Bilinear expansion of the photon quantum field and the emergence of classical mechanics from quantum field theory

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Abstract

A new quantum representation of the electro-magnetic field is introduced based on a bilinear expansion of the field in terms of quark creation and annihilation operators. This representation has definite advantages, leading automatically to the transversal nature of photons and eliminating the need for artificial gauge fixing. However, this representation seems unable to produce the correct expectation value of the energy of a photon. To cure this situation a new continuous quantum number is introduced, which entails new positional information of a classical nature. In standard applications of quantum field theory these degrees of freedom are hidden as they only lead to trivial phase factors. However, in many-body situations and when the state vectors are superpositions of momentum states, the phase factors can no longer be ignored and provide positional information about the particles. Hence, in situations typical of classical systems the "hidden" degrees of freedom emerge and are responsible for the classical positional information.

1. Bilinear quantum representation of the electromagnetic field

The free electro-magnetic (e.m.) field satisfies the free Maxwell equation:

$$-\partial_\mu \partial^\mu A^\nu(x) + \partial^\nu \left[\partial_\mu A^\mu(x)\right] = 0. \tag{1}$$

After imposing the Lorentz condition: $\partial_\mu A^\mu(x) = 0$ one arrives at the Klein-Gordon equations $-\partial_\mu \partial^\mu A^\nu(x) = 0$, which has the usual plane wave solutions e^{ikx} with $k_\mu k^\mu = 0$. The e.m. field is now quantized by expanding in terms of these classical solutions [1, p. 219], [2, p. 243]:

$$A_\mu(x) = (2\pi)^{-\frac{3}{2}} \int \frac{d^3k}{\sqrt{2k_0}} \sum_{\lambda=0}^{3} e_\mu^{(\lambda)}(\vec{k})[a_{(\lambda)}(\vec{k})\, e^{-ikx} + a_{(\lambda)}^\dagger(\vec{k})\, e^{ikx}], \tag{2}$$

where $k^\mu e_\mu^{(\lambda)}(\vec{k}) = 0$ and quantization is effected by imposing the commutation rules [2, p. 243]:

$$[\; a_{(\lambda)}(\vec{k}'), a_{(\lambda')}^\dagger(\vec{k}) \;] \propto \delta_{\lambda\lambda'} \delta^3(\vec{k}' - \vec{k}). \tag{3}$$

Various problems arise with this standard solution: (a) to derive the Klein-Gordon equation from the Lagrangian one needs to introduce a gauge fixing term in the Lagrangian. The resulting Lagrangian is no longer gauge invariant; (b) The quantum expansion requires 4 states, 2 of which are unphysical, as the physical photon has only two transverse degrees of freedom; (c) The unphysical components (time-like and longitudinal states) can be eliminated using techniques like the Gupta-Bleuler [3, 4] projection technique, but these methods are rather complex and unnatural; (d) it leads to an indefinite metric for the zeroth component of $e_\mu^{(\lambda)}(\vec{k})$; (e) this operator representation leads to an infinite vacuum energy, which can only be treated through heuristic means.

2. Electromagnetic field as a consequence of the existence of fermions

We propose an alternative representation of the electromagnetic field is possibly which gives more recognition to the original motivation for its introduction. The electromagnetic field is originally required to guarantee the invariance of fermion Lagrangians under local gauge transformations. $\psi(x) \to \exp i\alpha(x)\psi(x)$. Hence, instead of the original fermion Lagrangian one uses $L = i\bar{\psi}\gamma_\mu \partial^\mu \psi + eA^\mu \bar{\psi}\gamma_\mu \psi$, and subsequently adds the kinetic term $(1/4)F^{\mu\nu}F_{\mu\nu}$. The free field equation (1) for the e.m. field is then derived by ignoring the source term. However, a more basic quantum representation of this field is suggested by the original source term $e\bar{\psi}\gamma^\nu\psi$. If one expands the fermion fields ψ and $\bar{\psi}$ independently then one does not get the desired representation, as one ends up with a double integral representation , while the correct expansion, Eq.(2) only contains a single momentum integral. The solution is to collapse the momentum expansions of ψ and $\bar{\psi}$ to the same momentum variable, so that the fermion-anti-fermion state can be considered as a single elementary state. Using massless bare quarks as the basic fermion constituents, we get after imposing hermiticity:

