Z}_2 = \{\pm 1\}$	
phases	$U(1) = \{e^{i\phi}\}$	QED
colours	$SU(3) = \ldots$	QCD

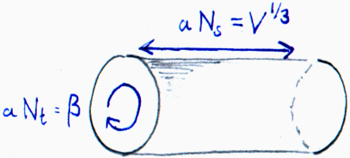
Aharanov-Bohm ring



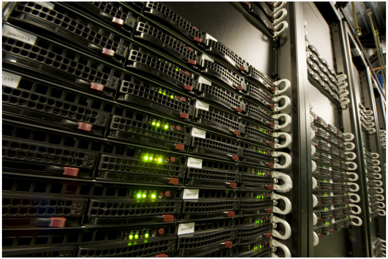
Lattice QCD

discretised & compactified

$$S = \sum_{\Box} \frac{1}{N} \Re \, \mathrm{tr} \left(\mathbf{1} - U_{\Box} \right)$$



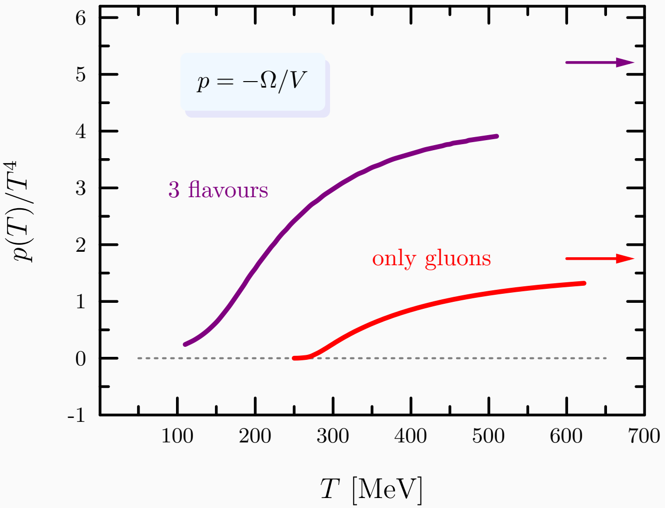
partition function $Z = \int \mathcal{D} A^{\mu} \mathcal{D} \bar{\psi} \mathcal{D} \psi \exp \left\{ -S[A^{\mu}, \bar{\psi}, \psi] \right\}$



$N_t \times N_s^3$ grid

continuum limit: $a \to 0$

thermodynamic : $V \to \infty$



(phase) transition

[Borsányi, *et al* (2012, 2014)]

How strong is strong?

"zeroth" order approximation

$n_f = 0$
(no fermions)

Harmonic Osc. $\mathcal{H} = \sum_{\boldsymbol{k},i} \left(n_i(\boldsymbol{k}) + 1/2\right)\omega_i(\boldsymbol{k})$ i = (pols, colour ...)

T.D. limit: $\sum_{\boldsymbol{k},i} \to d_g \cdot V \int \frac{\mathrm{d}^3 k}{(2\pi)^3}$

$d_{\text{gluons}} = 2 \times 8$

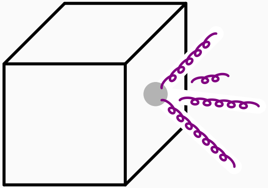
---

pressure: $p_0(T) = d_g \frac{\pi^2}{90} T^4$

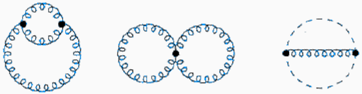
entropy density: $s(T) = \frac{\partial p}{\partial T} \sim T^3$

energy density: $\varepsilon(T) = -p + sT$

(Stefan-Boltzmann law)



LO: $\frac{p}{T^4} = \frac{p_0}{T^4}\left(1 - \frac{15}{4\pi}\alpha\right) + \mathcal{O}(\alpha^2?)$

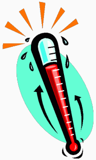


STRONG interactions $\qquad w(T) = T^5 \frac{\partial(p/T^4)}{\partial T} = T\frac{\partial p}{\partial T} - 4p$

