Thermal Models to High Energy Collisions

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Aim of this talk

- To present the influence of the Hagedorn spectrum on the hadronic yields, find the thermodynamic parameters.
- To discuss the nonextensive statistical distribution and its thermodynamic consistency.
- To present the Tsallis distribution fits of the transverse momentum distributions for p-p collisions together with estimates of the parameter q, temperature T and radius R.

Outline

- Introduction: The spatial evolution of heavy ion collision.

  $\rightarrow$ The idea of Hagedorn spectrum.

- The **H**adron **R**esonance **G**as **M**odel (**HRGM**) and its **E**xtension (**EHRGM**)

  $\rightarrow$ State the expressions for number, energy and entropy density and speed of sound for both models

- Show the results of **HRGM** and **EHRGM**

  $\rightarrow$ Discuss the results for particular thermodynamic quantities

- Introduction to nonextensive (Tsallis) statistical distribution.

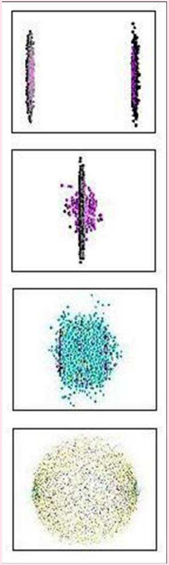
  $\rightarrow$ Nonextensive thermodynamics consistency

- Transverse Momentum Spectra - Tsallis distribution.

  $\rightarrow$ Results for p-p collision at 900 GeV and 7 TeV

- Summary and Conclusion.

Introduction: The Spatial evolution of a heavy ion collision



- Lorentz-contracted heavy ions approaching ...
  - Relativistic speeds cause the ions to appear disk-like
- Ions interpenetrate, individual particles scatter
- Deconfined quarks and gluons, plasma forms
  - Very short-lived, so not observable
- Formation of hadrons
  - Observable particles, analysis of these reveals information about QGP

Introduction: The Hagedorn spectrum

In 1965 Hagedorn [1] postulated that for large masses m the spectrum of hadrons grows exponentially, $\rho_H(m) \sim \exp(m/T_H)$.

The hypothesis was based on the observation increase of energy in collisions no longer raises the temperature of the formed fireball, but results in more and more particles being produced.

There exists uncertainty as to the value of the Hagedorn temperature, T_H which have two origins:

- Sparse information about hadronic resonances above 3 GeV and,
- The analytical form of the Hagedorn spectrum.

Recently, Hagedorn spectrum is rewritten as

$$\rho_H(m) = \frac{c}{(m^2 + m_0^2)^{5/4}} \exp\left(\frac{m}{T_H}\right). \tag{1}$$

This model uses $m_0 = 0.5$ GeV, it is adopted from *S. Chatterjee et. el. Phys.Rev.C81:044907*.

1) R. Hagedorn, Nuovo Cim. Suppl., 3:147186, 1965.

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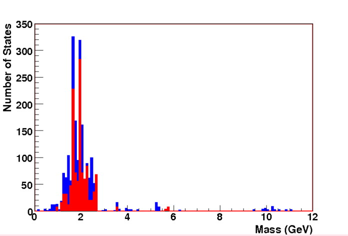
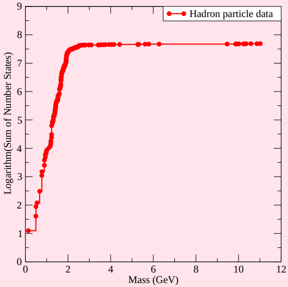
The number of states for particle data table arranged in terms of their masses with some hadron gas resonances. In figure below to the left side (blue: mesons and red: baryon).





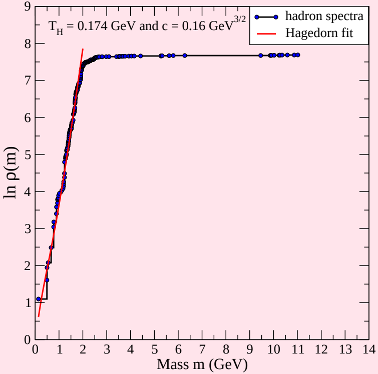
The spectrum of hadrons up to higher masses is given by,

$$\rho(m) = \sum_i g_i \delta(m - m_i), \tag{2}$$

where g_i is the degeneracy factor for hadron state i.

The best fit values for c and T_H parameters

The result of the parameters using Eq. 1 for the Hagedorn spectrum state given below:



$T_H = 0.174 \pm 0.011$ GeV and $c = 0.16 \pm 0.02$ GeV$^{3/2}$ (*J. Cleymans and D. Worku Mod.Phys.Lett.A26:11971209, 2011*).

