# Elemental magnetic vector charges linked to zero outward magnetic flux from any surface enclosing non-dipolar magnetic sources

**Max Chirwa**

Physics Department, Faculty of Science, Engineering and Technology, Walter Sisulu University, P/Bag X1, Nelson Mandela Drive, Mthatha 5117, Eastern Cape, South Africa

E-mail: mmchirwa@wsu.ac.za

**Abstract**. A harmonious formulation of the inverse-square laws for fields shows that elemental sources are rightfully represented as scalars for both gravitational and electric fields, but as vectors for magnetic fields. This permits an effective simple illustration that, unlike the gravitational or electric flux, the magnetic flux out of any closed surface is zero irrespective of whether the enclosed magnetic source is dipolar or non-dipolar. Then Gauss' law can be re-stated as: Out of any enclosing surface, if the enclosed source is a scalar quantity, the net flux is equal to the source itself, whereas if the enclosed source is any vector quantity, the net flux is the scalar zero, and thus independent of both the source and the enclosing surface.

## 1. Elemental scalar and vector charges

A harmonized representation of elemental sources of fields and the inverse-square laws [1] are used to express elemental fluxes of gravitational, electric and magnetic fields over an elemental surface. When distributed over a space of volume $v'$, the elemental *scalar* and *vector* sources of fields, that is a mass $dm'$ or an electric *scalar* charge $dq'$ and a magnetic *vector* charge $d\mathbf{Q}'$ which are physical attributes of materials, can be expressed in terms of their gravitational scalar $\rho'_m$, electric scalar $\rho'_e$ and magnetic vector $\mu_0\mathbf{J}' \equiv \hat{\mathbf{z}}\mu_0 J'$ volume densities as

$$dm' = \rho'_m dv' \tag{1a}$$

$$dq' = \rho'_e dv' \tag{1b}$$

$$d\mathbf{Q}' = \mu_0 \mathbf{J}' dv' \tag{1c}$$

where $\mathbf{J}' \equiv \hat{\mathbf{z}}J'$ is the electric current volume density in the $z$-direction. We show that the scalar or vector nature of the sources determines what the net elemental outward flux would be. Then by superposition the results are generalized to any distributed source.

## 2. Net elemental flux out of a spherical or cylindrical surface

If a particle characterized by the elemental sources $dm'$, $dq'$ and $d\mathbf{Q}'$ is located at the origin O of coordinates (see figures 1 and 2), then its free space elemental gravitational $\xi_0 d\mathbf{g}$, electric $d\mathbf{D} = \varepsilon_0 d\mathbf{E}$ and magnetic $d\mathbf{B} = \mu_0 d\mathbf{H}$ flux densities can be similarly expressed as [1]:

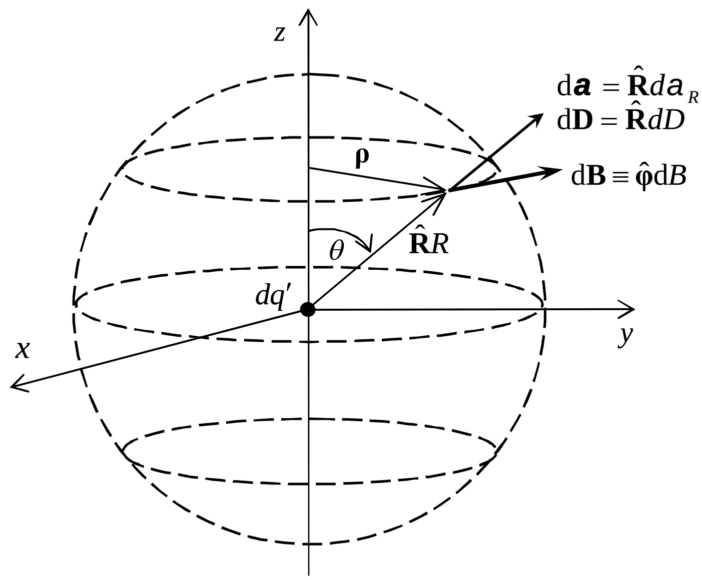


**Figure 1.** A spherical surface centrally enclosing a scalar ($dm'$ or $dq'$) or a vector ($d\mathbf{Q}'$) source.

$$\xi_0 d\mathbf{g} = \frac{\rho'_m dv'\hat{\mathbf{R}}}{4\pi R^2} \tag{2a}$$

$$d\mathbf{D} = \frac{\rho'_e dv'\hat{\mathbf{R}}}{4\pi R^2} = \hat{\mathbf{R}}dD \tag{2b}$$