BUT: QCD breaks scale inv. $\alpha_{\rm fix} \to \alpha(Q) = \frac{4\pi/11}{\log(Q^2/\Lambda^2)}$

[renorm] need running for $w \neq 0$!

guess scale dep. $\alpha(Q) \quad \ldots \quad Q \simeq 2\pi T$

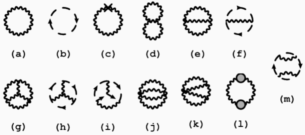


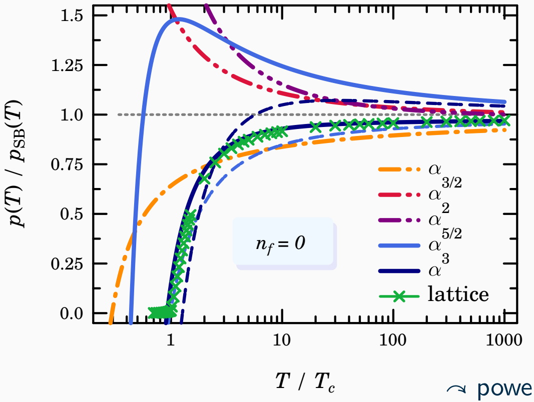
$\Rightarrow$ @LO $\quad w \sim \frac{\partial \alpha}{\partial T} \sim \alpha^2(T)\,T^4 \quad = 0$ **(formally)**

Ex: **pressure** (= thermodynamic potential)

[Kajantie, *et al* (2003)]

$p(T) =$





$\curvearrowright$ powers of $\alpha^{1/2}$ and $\log(\alpha)$?!

What is 'measured'??

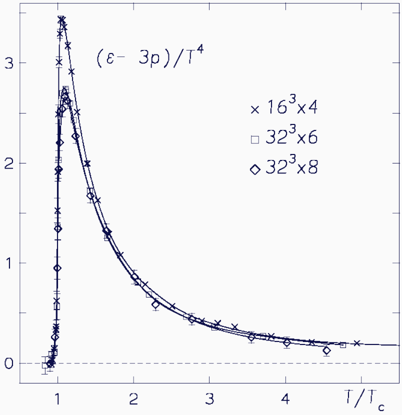
$$\frac{w}{T^4} \quad := \quad \frac{\varepsilon - 3p}{T^4}$$

vacuum ∞ cancelled :-)

$$T^{\alpha\beta} \quad = \quad \begin{pmatrix} \varepsilon & & & \\ & p & & \\ & & p & \\ & & & p \end{pmatrix}$$

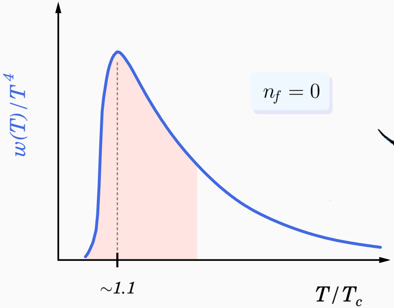
Trace *anomaly* $T^{\mu}_{\ \mu} = \varepsilon - 3p \stackrel{?}{=} 0$

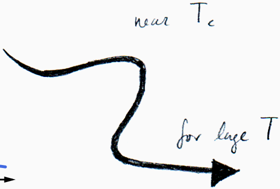
[Boyd *et al* (1996)]

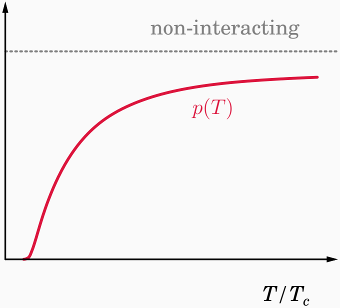


conformal (function of VT^3) ... ⇒ TRUE FOR INTERACTING SYSTEMS!