$$\begin{aligned} A^\mu(x) &= C\sum_{\alpha,\beta}\int d\vec{p}\ \left[\left(\bar{\phi}_{\alpha\,\vec{p}}\gamma^\mu\phi_{\beta\,\vec{p}}\right)b^\dagger_{\alpha\,\vec{p}}b_{\beta\,\vec{p}} + \left(\bar{\phi}^a_{\alpha\,\vec{p}}\gamma^\mu\phi^a_{\beta\,\vec{p}}\right)d_{\alpha\,\vec{p}}d^\dagger_{\beta\,\vec{p}}\right] \\ &+ C'\sum_{\alpha,\beta}\int\frac{d\vec{p}}{\sqrt{p_0}}\left[\left(\bar{\phi}_{\alpha\,\vec{p}}\gamma^\mu\phi^a_{\beta\,\vec{p}}\right)e^{2ipx}b^\dagger_{\alpha\,\vec{p}}d^\dagger_{\beta\,\vec{p}} + \left(\bar{\phi}^a_{\alpha\,\vec{p}}\gamma^\mu\phi_{\beta\,\vec{p}}\right)e^{-2ipx}d_{\alpha\,\vec{p}}b_{\beta\,\vec{p}}\right]. \end{aligned} \tag{4}$$

The fermion (quark) operators satisfy anti-commutation rules [2, p..223]:

$$\begin{aligned} \left\{b^\dagger_{\alpha\,\vec{p}}, b_{\beta\,\vec{p}'}\right\} &= b^\dagger_{\alpha\,\vec{p}}\,b_{\beta\,\vec{p}'} + b_{\beta\,\vec{p}'}\,b^\dagger_{\alpha\,\vec{p}} = \delta_{\alpha\beta}\delta^3(\vec{p}-\vec{p}'); \\ \left\{d^\dagger_{\alpha\,\vec{p}}, d_{\beta\,\vec{p}'}\right\} &= d^\dagger_{\alpha\,\vec{p}}d_{\beta\,\vec{p}'} + d_{\beta\,\vec{p}'}d^\dagger_{\alpha\,\vec{p}} = \delta_{\alpha\beta}\delta^3(\vec{p}-\vec{p}'). \end{aligned} \tag{5}$$

The quark matrix elements can be calculated using the free wave functions:

$$\phi_{\alpha\,\vec{p}} = \sqrt{\frac{E+m}{2m}}\begin{pmatrix} 1 \\ \dfrac{\vec{\sigma}\bullet\vec{p}}{E+m}\end{pmatrix}\chi_\alpha, \quad \phi^a_{\alpha\,\vec{p}} = \sqrt{\frac{E+m}{2m}}\begin{pmatrix} \dfrac{\vec{\sigma}\bullet\vec{p}}{E+m} \\ 1\end{pmatrix}\chi_\alpha \qquad m\downarrow 0, \tag{6}$$

normalized to $\bar{\phi}_{\alpha\,\vec{p}}\phi_{\beta\,\vec{p}} = \delta_{\alpha\beta}$ and $\bar{\phi}^a_{\alpha\,\vec{p}}\phi^a_{\beta\,\vec{p}} = -\delta_{\alpha\beta}$. The representation (4) does not have the disadvantages associated with the traditional representation. It also implies a certain unification of Nature, as it relates fermion and boson degrees of freedom in a natural way. The fundamental nature of quarks is elevated, while boson degrees of freedom are diminished.

Various other properties of this representation can be examined. For example, the photon propagator can be derived and has the form of a propagator for a massive particle, with $m\downarrow 0$. Hence, no gauge fixing terms are required in the Lagrangian to obtain this result (however, one could argue that the current Ansatz for the representation is a particular form of gauge fixing, but this form does not affect the gauge invariance of the Lagrangian itself). The representation can also be used to calculate expectation values for the photon. However, if one calculates the energy expectation value of the photon one gets extra terms which appear to be inconsistent. The resolution of this puzzle will lead to some dramatic consequences, which we now examine.

