A (severe) Condensation

of a

Century of Nuclear Theory

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In the beginning ....

from the word of Lord Rutherford
to the dawn of the computer revolution

1911 - 1950's

- Time-lines of a very selected series of discoveries
- The shell model in 1950
- Weisskopf's three stage prescription for nuclear reactions

A time-line of the ‘early’ years: 1911 -1932

- 1911: Rutherford: Small size of the nucleus, then found it contained protons

  Nuclear constituents thought to be: e^-, p, α

- 1920: Rutherford: Postulates neutron: nucleus with 1 proton and 1 electron

- 1924: Gamow: Quantum Mechanic treatment of α emission (tunnelling)

- 1928: Gamow: Analysed Rutherford (α, p) data

  $\longrightarrow$ nuclear radii $R = 1.2A^{\frac{1}{3}}$

  Dirac: Q.M. and relativity equation for fermions – and of antimatter

  Pauli: Identified (electron) states in atoms with $\ell, \frac{1}{2}, j, m_j$

- 1931: Pauli: Postulates existence of a massless neutral (neutrino)

  Wigner: Group theory - atomic structure — $\mathcal{D}$-matrix, W-E theorem

- 1932: Chadwick: Discovers the neutron

A time-line of the 'magic' years (part 1): 1933 - 1936

- 1933: Fermi: Theory of β-decay $\longrightarrow$ the weak force

- 1933: Wigner: Proposed that the p-n force should be short ranged and strong to explain the binding energies of the ^2H and of the α

  Proca: Field theory: massive vector bosons $\longrightarrow$ mesonic NN field

- 1935: Yukawa: The first meson exchange theory of the NN force

  Hahn-Meitner: Discover fission and isomerism

  Gamow: First liquid drop model of the nucleus

  Oppenheimer-Phillips: Propose (d, p) stripping as a spectroscopic tool

- 1936: Wigner: Noted the NN force has space, spin, and isospin components proposed the SU(4) scheme $\longrightarrow$ supermultiplet theory

A time-line of the 'magic' years (part 2): 1936 - 1940

- 1936 Bohr (N.): Compound nucleus theory of radiative capture resonances

  Breit-Wigner: A single level resonance formula

  based on a single particle motion assumption

- 1937 Weisskopf: Statistical (evaporation) model of nuclear reactions

  Kapur-Peierls: Dispersion theory of resonances

  ($\longrightarrow$ B-W formula – no single nucleon assumption used)

  Wheeler: Scattering specified in terms of the $\mathcal{S}$-matrix

  Digression: 1950 Lippmann-Schwinger: used similar variational methods to define

  the Lippmann-Schwinger equation for the $\mathcal{T}$-matrix, in terms

  of which $\mathcal{S}_{if} = \delta_{if} - 2\pi i \delta(E_i - E_f)\mathcal{T}_{if}$

- 1940 Bethe: Single particle effects in n-A scattering

A special note: targets, detectors, and accelerators

Theory moves beyond speculations when tested against critical, accurate data.

- targets: No targets – no reaction data. Some special targets, $Ca, SCRIT, etc.
- Detectors (1908-1930) Geiger counters, scintillation plates, cloud chambers

  1934-37 Photo-multiplier detectors: Using phosphors such as NaI, GeLi
- Projectile sources:

  Radiation sources were used, e.g. in 1934 Fermi: Ra-Be sources for (n, γ).

  Particle accelerators

  1928 Wilderoe: Idea of using alternating current for a LINAC

  1929-1933 van der Graaf: Creates his electrostatic generator

  1931 Lawrence: Makes the first cyclotron.

  1932 Cockcroft-Walton: Make their rectifier generator

The break-away years: 1941 - 1950's

Data observed at odds with Bohr's compound nucleus ideas

- 1941 Wigner: The $\mathcal{R}$-matrix theory of resonances
- 1942 Fermi: The first self-sustaining nuclear chain reaction
- 1949 Mayer-Jensen: Propose the shell model of nuclei
- 1950 Rainwater: Nuclei could be deformed
- 1951 Barschall: E averaged (n elastic) cross sections – not monotonic
- 1952 Bohr (A.)-Mottelson: Collective Hamiltonian – rotation/vibration spectra
- 1952 Hauser-Feshbach: Statistical theory of reactions
- 1954 Feshbach-Porter-Weisskopf: The nuclear optical potential
- 1956 Weisskopf: Rationalises reaction processes in a three stage system