the integral method





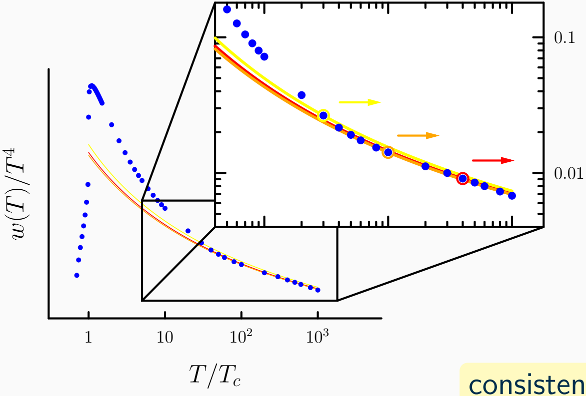


$$\frac{p}{T^4} = \frac{p(T_0)}{T_0^4} + \int^{T} \frac{\mathrm{d}\tau}{\tau} \cdot \frac{w(\tau)}{\tau^4}$$

scheme: $\mathcal{O}(\alpha^{5/2})$ pressure

2-loop $\alpha(2\pi T)$

[pQCD does not converge
... can still be useful!]



ADJUST parameter:

$\lambda = \Lambda / T_c$

match w @ $T^\star$ ...

(instead of n-pnt function)

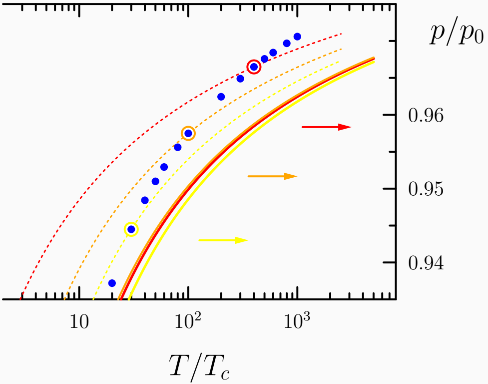
consistent results for *large* T

VERIFY model:

$\lambda = \Lambda / T_c$

compare to
another observable

slower approach to
free limit



Same applies for entropy & energy density!

Summary

perturbing analysis of lattice QCD trace anomaly

BUT: who cares about a 1% effect?

- equation of state, $\varepsilon = \varepsilon(p)$, crucial input for hydro
- large-T is where theory should be under control!
- error in $p(T)/p_0 \Rightarrow$ large differences in T/T_c

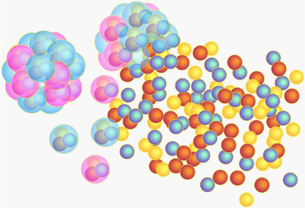
→ BACKUP SLIDES

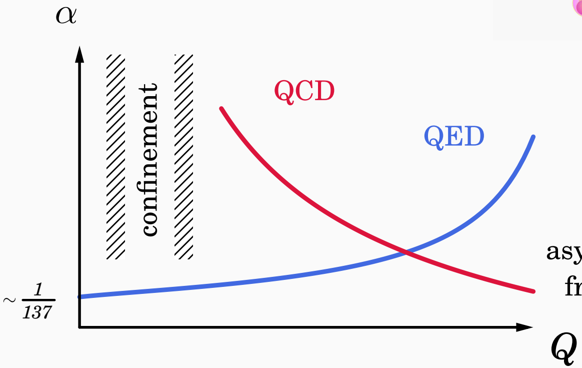
non-Abelian features

hadronic bound states (mesons/baryons)