3. Energy calculation of the photon

Within the quark representation the photon state can be written as follows:

$$\left|\vec{p},\varepsilon\right\rangle = \chi\sum_{\alpha\beta}(-1)^{½-\beta}\,\delta^{non-spin}_{\alpha\beta}\,C(½½1\left|\alpha-\beta\varepsilon\right)(-1)^\varepsilon\left|b^\dagger_{½\,\vec{p}\alpha}d^\dagger_{½\,\vec{p}\beta}\right|0\rangle, \tag{7}$$

where χ is a normalization factor. The sum goes over spin, color and isospin, but not over the generation (family) quantum number, as the massless bare quark states form an isospin doublet. There are two transversal states $\varepsilon = \pm 1$, and one longitudinal state $\varepsilon = 0$. One has to show that the latter state cannot exist in the limit $m \downarrow 0$. The desired outcome for the energy of transversal photons with $\varepsilon = \pm 1$ is:

$$\int d^3x \left\langle \vec{p}\varepsilon \right| T^{00}(x) \left| \vec{p}\varepsilon \right\rangle \Big/ \left\langle \vec{p}\varepsilon \| \vec{p}\varepsilon \right\rangle = p_0 = p = \hbar\omega. \tag{8}$$

We now calculate this expectation value in terms of quark operators. Because of the bilinear form of the photon state, Eq. (7), one gets two types of contributions from the operator matrix element. The first type results from contracting the operators appearing in T_{00} with the operators in the state vectors and is thus called the connected matrix element:

$$\begin{aligned} \left\langle \vec{p}', \varepsilon' \right| [T_{00}]_{connected} \left| \vec{p}, \varepsilon \right\rangle &= \quad \left\langle \vec{p}', \varepsilon' \right| \vec{p}, \varepsilon > [(p_0 - \frac{4m^2}{p_0})\bar{\delta}_{\varepsilon 0} + \frac{2m^2}{p_0}] \\ \rightarrow (m = 0) \quad &= \left\langle \vec{p}', \varepsilon' \right| \vec{p}, \varepsilon > p_0 \bar{\delta}_{\varepsilon 0} \end{aligned} \tag{9}$$

We see that it leads to the correct result for the transverse states ($\varepsilon = \pm 1$). For the longitudinal state we get zero, which is inconsistent with the desired result $p_0 = p = \hbar\omega$, showing that longitudinal states are not allowed physically.

Since we already obtained the correct result from the first type of contributions, it is desirable that the second set of contributions yield zero. However, these terms, which result from the internal contraction of the T_{00}, appear to be finite:

$$(T_{00})_{contracted} = -\frac{1}{2\Sigma} \int d\vec{p}\, (p_0 - \frac{m^2}{p_0}) \sum_{\alpha} (b^{\dagger}_{\alpha\vec{p}} b_{\alpha\,\vec{p}} + d^{\dagger}_{\alpha\,\vec{p}} d_{\alpha\,\vec{p}}) \ . \tag{10}$$

Here Σ is a sum over the quantum degrees of freedom of the fermions. The only way to eliminate this term is to ensure that Σ is infinite (this factor does not appear in the connected term). But this requires the introduction of a new set of infinitely many degrees of freedom. This is only possible if this degree of freedom ranges over a continuous space. We now discuss the possible form of this degree of freedom.

4. "Hidden" continuous degree of freedom

We suggest to add a continuous position variable $\vec{\xi}$ to the quantum number of the quark state:

$$b^{\dagger}_{\alpha\,\vec{p}} \rightarrow b^{\dagger}_{\alpha\,\vec{p}\vec{\xi}} \tag{11}$$