The Hadron Resonance Gas Model (**HRGM**)

The thermodynamic properties of the **HRGM** can be determined from the partition function

$$\begin{aligned} \ln Z(V,T,\mu) &= \int dm \, [\rho_M(m) \ln Z_b(m,V,T,\mu) \\ &+ \rho_B(m) \ln Z_f(m,V,T,\mu)] \,, \end{aligned} \tag{3}$$

where Z_b and Z_f are the partition functions for an ideal gas of bosons and fermions respectively with mass m, $\rho_M(m)$ and $\rho_B(m)$ are the spectral density of mesons and baryons.

The **HRGM** model takes the observed spectrum of hadrons up to some cutoff mass 2 GeV.

Extension of Hadron Resonance Gas Model (**EHRGM**)

In order to explore the stability of predictions from **HRGM**, we develop an **EHRGM** in which:

$$\rho(m) = \sum_{i}^{m_i \leq 2GeV} g_i \delta(m - m_i) + \rho_H(m), \tag{4}$$

where ρ_H is the Hagedorn spectrum which is given in Eq. 1.

Expression of thermodynamic quantities in **EHRGM**

Using HRGM, one can compute the thermodynamic quantities. We consider Boltzmann distribution for our calculation

$$\ln Z = \sum_{i=1}^{m_i \leq 2GeV} \frac{g_i V T m_i^2}{2\pi^2} K_2\left(\frac{m_i}{T}\right) \exp\left(\frac{\mu_i}{T}\right), \tag{5}$$

where K_2 is modified Bessel function and $\mu_i = S_i \mu_S + B_i \mu_B + Q_i \mu_Q$.

The particle number density, n using EHRGM is written as

$$\begin{aligned} n(T, \mu_B, \mu_S, \mu_Q) &= \frac{T}{2\pi^2} \sum_{i=1}^{m_i \leq 2GeV} \exp\left(\frac{\mu_i}{T}\right) \left[g_i m_i^2 K_2\left(\frac{m_i}{T}\right) \right. + \\ & \left. c \int_{m=2GeV}^{\infty} \frac{m^2}{(m^2 + m_0^2)^{5/4}} \exp\left(\frac{m}{T_H}\right) K_2\left(\frac{m}{T}\right) dm \right], \end{aligned} \tag{6}$$

Very often, it is considered to an isospin symmetric system, where μ_Q is zero.

The energy density

The energy density using EHRGM is given by

$$\varepsilon(T, \mu_B, \mu_S, \mu_Q) = \sum_{i=1}^{m_i \leq 2GeV} \left[\frac{T^2}{2\pi^2} \exp\left(\frac{\mu_i}{T}\right) \left(g_i m_i^2 \left[3K_2\left(\frac{m_i}{T}\right) + \frac{m_i}{T} K_1\left(\frac{m_i}{T}\right) \right] + A_1 \right) \right], \quad (7)$$

where

$$A_1 = c \int_m^\infty \frac{m^2}{(m^2 + m_0^2)^{5/4}} \exp\left(\frac{m}{T_H}\right) \left[3K_2\left(\frac{m}{T}\right) + \frac{m}{T} K_1\left(\frac{m}{T}\right) \right] dm.$$

The speed of sound

In hydrodynamic models, the speed of sound plays an important role in the evolution of a system and is an ingredient in the understanding of the effects of a phase transition.

In our extension we take the L.D. Landau condition, where s/n_B (particle number is not conserved but the ratio of entropy per baryon density should be constant) is fixed in order to define,

$$C_s^2(T,\mu_B,\mu_S) = \left(\frac{\partial P}{\partial \varepsilon}\right)_{s/n_B}, \tag{8}$$

hence C_s^2 can be rewritten as

$$C_s^2(T,\mu_B,\mu_S) = \frac{\left(\frac{\partial P}{\partial T}\right) + \left(\frac{\partial P}{\partial \mu_S}\right)\left(\frac{d\mu_S}{dT}\right) + \left(\frac{\partial P}{\partial \mu_B}\right)\left(\frac{d\mu_B}{dT}\right)}{\left(\frac{\partial \varepsilon}{\partial T}\right) + \left(\frac{\partial \varepsilon}{\partial \mu_S}\right)\left(\frac{d\mu_S}{dT}\right) + \left(\frac{\partial \varepsilon}{\partial \mu_B}\right)\left(\frac{d\mu_B}{dT}\right)}, \tag{9}$$

The first condition comes from keeping the ratio (s/n_B) constant.

$$d\left(\frac{s}{n_B}\right) = 0 \quad \rightarrow \quad n_B ds = s dn_B. \tag{10}$$

Rearranging the above equation in order to write $\frac{d\mu_B}{dT}$ in terms of $\frac{d\mu_S}{dT}$ one obtains

$$\frac{d\mu_B}{dT} = -\frac{1}{B}\left[A + C\frac{d\mu_S}{dT}\right]. \tag{11}$$