below $T_c \approx 200$ MeV

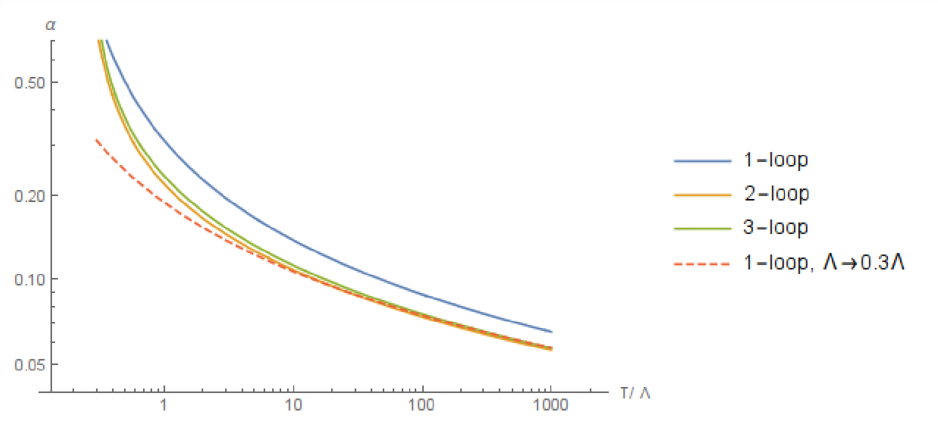
↓ (mostly pions)

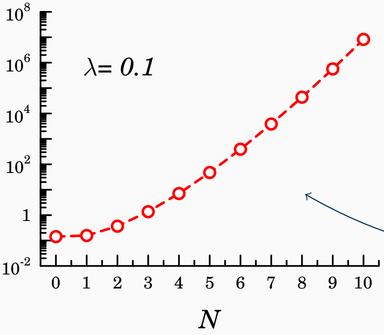




primordial 'soup'
of quarks & gluons

asymptotic
freedom


α
0.50
0.20
0.10
0.05
1
10
100
1000
T/ Λ
1-loop
2-loop
3-loop
1-loop, Λ→0.3Λ



ASYMPTOTIC *series* for $Z(\lambda)$:

$$\forall N \in \mathbb{N}, \quad \lim_{\lambda \to 0^+} \frac{1}{\lambda^N} \underbrace{\left| Z - \sum_{n=0}^{N} c_n \lambda^n \right|} = 0$$

Stirling approx. $\curvearrowright$ $c_n \approx \exp[n \log n]$ FAST GROWTH!

ratio test $\Rightarrow$ for *large* coupling, truncate at low order: $N^\star \sim \dfrac{1}{\lambda}$

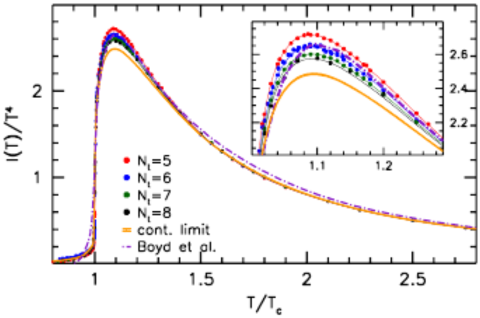


Figure 2. The trace anomaly on $N_s/N_t = 8$ lattices for various lattice spacings in the transition region. The result of a combined spline fit for each lattice spacing, together with the continuum extrapolation is shown by the colored lines and the yellow band, respectively. For comparison results with the standard Wilson action [14] are also shown by the dashed-dotted line. The continuum estimate of [14] in the inset has the same peak height as our $N_t = 6$ curve, which is about 7% higher than our continuum value.

[Borsányi, *et al* (2012)]

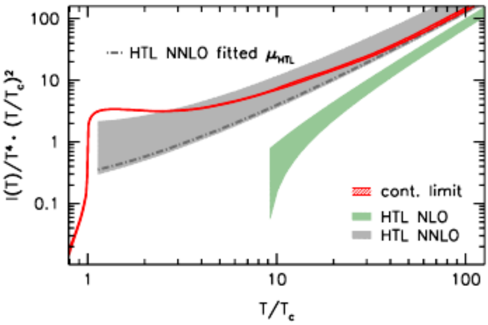


Figure 10. The trace anomaly in the continuum limit, compared to the NLO and NNLO HTL expansion with varied renormalization scale $0.5 < \mu_{\rm HTL}/2\pi T < 2$ (green and gray shaded regions). The dashed-dotted line represents the NNLO expansion with the fitted scale (see text).

[Borsányi, *et al* (2012)]