Physical models: A crucial link between reality and mathematical models

M Lemmer[1] and R Gunstone[2]

[1] School of Physical and Chemical Sciences, North-West University, Potchefstroom, South Africa
[2] Faculty of Education, Monash University, Clayton, Australia

miriam.lemmer@nwu.ac.za

Abstract. Physics describes real-life phenomena with the aid of models; mathematical modelling is a prime goal of physicists. All models, even abstract mathematical models, are embedded in real life experiences and Physics students should learn to look at the world through this lens and to handle modelling cycles with ease. Major processes of a modelling cycle are mathematical modelling of a physical system followed by mathematical processing from which the outcome is interpreted and validated in the physical system. It is argued here that it is crucial to pay attention during physics instruction to understanding of physical models (that incorporate physical systems) as an initial phase in the modelling process. Physical models involve simplifications of real life situations and the assumptions, features and limitations of physical systems; conceptual understanding of physics concepts, relations, basic principles, laws and theories and the ability to translate between various representations thereof as well as application of scientific causal, proportional and analogical reasoning. Research-based problems that students encounter when physics tuition commences with mathematical models or when these are directly build onto real life situations without sufficient attention to physical models are discussed. Teaching strategies to circumvent these problems are proposed.

1. Introduction

The central place of models and modelling in the evolution of the discipline of physics (and of science more generally) has been widely recognized by physicists. For example [1], writing for the International Commission on Physics Education, notes that “One of the main goals of physics is to develop plausible conceptual models, as they are called, in terms of which various physical phenomena can be described and explained” (p.14). The centrality of models and modelling has also been very commonly described by historians of physics (and science) (e.g. [2]), scholars of the philosophy of physics (and science) (e.g. [3]), and scholars of the nature of physics/science (e.g. [4]).

In physics, a “model” can be a physical and/or a mathematical and/or conceptual representation of a system of ideas, events or processes. Models are critical for the ways in which physicists seek to identify and understand patterns in our world. Models which also enable predictions are of greater epistemological value, and those that enable precise (mathematical) predictions are, in most areas of physics, the most highly valued (see [3] among many examples). In this paper, we use the term “modelling” to describe the constructing of and/or the using of appropriate models.

Our core purposes in this paper are about the learning of physics, and the ways models and modelling might be better considered in the development of student understanding. In such learning contexts, it can often be helpful to categorize models as “mental” models (that is the ways individuals represent complex ideas, events or processes in their thinking about these); “expressed models” (that is an explicit statement by an individual via word, speech, diagram etc of their mental model); and “consensus models” (that is an expressed model that has been subject to testing by physicists and consensus reached that the model has merit [5] because of its fit with data, its congruence with explanations of related phenomena, its transferability, and its power to enable questions, predictions and experiments [6]).

Central to our core purposes is the discrimination between “reality”, “physical models”, and “mathematical models”, and the ways these are relevant to learners’ development of the concepts and relationships of physics. Our use of these terms is quite conventional. Nevertheless it is appropriate to be clear as to this use. We also illustrate this use by reference to a specific example across the three.

Here “reality” is used to refer to the direct experiences that learners have with their world, relevant ways in which learners interact with their world (and so, “reality” will be variable across any given group of learners). In the broad area of Newtonian mechanics then, “reality” will include the alternative conceptions (sometimes labelled intuitive conceptions or, unfortunately, misconceptions) that a leaner has constructed through the everyday ways they have moved objects with forces they apply or seen others move objects, and whatever specific experiences have been provided in science/physics laboratories during their formal education. Alternatively, we could have described our meaning for “reality” as being the mental models held by a student as they enter our classroom. To emphasize this point, we use the term “real world” rather than “reality” from this point in the paper. We use “physical model” to refer to the core explanatory (conceptual) framework that physics has developed for a group of observations, phenomena, events (in the terms used above, the current “consensus” model). For the broad area of Newtonian mechanics, this can be expressed as “if the motion of an object changes [accelerates], then there must be a resultant force acting on the object”. A “mathematical model” then is the precise quantification (mathematization) of the physical model – in the case of our continuing example of Newtonian mechanics, “$\mathbf{F} = m\mathbf{a}$”.