The shell model in 1950

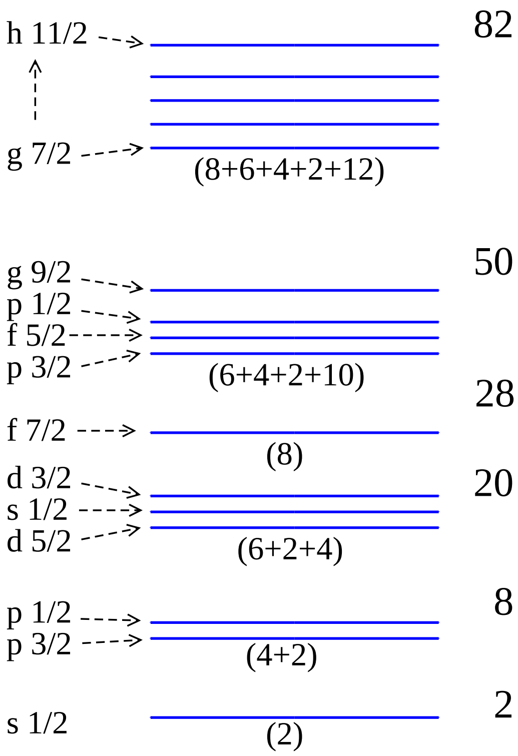
By 1950, evidence of shell structure for systems that had **magic** numbers,

viz. 2, 8, 20, 28, (40), 50, 82, 126, of protons and/or neutrons.

- Anomalously large binding energies
- Number of 'stable' nuclei for N/Z magic
- Energy gaps (to first excited state) were large

  $\Longrightarrow$ Mayer and Jensen: Need for a strong spin-orbit interaction $\longrightarrow$ s.p. levels $\longrightarrow$

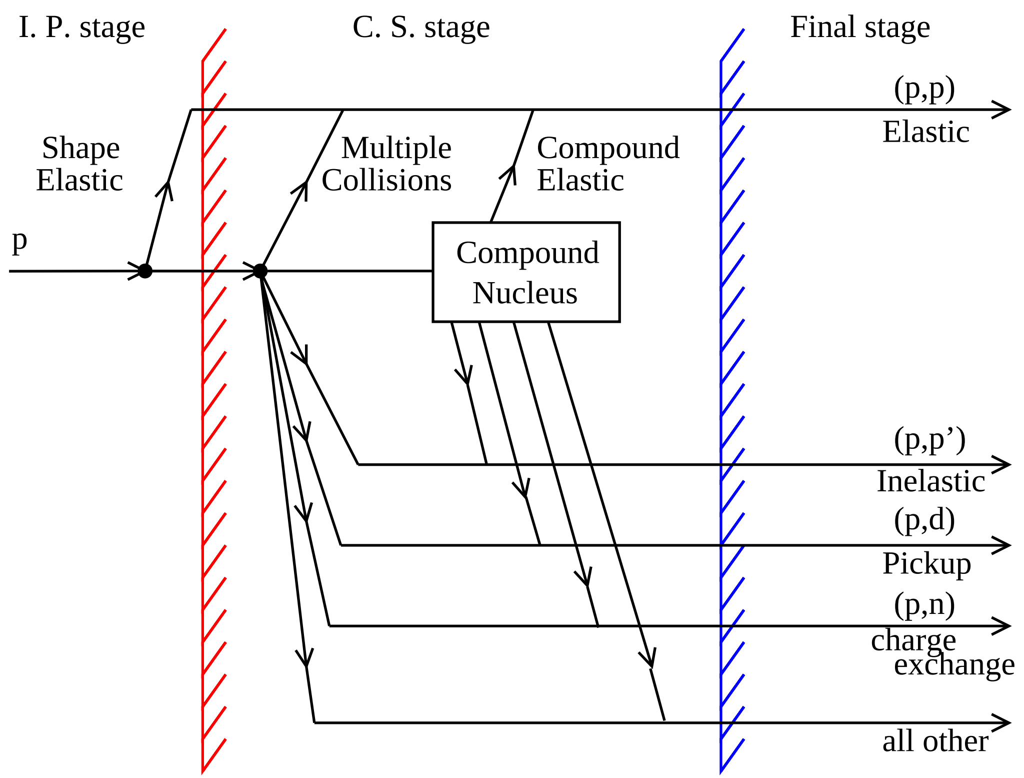
  Explained:
- Spins of some ground and low excited states
- The Schmidt lines for magnetic moments
- Isomeric states from the strong spin-orbit field
- Sign change of quad. moments (odd masses)