$$d\mathbf{B} = \frac{\hat{\mathbf{z}}\mu_0 J'dv' \times \hat{\mathbf{R}}}{4\pi R^2} \equiv \hat{\boldsymbol{\varphi}}\frac{\mu_0 J' \sin\theta\, dv'}{4\pi R^2} \equiv \hat{\boldsymbol{\varphi}}dB \tag{2c}$$

where the outward radial vector $\hat{\mathbf{R}}R \equiv \hat{\boldsymbol{\rho}}\rho + \hat{\boldsymbol{\varphi}}0 + \hat{\mathbf{z}}z$, in spherical and cylindrical coordinates, is from the source point to the field point. Here $4\pi G\xi_0 = -1$ and $c_0^2\mu_0\varepsilon_0 = +1$ relate the *gravitativity* $\xi_0 = -1.193\times10^9$ kg$^2$ m$^{-2}$ N$^{-1}$, permittivity $\varepsilon_0 = +8.854\times10^{-12}$ C$^2$ m$^{-2}$ N$^{-1}$ and permeability $\mu_0 = +4\pi\times10^{-7}$ Wb$^2$ m$^{-2}$ N$^{-1}$ in vacuum to the universal gravitational constant $G$ and the speed $c_0$ of light. Since $dm'$ is a positive scalar quantity, $\xi_0 d\mathbf{g}$ and its elemental gravitational flux are always outward. This is consistent with the $\varepsilon_0 d\mathbf{E}$ of a positive $dq'$.

On a spherical surface of area $a_R$ (see figure 1), the elemental area vector is

$$d\boldsymbol{a}_R = \hat{\mathbf{R}}R^2 \sin\theta\, d\theta\, d\phi \equiv \hat{\mathbf{R}}da_R \tag{3}$$

so that the respective elemental gravitational, electric and magnetic fluxes are

$$\xi_0 d\mathbf{g}\cdot d\boldsymbol{a}_R = \hat{\mathbf{R}}\cdot\hat{\mathbf{R}}\xi_0 dg da_R \neq 0, \tag{4a}$$

$$d\mathbf{D}\cdot d\boldsymbol{a}_R = \hat{\mathbf{R}}\cdot\hat{\mathbf{R}}dD da_R \neq 0, \tag{4b}$$

$$d\mathbf{B}\cdot d\boldsymbol{a}_R = \hat{\boldsymbol{\varphi}}\cdot\hat{\mathbf{R}}dB da_R = 0, \tag{4c}$$

as $\xi_0 d\mathbf{g}$ and $d\mathbf{D}$ are collinear with $\hat{\mathbf{R}}$, but $d\mathbf{B}$ is normal to it and to $d\mathbf{Q}'$. Ultimately this is simply because $dm'$ and $dq'$ are scalars while $d\mathbf{Q}'$ is a vector.

For magnetic flux, a closed cylindrical surface axially concentric with the enclosed elemental magnetic vector charge $d\mathbf{Q}'$ (as in figure 2) is equally suitable for evaluating the net outward flux. This composite closed surface has three open surfaces which are typified by the elemental area vectors