Our motivations for writing this paper are twofold:

1) To argue the central place of models in physics, and therefore in physics learning
2) To describe and justify an ongoing research programme focussed on models and physics learning.

In a manner consistent with the northern European construct of *didactics* (see for example [7]), this research programme takes two significantly interrelated beginning points – the discipline of physics and the learner of physics.

2. Theoretical framework

Models and modelling are central in the discipline of physics and consequently should also be in the learning and teaching of physics (e.g. [8, 9]). Models describe key characteristics of observed phenomena, events or processes by using scientific representations in order to make explanations and testable predictions. Essential components of a model are the target phenomenon or system, and assumptions and simplifications used to focus on relevant features and representations that depict scientific concepts, relations and principles in ways that create a model with explanatory and predictive power.

Modelling is the “dynamic process of constructing and using models” [9]. This is widely recognized in the content development of physics; it is also critical to students’ conceptual development in physics. [10] added to the elements of construction and deployment in the practice of modelling also the evaluation and revision thereof. They further emphasized that students should understanding the nature and purpose of models that guide and motivate the practice of physics.

Since [8] advocated modelling as instructional method it has developed into an efficient approach towards meaningful science learning in which students’ existing mental models are re-constructed systematically and intentionally towards the consensus models of the scientific theory (mathematized scientific models) [6].

Working from constructivist and socio-cultural theories, [6] derived six criteria for pedagogical usefulness of teaching models: The models must be intelligible, plausible and fruitful to students;

contain meaningful causal mechanisms; bring to the fore and address students' conceptual difficulties; engage students effectively; advance students' understanding of consensus models and also the nature, purpose, assumptions and limitations of all models. Conceptual refinement instructional approaches can guide students in refining and advancing their experiential resources towards a conceptual understanding of generalized physics principles and laws before formalising it as mathematical expressions and representations ([11]).

Modelling cycles intended to promote students' understanding of consensus scientific models have been proposed by science education researchers (e.g. [12, 13]). Most modelling cycles distinguish between the real world, physical models and mathematical models and describe *translation processes* between them. Physical models are abstracted from real world situations through processes of *simplification* and structuring representations. Integration of mathematics knowledge aids in translating physical models into mathematical models, a process called *mathematization*. Conversely, mathematical models are *interpreted* in physical models and the results are *validated* in the real world. It is important to realise that modelling cycles are structured pedagogical tools that help advance students' understanding, but are not necessarily chronological and are not identical for each student in a given context of physics.

3. Differences between real world and physical models

Although authors of proposed modelling cycles recognize that scientific models are embedded in the Real World, these cycles tend to focus more on construction of mathematical models than the development of physical models from real world situations. In this section we argue that many of students' conceptual difficulties reported by physics education researchers (refer to [14] for examples) may result from differences in how learners perceive concepts, solve problems, explain events and apply reasoning in their real world as compared to the way physicists do these tasks when using physical models. A physical model can be perceived as an encoding of a target system that is embedded in a complex real world situation.

Other Physical models differ in various aspects from everyday observations and descriptions of situations in the real world, as we show in table 1. For example, physics concepts are uniquely defined, usually in relation to previously defined concepts, while concepts in everyday life are often perceived as contextually or functionally related. While physicists seek an underlying framework of principles and laws that explains various phenomena, students' intuitive explanations and reasoning depend on the situation or event. We now illustrate the ways beginning students derive concepts and intuitive explanations and reasoning – "alternative conceptions" – from their everyday experiences by considering the case of normal reaction.