Weisskopf's three stages for reactions

In 1956 there were two extreme models (compound nucleus and cloudy crystal ball) to interpret nuclear reaction data. Weisskopf reconciled these with

Figure 1: A schematic of proton initiated reactions



The middle age of Nuclear Theory ...

From the (ongoing) computer revolution to a resurrection ... flights on the plane of instability

1950's - 1990's

During these years, there was a veritable explosion of ideas, models, and applications in nuclear theory. There are far too many things to cover in a talk.

- Abridged catalogues of structure models and reactions
- The advent of direct reaction theory
- The optical model
- Two specific theories of nucleon scattering:

MCAS and g-folding

An abridged catalogue of nuclear structure models

Since the first shell model, nuclear structure theories have burgeoned.

Some groupings (though some items fit more than one) are:

- Collective models:– e.g. the liquid drop model, quantised rotors and vibrators, etc.
- Shell Models:– e.g. standard shell model, deformed (Nilsson) shell model, random phase approximation models, etc.
- Mean field theories:– e.g. Hartree-Fock (HF), Hartree-Fock-Bogoliubov (HFB) time dependent HF, etc.
- Cluster models:– e.g. cluster-orbital model, generator coordinate methods antisymmetrized molecular dynamics model, etc.
- Few body models:– e.g. Fadde'ev three body, quantum Monte Carlo models, etc.
- Group theoretic models:– Wigner supermultiplet theory, IBMs (IBA and IBF), etc.

An abridged catalogue of nuclear reactions

Myriads of nuclear reactions have been studied (and still are being). Some are

- Fusion and capture reactions:– Low energy usually – astrophysics cases
- Fission reactions:– Low energy again – some are spontaneous
- Spallation:– Intra-nuclear cascade, evaporation models
- Relativistic heavy ion collisions:– Seeking the quark-gluon plasma
- Direct reactions:– Many reactions, elastic, inelastic, particle transfer, etc.

There are almost as many reaction theories. Some are

- Hauser-Feshbach:– Low energy compound nuclear reactions
- $\mathcal{R}$-matrix theory:– Classify resonance structures
- Transport theories:– Spallation, heavy ion fragmentation
- Direct reactions:– The optical model, DWA, multi-step FKK theory

Direct reactions and the computer revolution

By 1950's:

Accelerators forming beams with energies tens of MeV

The Oppenheimer-Phillips suggestion taken seriously

i.e. to use (d,p) reactions as spectroscopic tools

- 1957 Butler: Direct reaction theory of stripping $\frac{d\sigma}{d\Omega} \propto \frac{j_\ell^2(qR)}{[q^2+k_d^2]}$
- Reaction localised: to have the residual nucleus in a unique state,

  shape of the cross section $\longrightarrow$ properties of residual nuclear state.
- (d,p) data: A spectroscopic tool. Identified transfer angular momentum ℓ.
- Transistorised computers: DWA calculations with deuteron D-state $\rightarrow \ell, j$.

So began a spectroscopy hunt.