This quantum number can be incorporated into the wave function without effect on quantum mechanical transition matrix elements if it appears as a phase factor:

$$e^{ipx} \quad \rightarrow \quad e^{ip(x-\xi)}. \tag{12}$$

which vanishes after squaring the matrix elements. The new representation of the electromagnetic field now reads (we suppress the less relevant constant part):

$$\begin{aligned} A^{\mu}(x) = C \sum_{\alpha,\beta} \int \frac{d\vec{p}}{\sqrt{p_0}} \left[\left(\bar{\phi}_{\alpha\,\vec{p}} \gamma^{\mu} \phi^{a}_{\beta\,\vec{p}} \right) e^{2ipx} b^{\dagger}_{\alpha\,\vec{p}} d^{\dagger}_{\beta\,\vec{p}} + \left(\bar{\phi}^{a}_{\alpha\,\vec{p}} \gamma^{\mu} \phi_{\beta\,\vec{p}} \right) e^{-2ipx} d_{\alpha\,\vec{p}} b_{\beta\,\vec{p}} \right] \rightarrow \\ D \sum_{\alpha,\beta} \int \frac{d\vec{p}}{\sqrt{p_0}} \int d\vec{\xi} \left[\left(\bar{\phi}_{\alpha\,\vec{p}} \gamma^{\mu} \phi^{a}_{\beta\,\vec{p}} \right) e^{2ip(x-\xi)} b^{\dagger}_{\alpha\,\vec{p}\vec{\xi}} d^{\dagger}_{\beta\,\vec{p}\vec{\xi}} + \left(\bar{\phi}^{a}_{\alpha\,\vec{p}} \gamma^{\mu} \phi_{\beta\,\vec{p}} \right) e^{-2ip(x-\xi)} d_{\alpha\,\vec{p}\vec{\xi}} b_{\beta\,\vec{p}\vec{\xi}} \right]. \end{aligned} \tag{13}$$

The new anti-commutation rules read:

$$\left\{ b^{\dagger}_{\alpha\,\vec{p}\vec{\xi}}, b_{\beta\,\vec{p}'\vec{\xi}'} \right\} = \delta_{\alpha\beta}\delta^3(\vec{p}-\vec{p}')\delta^3(\vec{\xi}-\vec{\xi}'), \left\{ d^{\dagger}_{\alpha\,\vec{p}\vec{\xi}}, d_{\beta\,\vec{p}'\vec{\xi}'} \right\} = \delta_{\alpha\beta}\delta^3(\vec{p}-\vec{p}')\delta^3(\vec{\xi}-\vec{\xi}'). \quad (14)$$

One of the many desirable features of the new operators $b^{\dagger}_{\alpha\,\vec{p}\vec{\xi}}$ and $d^{\dagger}_{\alpha\,\vec{p}\vec{\xi}}$ is that they are now dimensionless, just like they are in the case when they are defined over a discrete set of quantum numbers. The introduction of this new quantum variable $\vec{\xi}$ ensures that Σ is infinite, so that the matrix element $(T_{00})_{contracted}$ vanishes. This resolves the consistency problem with the energy expectation value in the quark representation of the electromagnetic field. But what is its physical meaning and justification of this new quantum number? This question will be examined now.

5. Classical nature of hidden degrees of freedom

To examine the nature of the new degree of freedom we consider the phase factor $e^{ip(x-\xi)}$ in more detail:

$$p(x-\xi) = E(t-\xi_0) - \vec{p}\bullet(\vec{x}-\vec{\xi}) = -\vec{p}\bullet\left[\vec{x}-\vec{\xi}-\frac{\vec{p}}{E}(t-\xi_0)\right] \quad (15)$$

Hence, the introduction of $\vec{\xi}$ enables us to define a definite reference point $\vec{R} = \vec{\xi} + \frac{\vec{p}}{E}(t-\xi_0)$ for the spatial variable $\vec{x}$. However, $\vec{R}(t)$ can also be interpreted as the classical position of a particle at time t if it started in $\vec{\xi}$ at time ξ_0 and moves with velocity $\vec{v} = \vec{p}/E$. If we choose all initial points at a common time ξ_0 then $\vec{\xi}$ and $\vec{p}$ fully describe the classical movement of all particles specified by the universal state vector with time. The natural choice for the common time ξ_0 is the big bang, as this is certainly a unique event in the universe. The exact nature of the variables $\vec{\xi}$ and ξ_0 in the context of general relativity will not be discussed here but insights in the nature of different time and spatial variables in an expanding universe are discussed in [5]. The proposed theory also leads to a certain degree of unification between cosmology and particle physics. Intuitively one can think of $\vec{\xi}$ as the equivalent birthplace of the particle at the big bang. The new quantum number also explains why the Pauli principle is effective within one atom, where the particles share the same quantum number $\vec{\xi}$, while distinct atoms do not need to have different internal quantum numbers, as the external quantum number $\vec{\xi}$ is different. So we can extend the validity of the Pauli principle to the state vector of the whole universe.