4. The difficulties we know beginning physics students have with normal reaction

The term "normal reaction" itself leads some students to construct alternative conceptions. If the term "normal" is not explicitly linked with the mathematical concept of orthogonality, then a meaning can be constructed that there is somewhere an "abnormal" force. Unless the matter is explicitly considered with students, many will make most unfortunate links with Newton's Third Law through the term "reaction", and conclude that gravity and normal reaction for a book on a table are an action-reaction pair. (This incorrect construction is also made by some teachers, and even the occasional school physics textbook writer.) Other alternative conceptions likely to be found in beginning physics students are: in any system that a physicist would describe as in 'equilibrium' there are no forces of any form (crudely, 'no motion means no force'); the only force involved with a book on a table is gravity, the book remains at rest because the table is just "rigid" or "in the way"; when the book is placed on a rigid table nothing about the table changes so it makes no sense to even think about forces; gravity exerts the same force on everything (and so there is no mystery in the table being able to support either one book or many books seemingly without bending); gravity must be stronger than any upwards force or the book would float. Also a significant difficulty, although hardly an alternative conception, is a common tendency to <u>not</u> see the need to describe a force in terms of what object exerts the force and what object the force is exerted on (e.g. [15, 16, 17]).

Table 1. Some differences between real world and physical models.

	Real world	Physical model
Concepts and relations	Different meanings can be attached to a concept name. Concepts are related on basis of observable or functional correspondences.	Concepts are uniquely defined. Concepts are related mathematically to other concepts.
Problem solving	Focus on what seems to be relevant Contextual features of the problem setting play a role, sometimes only these.	Consider concepts, principles and representational formats of a specific physical system (model of reality).
Explanations of events or phenomena	Social and cultural views and situational aspects are taken into account.	Scientific principles, theories or laws are used; these are independent of situational aspects, including social and cultural views.
Findings/Results	Findings may depend on the situation or context	Results are required to be repeatable, valid and reliable and independent of context.
Representations	Realistic diagrams of phenomena or events (i.e. reproductions of the reality).	There are multiple scientific acceptable representations of events, e.g. diagrams, graphs, mathematical expressions, etc.
Reasoning	Intuitive cause-effect and analogical reasoning.	All of causal, proportional, analogical, mathematical reasoning are used.

5. A modelling framework to discuss, explore and explain physics students' difficulties in understanding consensus models

Because of the difficulties that students experience in translating between real world situations and physical models we argue that physical models provide an important connection between students' mental models derived from real world experiences and the mathematical models that are the endpoint learning goals of these students. A modelling cycle is suggested in figure 1 followed by an example of implementation in a sequence on normal reaction.

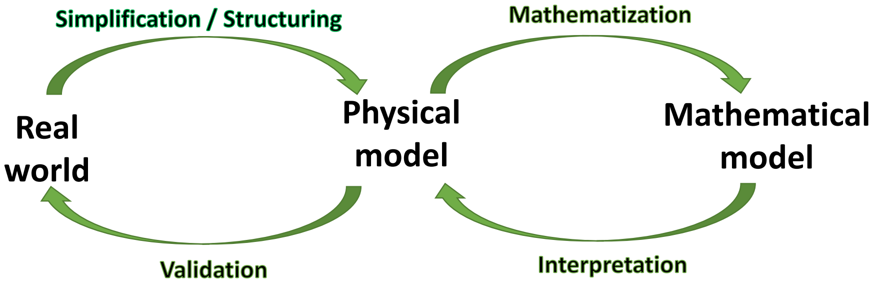

Figure 1. Proposed modelling cycle.

This modelling cycle is derived from literature described in the theoretical framework above. For example, the cycle of [12] involves transfer between the physical and mathematical models. [13] argue that this transfer should be perceived as a continuum. We include the transfer between the real world and physical models on an equal footing to the transfer from physical to mathematical models and perceive this first cycle also as a continuum. The processes that connect the real world with physical models (simplification / structuring a physical model from the real world, validation of findings from the physical model in real world situations) are profoundly neglected in physics instruction, resulting in learning difficulties such as those on normal reaction discussed above.

6. An example of the application of the Modelling Framework to the development of student understanding

Our example is, again, the concept of normal reaction. The approach outlined here has been evolving over many years in the teaching of the second author, and of a high school science teacher. Its early forms derived much from the work of [16]. Versions of the sequence close to its current form exist, as used with BSc graduates with biology majors preparing to teach integrated science to Grade 10 ([17]) and with Grade 10 science classes ([15]). The sequence has also been used in undergraduate physics. Other research has shown the same broad features of student difficulties before undergraduate instruction in this topic (e.g. [18]).