The second condition comes from overall strangeness neutrality, which is

$$n_S = n_{\bar{S}} \quad \rightarrow \quad d(n_S) = d(n_{\bar{S}}) \tag{12}$$

where n_S and $n_{\bar{S}}$ are the strange and antistrange particle densities. The final expression for condition two become

$$\frac{d\mu_B}{dT} = -\frac{1}{E}\left[D + F\frac{d\mu_S}{dT}\right]. \tag{13}$$

Finally, by equating Eq. (11) and Eq. (13) we find

$$\frac{d\mu_S}{dT} = \frac{A*E - B*D}{B*G - C*E} \text{ and } \frac{d\mu_B}{dT} = \frac{C*D - A*G}{B*G - C*E}, \tag{14}$$

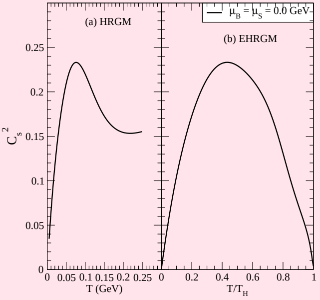
Cont'd

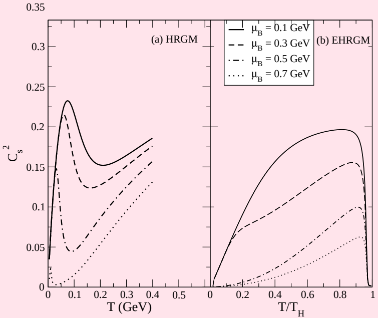
where

$A = n\frac{\partial s}{\partial T} - s\frac{\partial n}{\partial T}$, $B = n\frac{\partial s}{\partial \mu_B} - s\frac{\partial n}{\partial \mu_B}$, $C = n\frac{\partial s}{\partial \mu_S} - s\frac{\partial n}{\partial \mu_S}$, $D = \frac{\partial L}{\partial T} - \frac{\partial R}{\partial T}$,
$E = \frac{\partial L}{\partial \mu_B} - \frac{\partial R}{\partial \mu_B}$, $F = \frac{\partial L}{\partial \mu_S} - \frac{\partial R}{\partial \mu_S}$.

We define $L = n_S^B + n_S^M$ and $R = n_S^{\bar{B}} + n_S^{\bar{M}}$, which represents the strangeness and antistrangeness density for baryons and mesons.

HRGM and **EHRGM**: The speed of sound versus temperature

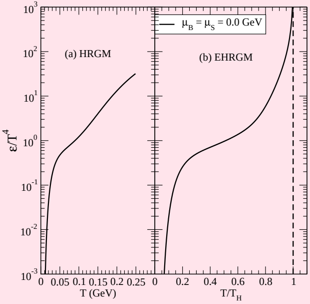


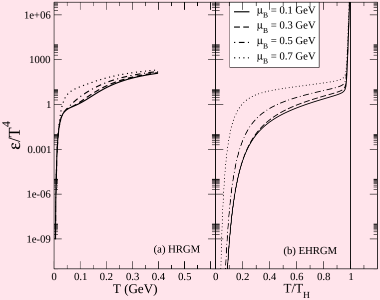


→ The value of the squared speed of sound, C_s^2 remains well below the ideal-gas limit for massless particles $C_s^2 = 1/3$.

→ It showed that sharp dip of C_s^2 in the critical region and can be considered as an evidence for the phase transition in the system.

HRGM and **EHRGM**: Energy density versus temperature

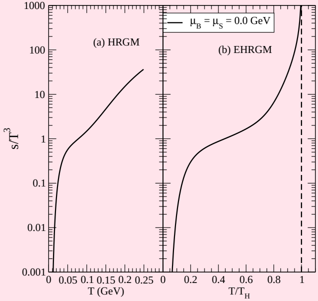


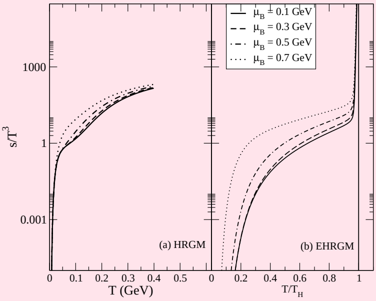


→ The result using **EHRGM** as a function of the temperature show a sudden change and start to increase rapidly at a particular temperature, (i.e. T_H).

→ In a similar way, in *Phys.Rev.C81:044907 and Eur.Phys.J.C66:207-213* presented with few hadron resonance gases and the shape of graphs shown are similar to our results.

HRGM and **EHRGM**: The entropy versus temperature





We observe that around $T/T_H \simeq 0.8$, resonances come significantly into play, so that ε and s begin to increase until reach to T_H in these case the resonances provide the dominant part for the those thermodynamic quantities.