Many physicists have played with the idea of hidden variables to bring back part of reality, the most well-known one being Einstein. He, Podolsky and Rosen argued that "elements of reality" (hidden variables) must be added to quantum mechanics to explain entanglement without action at a distance [6]. One reason why these efforts have failed is that one typically uses the variable $\vec{R}$, rather than a quantum number $\vec{\xi}$, to characterize the classical movement. This is partly due to the fact that in most treatments of the quantum-classical transition one use non-relativistic quantum mechanics (NRQM). In NRQM the characterization of the state is through a time dependent wave function, whereas in quantum field theory one specifies the state with a state vector and carries most time evolution in the field operators. This use of $\vec{R}$ rather than $\vec{\xi}$ may also explain why Bell's proof [7] of the impossibility of hidden variables does not apply.

The current formulation clearly demonstrates why the classical variable $\vec{R}$ plays no significant physical role in quantum processes, except when one would link the interacting particles explicitly to the macroscopic classical measurement equipment. However, the classical variables $\vec{R}$ and $\vec{\xi}$ start playing a role in many body systems which are typically considered as classical. We now illustrate how these classical properties emerge.

6. How can Nature appear classical in a quantum world?

Consider a typical quantum process: fermion - photon scattering . One amplitude (Feynman diagram) for this process can be written as follows in the operator language:

$$S^{(2)} = \frac{e^2}{2!}\int_{-\infty}^{+\infty} d^4x_1 \int_{-\infty}^{+\infty} d^4x_2 \{N[(\overline{\psi}\gamma^{\nu}\overleftrightarrow{\psi A_{\nu})_{x_2}(\overline{\psi}}\gamma^{\mu}\psi A_{\mu})_{x_1}] + N[(\overleftrightarrow{\overline{\psi}\gamma^{\nu}\psi A_{\nu})_{x_2}(\overline{\psi}\gamma^{\mu}\psi} A_{\mu})_{x_1}]\} =$$
$$= e^2 \int_{-\infty}^{+\infty} d^4x_1 \int_{-\infty}^{+\infty} d^4x_2 N[(\overline{\psi}\gamma^{\nu}\overleftrightarrow{\psi A_{\nu})_{x_2}(\overline{\psi}}\gamma^{\mu}\psi A_{\mu})_{x_1}] \tag{16}$$

where the symbol N stands for normal ordering, and the overline implies a contraction of operators. After evaluation of this amplitude there is an additional phase factor in the new formulation:

$$P = \exp\left(ip_i\xi_i + ip'_i\xi'_i - ip_f\xi_f - ip'_f\xi'_f\right) == \exp\left(-i\vec{p}_i \bullet \vec{\xi}_i - i\vec{p}'_i \bullet \vec{\xi}'_i + i\vec{p}_f \bullet \vec{\xi}_f + i\vec{p}'_f \bullet \vec{\xi}'_f\right) \tag{17}$$

where the meaning of the variables should be obvious. Clearly if we calculate the Feynman diagram we square P and there is no effect. However, if the amplitude is part of the time evolution of the state, and not calculated as a transition matrix element, then the phase factors will remain. Rewriting the phase factor for this amplitude as follows:

$$P = \exp\left[-i\vec{p}_i \bullet (\vec{R}_i) - \frac{\vec{p}_i}{p_{i0}}t - i\vec{p}'_i \bullet (\vec{R}'_i - \frac{\vec{p}'_i}{p'_{i0}}t) + i\vec{p}_f \bullet (\vec{R}_f - \frac{\vec{p}_f}{p_{f0}}t) + i\vec{p}'_f \bullet (\vec{R}'_f - \frac{\vec{p}'_f}{p'_{f0}}t)\right] \tag{18}$$