The beginning point is a book on a horizontal table and the question "why doesn't the book fall?". This Real world situation is placed in front of the students, and throughout the following discussion is considered only in terms of book and table. That is, the whole system of book/table/floor/ building/ earth is simplified to the limited system of book/table. As a further simplification, only this situation is considered, and the vector nature of force is only addressed via terms "up", "down", "push", "pull".

A range of related real situations is used to seek to reveal the ways students already make sense of their real world, and to challenge these ideas. These include: a volunteer has one arm horizontal, one book is placed on the hand (figure 2 left), the sensory consequence described and class explanations briefly discussed, then several books are placed on the hand (figure 2 right); similar use of one book and several books suspended from a strong rubber band; volunteers of obviously different weight successively sit on the same chair; a metre ruler is supported at each end by bricks and increasing numbers of books placed in the middle (figure 3). This latter situation is particularly powerful as a direct illustration of what physically happens to a (rigid) table when a book is placed on it.

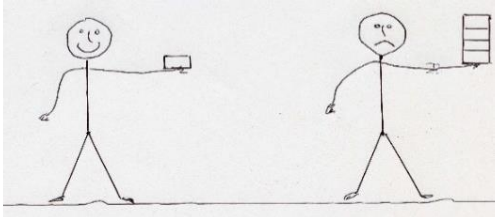

Figure 2. Sensory experience of holding one and more books on the hand.

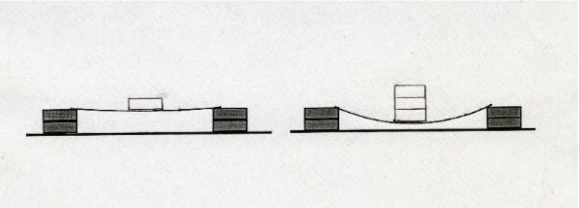

Figure 3. Illustration of the effect of an increasing number of books on a metre rule.

This set of experiences is used to lead to the Physical Model of a system of an object placed on another object and remaining stationary while being acted on by gravity: distortion resulting in a force opposite to the gravitational force. It is noted that the initial case of the book on the table is as yet not explained (there is as yet no evidence of distortion of that table). Discussion quite quickly leads to most students accepting that if one could actually show distortion in the table, then this situation too is explained by the same Physical Model. That demonstration is surprisingly easy for many tables (so one chooses the initial table with some care!). Unless the table has a metal frame or other stabilising feature, the distortion is usually shown clearly by (i) placing a mirror in the middle of the table, (ii) shining a bright beam of light (e.g. slide projector) onto the mirror at an angle, so a reflection appears on a side wall, and (iii) leaning on the table and showing the image of the light beam being displaced. It is very useful to explore with students how they predict the beam would move when the table is lent on at a specific point – this predicting makes engagement with the observation stronger and more cognitively meaningful.

In this example, the more difficult matter in developing student understanding is helping students simplify and structure as they move from their Real World to the Physical Model. The further shift to the Mathematical Model is relatively much easier. Indeed, aspects of the initial experiences described above also show that increased gravity force results in increased force in the opposite direction in order to continue the stationary state. The more precise quantification that is the final Mathematical Model is relatively clear by using rubber bands or springs where there is a linear relationship between extensions and upwards force. The completion of the sequence is to work back to the Real World and to explain the initial experiences in terms of both the Physical Model and the Mathematical Model.

7. Conclusions and implications for physics instruction

Ignoring physical models and trying to connect mathematical models directly to real life problems often results in students' alternative conceptions. To a considerable extent this is because no attempts are made to link/bridge/reconcile the Reality of the student that is brought to the study of physics with the Physical Model that is central to the mathematical model the physicist has developed.

Students should obtain first-hand experience with analysing differences between real world, physical and mathematical models when doing experiments and solving problems. Physics instructors should carefully introduce, motivate and explain the construction and use of models to their students, constantly revise and refine their understanding until the students are encultured in physics.

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