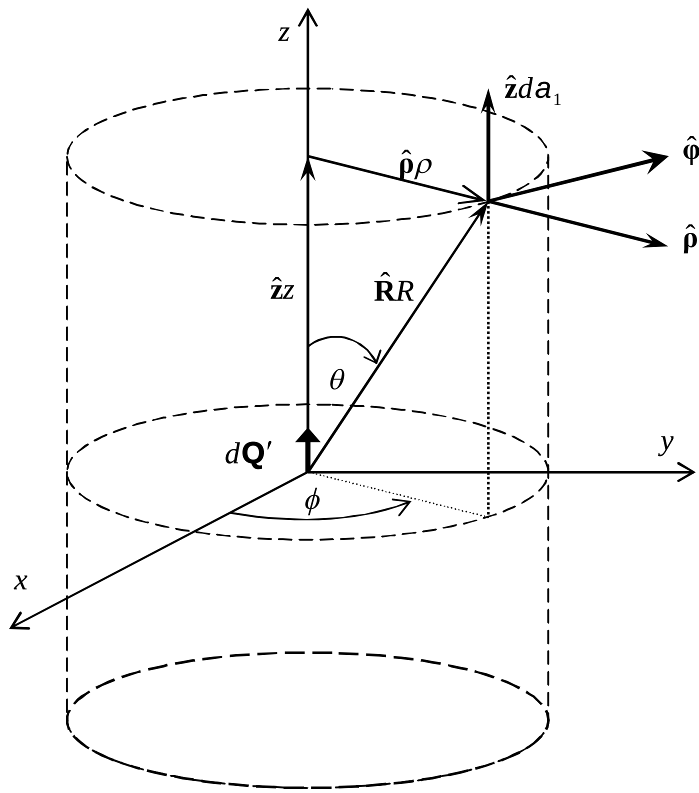


**Figure 2.** A cylindrical surface axially concentric with an enclosed magnetic vector charge $d\mathbf{Q}'$.

$$d\boldsymbol{a}_1 = +\hat{\mathbf{z}}\rho d\rho d\phi = +\hat{\mathbf{z}}da_1 \tag{5}$$

$$d\boldsymbol{a}_2 = +\hat{\boldsymbol{\rho}}\rho d\phi dz = +\hat{\boldsymbol{\rho}}da_2 \tag{6}$$

$$d\boldsymbol{a}_3 = -\hat{\mathbf{z}}\rho d\rho d\phi = -\hat{\mathbf{z}}da_3 \tag{7}$$

Since each of these elemental area vectors is normal to $d\mathbf{B}$, that is, at any point on the surface the vector $d\mathbf{B}$ lies within the surface, the corresponding elemental magnetic fluxes vanish:

$$d\mathbf{B}\cdot d\boldsymbol{a}_1 = +\hat{\boldsymbol{\varphi}}\cdot\hat{\mathbf{z}}da_1 dB = +0 \tag{8}$$

$$d\mathbf{B}\cdot d\boldsymbol{a}_2 = +\hat{\boldsymbol{\varphi}}\cdot\hat{\boldsymbol{\rho}}da_2 dB = +0 \tag{9}$$

$$d\mathbf{B}\cdot d\boldsymbol{a}_3 = -\hat{\boldsymbol{\varphi}}\cdot\hat{\mathbf{z}}da_3 dB = -0 \tag{10}$$

Evidently, each of the elemental sources cited above is a single non-dipolar source [2 – 4]. From equations (4a) to (4c), their individual net outward gravitational, electric and magnetic fluxes through the enclosing spherical surface are

$$d\Phi_{\mathrm{g}} = \oint_{\text{sphere}} \hat{\mathbf{R}}\cdot\hat{\mathbf{R}}\xi_0 dg\, da_R = dm' \tag{11a}$$

$$d\Phi_{\mathrm{e}} = \oint_{\text{sphere}} \hat{\mathbf{R}}\cdot\hat{\mathbf{R}}dD\, da_R = dq' \tag{11b}$$

$$d\Phi_{\mathrm{m}} = \oint_{\text{sphere}} \hat{\boldsymbol{\varphi}}\cdot\hat{\mathbf{R}}dB da_R = 0 \tag{11c}$$

Similarly a combined integration of equations (8) to (10) yields the net magnetic flux from the closed cylinder

$$d\Phi_{\mathrm{m}} = \oint_{\mathrm{cyl}} d\mathbf{B} \cdot d\boldsymbol{a}_{\mathrm{cyl}} = \int_{a_1} \hat{\boldsymbol{\varphi}} \cdot \hat{\mathbf{z}} da_1 dB + \int_{a_2} \hat{\boldsymbol{\varphi}} \cdot \hat{\boldsymbol{\rho}} da_2 dB - \int_{a_3} \hat{\boldsymbol{\varphi}} \cdot \hat{\mathbf{z}} da_3 dB = 0 + 0 - 0 \tag{11d}$$

**3. Net flux out of any enclosing surface**

Equations (11a) to (11d) mean that, as $R$ is arbitrary, any elemental source can be regarded as being at the centre of an arbitrary spherical or cylindrical surface that fully encloses it. Then the net flux from such a closed surface is given by these respective equations. That is, only one closed surface out of a family of concentric closed surfaces contributes to the net flux.