Introduction to Nonextensive Statistical Distribution

- The generalization that concerned in non-extensive thermostatistics, which recovers the extensive, Boltzmann-Gibbs (BG) one as particular case, was proposed in 1988 by *C. Tsallis. J. Stat. Phys. 52:479, 1988*.
- There has been a growing tendency to use the non-extensive statistical formalism. From 1988 up to the present, numerous theories of statistical thermodynamics have been presented in the framework of the Tsallis formalism.
- The generalized statistical mechanics based on the Tsallis distribution, termed as nonextensive statistical distribution, is now under active investigation.

Nonextensive Statistical Distribution

The postulated form of the generalized entropy is:

$$S_q \equiv \frac{1 - \sum_{i=1}^{W} P_i^q}{q - 1}, \ (q \in \Re; \ \sum_{i=1}^{W} P_i = 1), \tag{15}$$

or, for continuum distributions

$$S_q \equiv \frac{1 - \mathtt{Tr}\rho^q}{q - 1}, \tag{16}$$

where q is a positive constant, P_i are the probabilities of microscopic states, and ρ the corresponding density operator. The parameter q is called non-extensivity parameter, entropic index or simply Tsallis parameter. It is a measure of the non-extensivity of the system of interest.

The true nature of the nonextensive parameter q in Tsallis statistics has not been fully understood yet by us.

This new entropy formula recovers the usual BG entropy in the limit where q tends to 1. The reason that q is called the non-extensivity parameter is due to the following pseudo additivity rule; is given by

$$S_q(I+J) = S_q(I) + S_q(J) + (1-q)S_q(I)S_q(J), \tag{17}$$

where I and J are two independent systems in the sense that the system $I+J$ factorises into those of I and J.

Since S_q is greater or equal to 0 for all values of q, we can say that $q < 1$, $q = 1$ and $q > 1$ correspond to:

- superextensivity (entropy of the whole system is greater than the sum of its parts or superadditive),
- extensivity (entropy is additive) and
- subextensivity (entropy of the whole system is smaller than the sum of its parts or subadditivity) respectively.

Tsallis Distribution for Particle Multiplicities

- The standard Fermi-Dirac distribution is given by

$$n^{FD}(E) \equiv \frac{1}{1 + \exp\left(\frac{E-\mu}{T}\right)}. \quad (18)$$

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- The Tsallis form of Fermi-Dirac distribution proposed which uses

$$n_T^{FD}(E) \equiv \frac{1}{1 + \exp_q\left(\frac{E-\mu}{T}\right)}. \tag{19}$$

where the function $\exp_q(x)$, for $q > 1$ is defined as

$$\exp_q(x) \equiv \begin{cases} [1 + (q-1)x]^{1/(q-1)} & \text{if} \quad x > 0 \\ [1 + (1-q)x]^{1/(1-q)} & \text{if} \quad x \leq 0. \end{cases} \tag{20}$$

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- The extensive form of mathematical properties of $\exp_q(x)$ only occurred in the limit where $q \to 1$ reduces to the standard exponential:

$$\lim_{q\to 1} \exp_q(x) \to \exp(x).$$

Tsallis Distribution for Particle Multiplicities

- All forms of the Tsallis distribution introduce a new parameter q. In practice this parameter is always close to 1.
- The results obtained by the ALICE and CMS collaborations (*Eur. Phys. J., C71:1655, 2011 & JHEP, 05:064, 2011*) typical values for the parameter q can be obtained from fits to the transverse momentum distribution for identified charged particles are in the range 1.1 to 1.2.
- The measured spectra range in transverse momentum by STAR, PHENIX, ALICE and CMS collaborations for hadron particles a form that has been suggested is given by called Tsallis-Lévy distribution

$$\frac{1}{2\pi p_T}\frac{d^2N}{dydp_T} = \frac{dN}{dy}\frac{(n-1)(n-2)}{2\pi nC[nC+m_0(n-2)]} \times\left(1+\frac{m_T-m_0}{nC}\right)^{-n} \tag{21}$$

where T, p_0, n, $\frac{dN}{dy}$, C, m_T and m_0 are fitting parameters. Thus, No (direct) connection with Tsallis distribution.

- We will clarify the theoretical background for non-extensive statistics and discuss its thermodynamical consistency.

Thermodynamics Consisitency: Tsallis Distribution

- The explicit form which we use for the transverse momentum distribution in relativistic heavy ion collisions is:

$$\frac{d^2N}{dp_T\ dy} = gV\frac{p_T m_T \cosh y}{(2\pi)^2}\left[1+(q-1)\frac{m_T\cosh y-\mu}{T}\right]^{q/(1-q)}, \qquad (22)$$

where p_T, m_T, y, T, μ, V and $g=(2I+1)(2s+1)$ are: transverse momentum, transverse mass, rapidity, temperature, chemical potential, volume and spin-isospin degeneracy factor respectively.