Pursued today in the quest for properties of exotic nuclear systems;

(even of ones beyond the nucleon drip lines)

The optical model potential (OMP): a brief history

- 1940 Bethe: Noted data $\longrightarrow$ the concept of a complex optical potential

  All early optical potentials were phenomenological

  At high E, N-A OMPs sought from free NN scattering amplitudes

- 1959 Kerman, McManus, and Thaler: Developed a multiple scattering theory

- 1960 $\longrightarrow$ Arndt: Reliable NN scattering phase shift analyses – NN potentials

- Semi-microscopic models e.g. the JLM model (phenomenological imag. part)

- 1990 $\longrightarrow$ the g-folding model with medium modifications of the NN force

  (due to *Pauli blocking* and *mean field effects*)

  Antisymmetrization $\longrightarrow$ direct and (knock out) exchange amplitudes

  gives a complex, energy dependent, non-local potential

- Relativistic derivations of the optical potential and reactions

The optical model potential — formally

Feshbach formalism:

Projection operators, P and Q where $P + Q = \mathbf{1}$. The Schrödinger equation

$$[E - H] \left|\Psi^+\right\rangle = [E - H] (P + Q) \left|\Psi^+\right\rangle = 0$$

Projection operator algebra

$$P^2 = P \;\; ; \;\; Q^2 = Q \;\; ; \;\; PQ = QP = 0$$

Let P project the elastic channel, Q everything else. Apply P and Q separately

$$P [E - H] (P + Q) \left|\Psi^+\right\rangle = [EP - PHP - PHQ] \left|\Psi^+\right\rangle = 0$$
$$Q [E - H] (P + Q) \left|\Psi^+\right\rangle = [EQ - QHP - QHQ] \left|\Psi^+\right\rangle = 0$$

With $H_{XY} = XHY$, and using the algebra, these two equations are

$$[E - H_{PP}] P \left|\Psi^+\right\rangle = H_{PQ} Q \left|\Psi^+\right\rangle \;\; ; \;\; [E - H_{QQ}] Q \left|\Psi^+\right\rangle = H_{QP} P \left|\Psi^+\right\rangle$$

The optical model potential — formally ctnd.

The second equation,

$$[E - H_{QQ}]\, Q \left|\Psi^+\right\rangle = H_{QP} P \left|\Psi^+\right\rangle \implies Q \left|\Psi^+\right\rangle = [E - H_{QQ}]^{-1} H_{QP} P \left|\Psi^+\right\rangle$$

used to eliminate $Q\left|\Psi^+\right\rangle$ from the first

$$\left[E - H_{PP} - H_{PQ} \underbrace{[E - H_{QQ} + i\epsilon]^{-1}}_{} H_{QP}\right] \left|\Psi^+\right\rangle = 0$$

With the underbrace being $G^{(+)}_{QQ}$ and $H = H_0 + V$ so that $H_{QP} = H_{PQ} = V$, the ground state expectation leads to a one body equation

$$\left[E - H_0 - \langle \Phi_{gs} | V | \Phi_{gs} \rangle - \left\langle \Phi_{gs} \left| V G^{(+)}_{QQ} V \right| \Phi_{gs} \right\rangle\right] \left|\chi^+\right\rangle = 0$$

The intermediate state propagator is complex (poles) and the OMP is

$$U_{OM}(E) = \langle \Phi_{gs} | V | \Phi_{gs} \rangle + \left\langle \Phi_{gs} \left| V G^{(+)}_{QQ} V \right| \Phi_{gs} \right\rangle$$

The second term means that it is non-local, complex, and energy dependent.

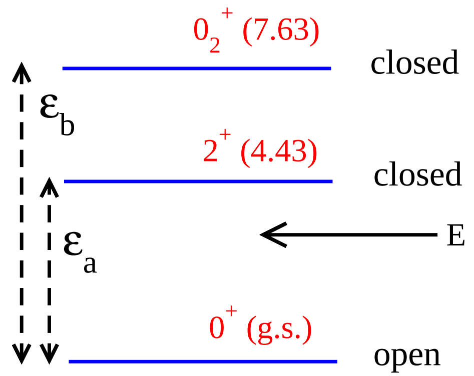
Two specific theories of nucleon scattering

- Low energy scattering:

  Typically $E < 6$ MeV

  — a coupled channels problem

  (note: 4.43 MeV $\longrightarrow T_9 \sim 50$)



- Higher energy scattering:

  Typically for

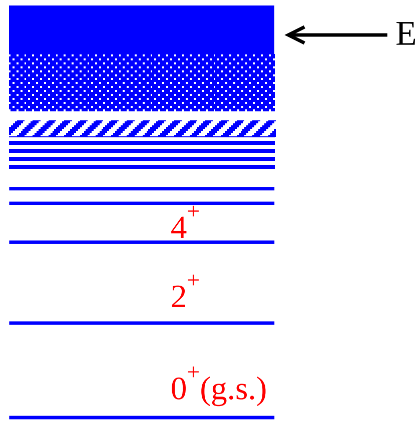
  $\epsilon_{\rm GR} < E < 300$ MeV

  mean field theories

  Microscopic models of reactions

  g-folding model (elastic)

  the DWA (inelastic)



Low energy nucleon scattering

(consider nuclei A and B colliding with C their compound)

- For projectile energies thermal to $\sim$ 6 MeV

  nuclear scattering is a coupled channels problem

- The basic first requirement: (whatever reaction cross section needed)

  is to explain elastic scattering.

- The spectrum of the compound nucleus C should be determined

  both subthreshold and resonance states

- Physics principles (e.g. the Pauli principle) must not be violated

  (at least, not too much)

- A method exists – the Multi-Channel Algebraic Scattering method (MCAS):

  A method to solve multi-channel Lippmann-Schwinger equations

MCAS and multi-channel $\mathcal{T}$-matrices: $\mathcal{T} = \mathcal{T}_{cc'}(p, q; E)$

- Coupled Lippmann-Schwinger equations   channels: $c = (j_i \otimes I_k) J_c \quad I_k =$ target state

$$\mathcal{T}_{cc'} = V_{cc'}(p,q) - \mu \sum_{c"=1}^{\text{closed}} \int_0^\infty V_{cc"}(p,x) \frac{1}{h_{c"}^2 + x^2} \mathcal{T}_{c"c'}(x,q;E)\, x^2\, dx$$
$$+ \mu \sum_{c"=1}^{\text{open}} \int_0^\infty V_{cc"}(p,x) \frac{1}{k_{c"}^2 - x^2 + i\epsilon} \mathcal{T}_{c"c'}(x,q;E)\, x^2\, dx$$

- Expand the potential matrix

$$V_{cc'}(p,q) \sim V_{cc'}^{(N)}(p,q) = \sum_{n=1}^{N} \hat{\chi}_{cn}(p)\, \eta_n^{-1}\, \hat{\chi}_{c'n}(q)$$

- Optimal functions, $\hat{\chi}_{cn}(q)$, involve Sturmians, $|\Phi_{c'n}\rangle$

$$|\hat{\chi}_{cn}\rangle = \sum_{c'} V_{cc'} |\Phi_{c'n}\rangle \;\; ; \;\; \sum_{c'} G_c^{(0)} V_{cc'} |\Phi_{c'n}\rangle = -\eta_n |\Phi_{cn}\rangle$$

Multi-channel $\mathcal{S}$-matrices

- Separable expansion of multi-channel $V_{cc'} \Longrightarrow$ multi-channel $\mathcal{S}$- matrix

  (c, c' are open channels; each J^π understood)