Now, any surface enclosing a source can be built up from a suitable combination of spherical surfaces, a number of which are centred at elemental sources in the distribution. Then, since each family of concentric spheres contributes to the flux in accordance with equations (11a) to (11c), the superposition principle yields the net gravitational

$$\Phi_{\mathrm{g}} = \int_{m'} d\Phi_{\mathrm{g}} = \int_{m'} \oint_{\mathrm{sphere}} \hat{\mathbf{R}} \cdot \hat{\mathbf{R}} \xi_0 dg \, da_R \equiv \oint_{\mathrm{sphere}} \int_{m'} \left( \hat{\mathbf{R}} \xi_0 dg \right) \cdot \hat{\mathbf{R}} \, da_R \equiv \oint_{\mathrm{sphere}} \xi_0 \mathbf{g} \cdot d\boldsymbol{a}_R = m', \tag{12a}$$

electric

$$\Phi_{\mathrm{e}} = \int_{q'} d\Phi_{\mathrm{e}} = \int_{q'} \oint_{\mathrm{sphere}} \hat{\mathbf{R}} \cdot \hat{\mathbf{R}} dD \, da_R \equiv \oint_{\mathrm{sphere}} \int_{q'} \left( \hat{\mathbf{R}} dD \right) \cdot \hat{\mathbf{R}} \, da_R \equiv \oint_{\mathrm{sphere}} \mathbf{D} \cdot d\boldsymbol{a}_R = q' \tag{12b}$$

and magnetic

$$\Phi_{\mathrm{m}} = \int_{\mathbf{Q}'} d\Phi_{\mathrm{m}} = \int_{\mathbf{Q}'} \oint_{\mathrm{sphere}} \hat{\boldsymbol{\varphi}} \cdot \hat{\mathbf{R}} dB da_R \equiv \oint_{\mathrm{sphere}} \int_{\mathbf{Q}'} \left( \hat{\boldsymbol{\varphi}} dB \right) \cdot \hat{\mathbf{R}} da_R \equiv \oint_{\mathrm{sphere}} \mathbf{B} \cdot d\boldsymbol{a}_R = 0 \tag{12c}$$

fluxes out of *any* spherical surface enclosing *any* related net source $m' \neq 0$, $q' \neq 0$ and $\mathbf{Q}' \neq \mathbf{0}$, that is any non-dipolar source, paired or not [2, 4]. These results show that the size, shape or position of the enclosure does not affect the net flux out of it. As $dv'$ and $da$ are independent, the order of integration was reversed to recover the usual Gaussian formats for the fluxes.

The above treatise clearly demands a more suitable interpretation of Gauss' law for any source type. Out of any enclosing surface, if the enclosed source is a scalar quantity, the net outward flux is equal to the source itself, whereas if the source is a vector quantity, the net flux is equal to the scalar zero, and thus independent of both the source and the enclosing surface.

**References**


[1] Chirwa M 2009 A new theory of elemental electromagnetic scalar and vector charges, *Brochure, 54th Annual Conference of South African Institute of Physics, SAIP 2009* (Durban, 6th – 10th July 2009), abstract # 73 G.

[2] Chirwa M 2010 Status of electric and magnetic dipole moments in a complete characterization of electromagnetic first order moments, *Proc. 2nd WSU International Research Conference* (East London, 25th – 27th August 2009) pp. 88–96, ISBN 978 0 620 47102 2.

[3] Griffiths D J 1999 *Introduction to Electrodynamics*, 3rd edition (London, John Wiley & Sons) pp. 146–165 and 243–258, ISBN 0 13 919960 8.

[4] Chirwa M 2010 An authentic depiction and classification of elemental electric and magnetic dipole moments, *Proc. 55th Annual Conference of South African Institute of Physics, SAIP 2010* (Pretoria, 27th September – 1st October 2010) paper SAIP2010_187, ISBN 978 0 620 46211 2.