- Hence, thermodynamic consistency requires that the following relations must be satisfied

$$T=\left.\frac{\partial\epsilon}{\partial s}\right|_n, \quad \mu=\left.\frac{\partial\epsilon}{\partial n}\right|_s, \quad n=\left.\frac{\partial P}{\partial\mu}\right|_T, \quad s=\left.\frac{\partial P}{\partial T}\right|_\mu. \qquad (23)$$

Tsallis Fit Details

- The total number of particles is given by the integral version

$$N = gV \int \frac{d^3p}{(2\pi)^3} \left[1 + (q-1)\frac{E-\mu}{T}\right]^{q/(1-q)} . \qquad (24)$$

- The extra power q is necessary for thermodynamic consistency. The corresponding (invariant) momentum distribution is given by

$$E\frac{dN}{d^3p} = gVE\frac{1}{(2\pi)^3} \left[1 + (q-1)\frac{E-\mu}{T}\right]^{q/(1-q)} , \qquad (25)$$

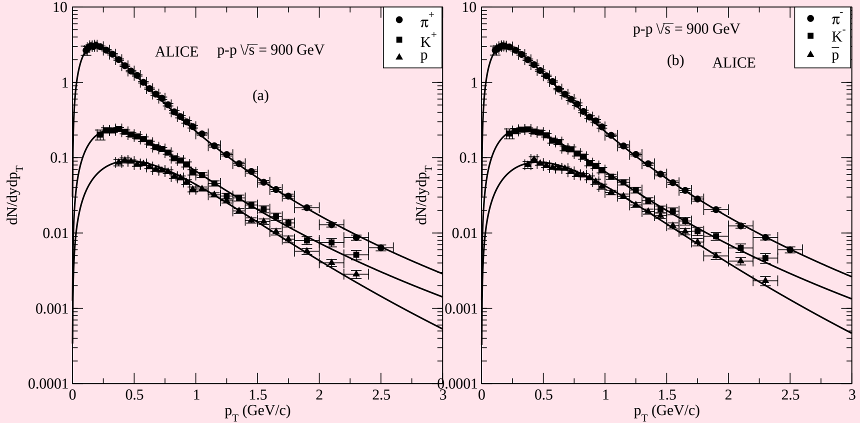
- In terms of the rapidity and transverse mass variables, at mid-rapidity, $y = 0$, reduces to the following expression

$$\left.\frac{dN}{p_T dp_T\ dy}\right|_{y=0} = gV\frac{m_T}{(2\pi)^2} \left[1 + (q-1)\frac{m_T}{T}\right]^{q/(1-q)} , \qquad (26)$$

- Comparison of Tsallis in Eq. (28) with STAR, ALICE, CMS distributions in Eq. (23)

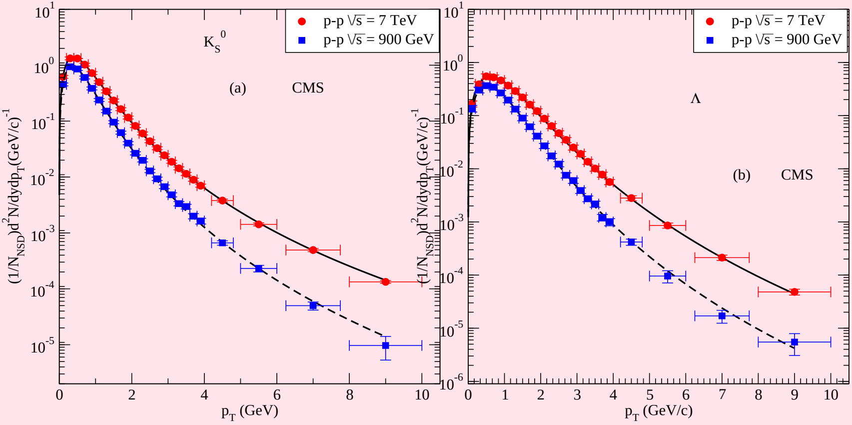
$$n \rightarrow \frac{q}{q-1} \quad \text{and} \quad nC \rightarrow \frac{T + m_0(q-1)}{q-1} . \qquad (27)$$

Transverse Momentum Spectra: Tsallis-Boltzmann fits



Transverse momentum distribution for π^+, K^+, p, π^-, K^- and $\bar{p}$ as measured by the ALICE collaboration in p-p collision and the Tsallis-B distribution.