one can rewrite this as a product of two phase factors:

$$P_1 = \exp\left(-i\vec{p}_i \bullet \vec{R}_i - i\vec{p}'_i \bullet \vec{R}'_i + i\vec{p}_f \bullet \vec{R}_f + i\vec{p}'_f \bullet \vec{R}'_f\right), \tag{19}$$

and

$$P_2 = \exp\left[it\left(\vec{p}_i \bullet \frac{\vec{p}_i}{p_{i0}} + \vec{p}'_i \bullet \frac{\vec{p}'_i}{p'_{i0}} - \vec{p}_f \bullet \frac{\vec{p}_f}{p_{f0}} - \vec{p}'_f \bullet \frac{\vec{p}'_f}{p'_{f0}}\right)\right]. \tag{20}$$

The second phase factor reduces to unity if we impose energy conservation. The first phase factor can be rewritten as follows, after imposing momentum conservation:

$$P_1 = \exp i\left\{-\vec{p}_i \bullet \left(\vec{R}_i - \vec{R}'_f\right) - \vec{p}'_i \bullet \left(\vec{R}'_i - \vec{R}'_f\right) + \vec{p}_f \bullet \left(\vec{R}_f - \vec{R}'_f\right)\right\}. \tag{21}$$

If the initial and final states are mixtures of momentum states (e.g. when they are part of a many-body state) then the maximal contributions of the amplitude considered come from stationary phase terms. Hence, in that case we can demand that the individual phase factors are approximately stationary under variations of the momenta, so that:

$$\vec{\nabla}_{\vec{p}}\left[\vec{p}_i \bullet \left(\vec{R}_i - \vec{R}'_f\right)\right] = 0 \quad \rightarrow \quad \vec{R}_i - \vec{R}'_f = 0\,. \tag{22}$$

Applying this demand to all free momenta we have:

$$\vec{R}_i = \vec{R}'_i = \vec{R}_f = \vec{R}'_f\,. \tag{23}$$

So in an environment with a continuous range of initial momenta, instead of specific momentum states, coherence requires the classical coordinates to be approximately the same. Hence, only the quantum states satisfying Eq. (23) survive the many-body probability contest and thus the classical world emerges from the quantum world. So coherence is responsible for the classical phenomena, not decoherence. Detailed considerations of the time-evolution of the many-body quantum states are required to decide when a classical description becomes a good approximation to the quantum description. Reduction theories like those by Ghiraldi, Rimini and Weber [8] may well play a role in linking the quantum field transitions to the classical picture of macroscopic physics.

7. Various consequences and properties of the new classical quantum number and further work required

Because the phase factors do not affect the usual quantum calculations the presence of the classical quantum numbers is not in violation with the Heisenberg uncertainty principle. The phase factors only become important in a many-body environment because of the dominance of coherent states in many-body configurations. Only the relative value of the classical position variable $\vec{R}$ is of relevance as the phase factor plays no role for individual particles. This is to be expected from a proper physical theory. When we look at coherence arguments it is the relative local classical position $\vec{R}(t)$ which is relevant, not the big bang coordinate $\vec{\xi}$, which can undergo massive changes in quantum transitions.

The combined set of three position and three momentum variables reminds us of the phase space variables in statistical mechanics, however, the big bang quantum numbers $\vec{\xi}$ have no continuity in collisions (quantum transitions) and one first must make the link to the classical positional variable $\vec{R}$, before relating this theory to statistical mechanics. To realize this connection in a consistent way should be an interesting theoretical challenge.

8. Postscript

The relationship between classical and quantum physics has been the subject of many papers and of many debates between prominent physicists. Einstein asserted in 1931 that Quantum Mechanics is incomplete and this paper shows that he well may have been right. The current study indicates that standard quantum field theory QFT is incomplete in its usual form, although the generalization required is probably not along the lines Einstein was thinking of at the time.

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