Transverse Momentum spectra: Tsallis-Boltzmann fits



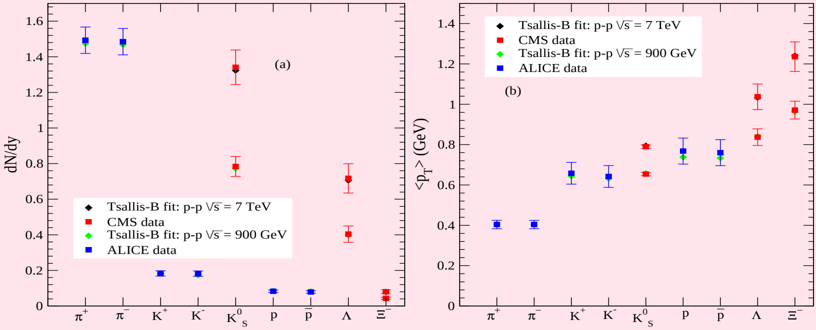
Comparison between the measured transverse momentum distribution for K_S^0 and Λ as measured by the CMS collaboration in p-p collisions and the Tsallis-Boltzmann distribution.

Thermal fit parameters (q, T and R)

Particle	q	T (GeV)	R (fm)
π^+ (0.9 TeV)	1.1661 ± 0.0041	0.0649 ± 0.0010	9.4945 ± 0.1184
π^- (0.9 TeV)	1.1620 ± 0.0031	0.0648 ± 0.0008	9.2841 ± 0.0844
K^+ (0.9 TeV)	1.1446 ± 0.0076	0.0776 ± 0.0044	5.8065 ± 0.3636
K^- (0.9 TeV)	1.1425 ± 0.0029	0.0769 ± 0.0013	5.9307 ± 0.1171
K_S^0(0.9 TeV)	1.1352 ± 0.0024	0.0919 ± 0.0015	7.5915 ± 0.1325
K_S^0(7 TeV)	1.1608 ± 0.0025	0.0986 ± 0.0017	7.4900 ± 0.3852
p (0.9 TeV)	1.0880 ± 0.0042	0.0949 ± 0.0025	7.3151 ± 0.3079
$\bar{p}$(0.9 TeV)	1.0961 ± 0.0177	0.0973 ± 0.0015	6.2010 ± 1.2324
Λ(0.9 TeV)	1.1060 ± 0.0045	0.0796 ± 0.0048	18.8639 ± 1.7883
Λ(7 TeV)	1.1401 ± 0.0042	0.0747 ± 0.0047	19.3363 ± 1.3169
Ξ^- (0.9 TeV)	1.0807 ± 0.0043	0.1262 ± 0.0037	4.9138 ± 0.2201
Ξ^- (7 TeV)	1.1188 ± 0.0029	0.1499 ± 0.0012	3.2533 ± 0.0158

The calculated values of the q, temperature, T and radius, R for different hadron species measured by the ALICE and CMS collaborations in p-p collisions at 900 GeV and 7 TeV using the Tsallis-B form for the momentum distribution.

Particle Yield, dN/dy and $\langle p_T \rangle$



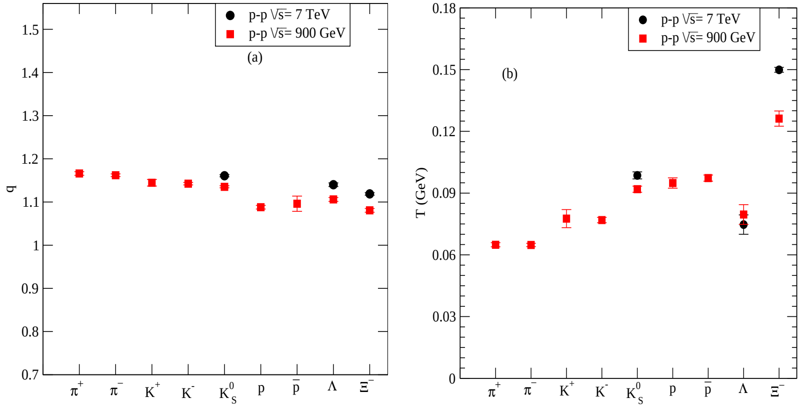
The particle yield at mid-rapidity, dN/dy and the average transverse momentum, $\langle p_T \rangle$ for different hadron species.

$$\left.\frac{dN}{dy}\right|_{y=0} = a \int_0^\infty dp_T m_T p_T \left[1 + (q-1)\frac{m_T}{T}\right]^{q/(1-q)},$$

and

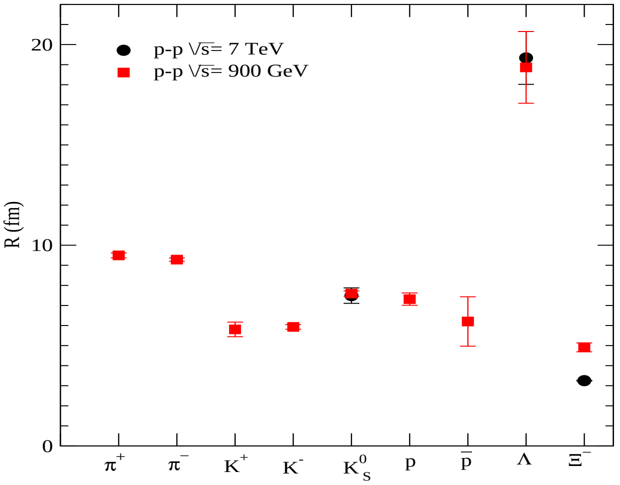
$$< p_T > = \frac{a \int_0^\infty dp_T m_T p_T^2 \left[1 + (q-1)\frac{m_T}{T}\right]^{q/(1-q)}}{\frac{dN}{dy}},$$

respectively, where a is the normalization constant.



The calculated values of q and T in p-p collisions at 7 TeV and 900 GeV results for different hadron species. The q parameter values are lying in the range between 1.1 to 1.2.

However, the temperatures increasing significantly. The resulting values of the temperature show a wider spread around 70 MeV.



The calculated value of the radius, R in p-p collisions at 900 GeV and 7 TeV, for different hadron species. J.Cleymans and D. Worku Eur. Phys. J. (2012) 48: 160.

Summary

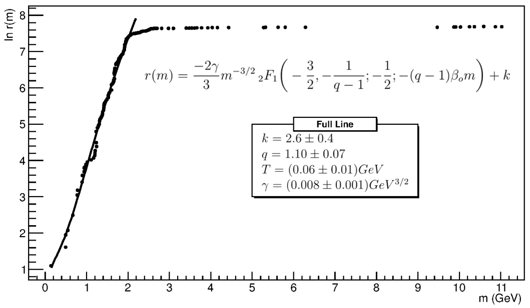
- We presented the **HRGM** to investigate the thermodynamic properties of hadrons, we further extended to the **EHRGM** by involving the Hagedorn spectrum.
- The Hagedorn temperature is determined from the number of hadronic resonances lead to a stable result which is consistent with the critical and the chemical freeze-out temperatures at zero chemical potential.
- We calculated C_s^2 and relevant thermodynamic quantities for a wide range of baryon chemical potentials following the chemical freeze-out curve.
- Tsallis Thermodynamics (TT) has also been successfully applied to some of the physical systems where BG formalism is known to fail, since these achievements can be thought of as verifications of TT.
- Tsallis distribution considered the thermodynamic consistency of the resulting distribution. We emphasized that an additional power of q is needed to achieve consistency with the laws of thermodynamics.
- Tsallis distribution has been analysed using the transverse momentum distribution to p-p collisions at 900 GeV and 7 TeV.

Conclusions

- The **EHRGM** results show unique behavior at the critical point, T_H.
- The thermodynamic quantities obtained using **EHRGM** start rising rapidly at a temperature of about T_H.
- For $T \leq T_H$ hadronic resonances are indeed the most important degrees of freedom in the confined phase.
- The result of C_s^2 can be considered as a sensitive indicator of critical behavior in strongly interacting matter.
- The resulting distribution, called Tsallis-B distribution, was compared with recent measurements from the ALICE and CMS collaborations at the LHC and good agreement was obtained.
- Based on the Tsallis-B distribution, we have determined from the fit; the estimates of the parameter q, the temperature T and the radius factor R that comes from the volume factor V.
- Hence, the resulting parameter q which is a measure for the deviation from a standard Boltzmann distribution was found to be between 1.1 and 1.2. The temperature show a wider spread around 70 MeV.

Future plans

- It will be very interesting to study further in future plan, to establish clearly the the combination of both Hagedorn spectrum and Tsallis distribution to form a new distribution that should apply in high energy physics.
- Recently, L. Marques, E. Andrade-II and A. Deppman (arXiv:1210.3456 [hep-ph]) discuss the cumulative of hadron-mass spectrum with nonextensive statistical distribution.



Acknowledgements

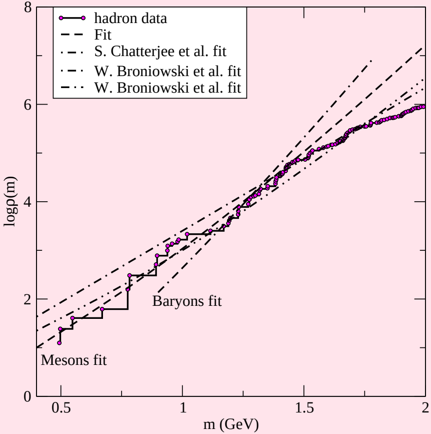
1. Department of Physics and Mathematics, Cape Peninsula University of Thechnology (CPUT).
2. The organizers of SAIP 2016
3. Em. Prof. J. Cleymans and Prof. A. Peshier, UCT and Prof. A. Muronga, UFS

Relevant References

For further explanations about this talk, please see the following references.

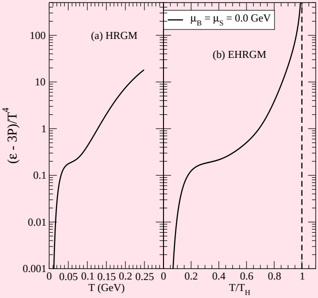
1. J. Cleymans and D. Worku, Mod.Phys.Lett.A26:1197-1209, 2011; *arXiv:1103.1463 [hep-ph]*.
2. J. Cleymans and D. Worku, *arXiv:1106.3405 [hep-ph] (2011)*.
3. J. Cleymans and D. Worku J. Phys. G39 (2012) 025006; *arXiv:hep-ph/1110.5526, 2011*.
4. J. Cleymans and D. Worku, Eur. Phys. J. A (2012) 48: 160; *arxiv:hep-ph/1203.4343, 2012*.

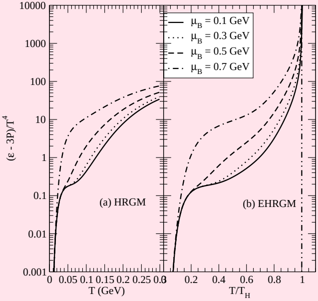
Backup Slides: The hadron spectrum fits of different equations



Cumulative number of hadronic resonances as a function of *m*. The hadron data are included up to 2.0 GeV, including baryons and mesons from the fits of different authors, our fit is also shown here (dashed line).

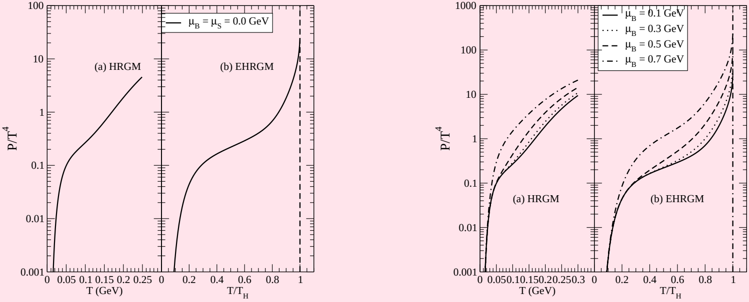
The interaction measure $\frac{(\varepsilon-3P)}{T^4}$ at various μ





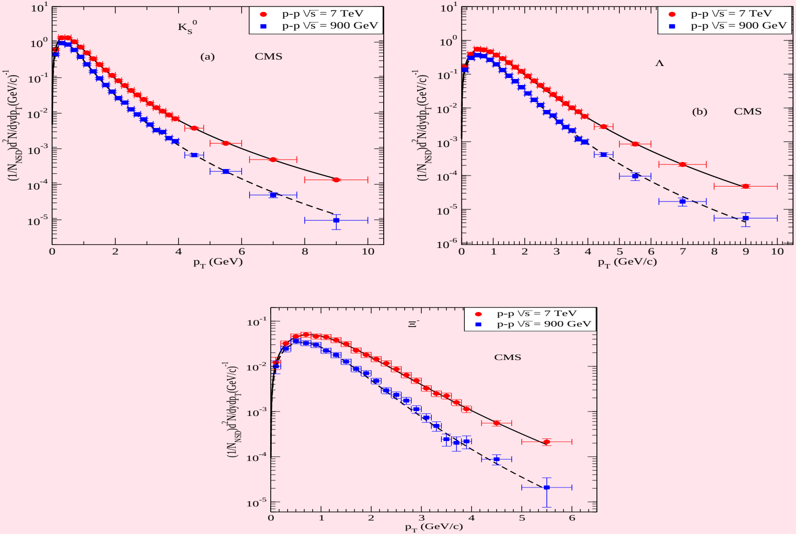
(a) The interaction measure, $(\varepsilon - 3P)/T^4$ in units of T^4 calculated using the **HRGM** as a function of the temperature T at $\mu_B = \mu_S = 0$ GeV. (b) The interaction measure, $(\varepsilon - 3P)/T^4$ in units of T^4 for **EHRGM** as a function of the temperature scaled by the Hagedorn temperature T_H.

HRGM and **EHRGM**: The pressure versus T at various μ



(a) The pressure, P, in units of T^4 calculated using the **HRGM** as a function of the temperature T at various μ_B. (b) The pressure in units of T^4 for **EHRGM** as a function of the temperature scaled by the Hagedorn temperature T_H.

Transverse Momentum Distribution: Tsallis-B fit



Transverse momentum distribution for K_S^0, Λ and Ξ^- as measured by the CMS collaboration in p-p collision and the Tsallis-B distribution.