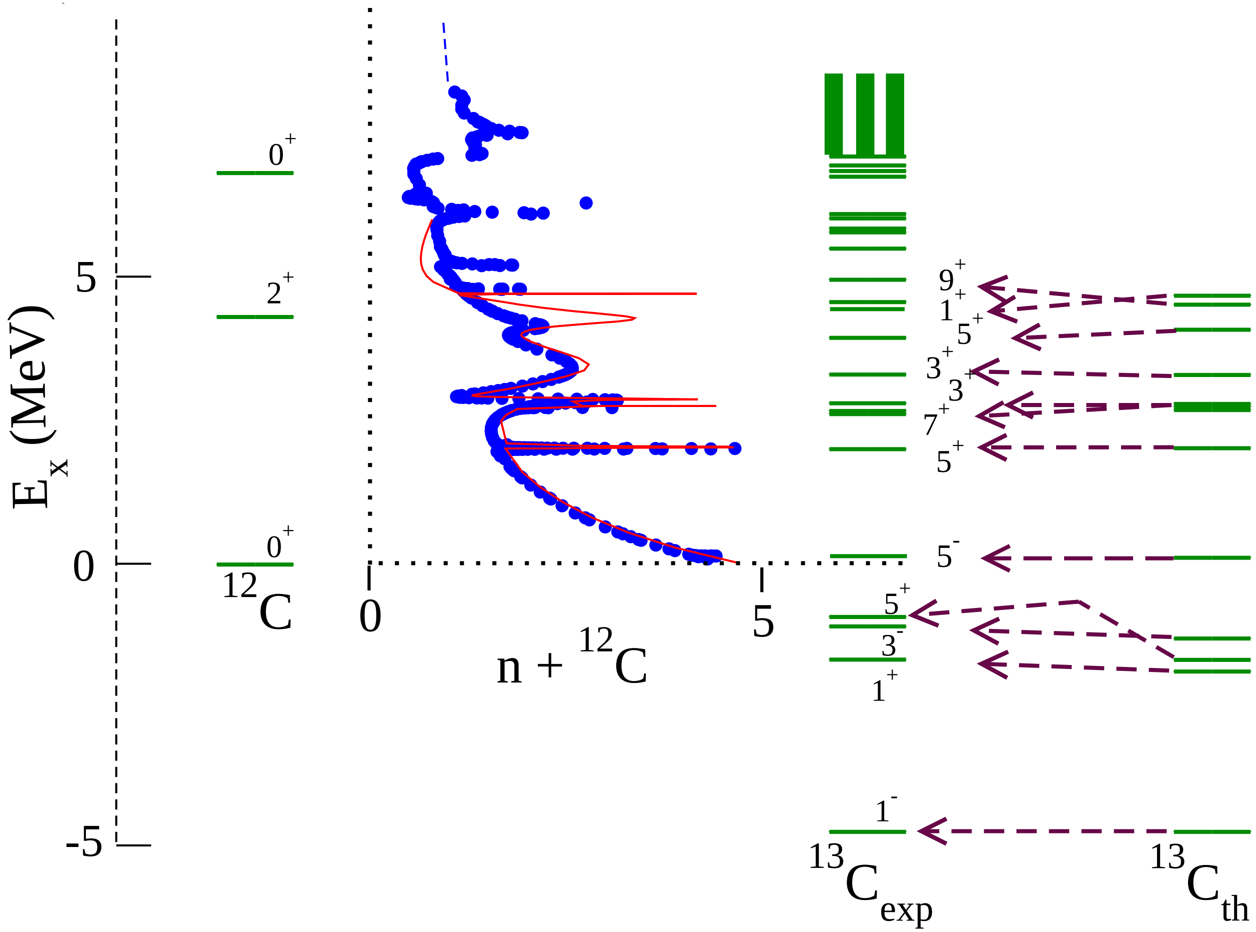
$$\begin{aligned} \mathcal{S}_{cc'} &= \delta_{cc'} - i\pi\mu\sqrt{k_c k_{c'}}\,\mathcal{T}_{cc'} \\ &= \delta_{cc'} - i\,\pi\mu \sum_{n,n'=1}^{N} \sqrt{k_c}\hat{\chi}_{cn}(k_c)\left([\boldsymbol{\eta} - \mathbf{G}_0]^{-1}\right)_{nn'} \hat{\chi}_{c'n'}(k_{c'})\sqrt{k_{c'}} \end{aligned}$$

- Matrix elements (Sturmian basis) and with $[\eta]_{nn'} = \eta_n\,\delta_{nn'}$

$$\begin{aligned} [\mathbf{G}_0]_{nn'} &= \mu\left[\sum_{c=1}^{\text{open}} \int_0^\infty \hat{\chi}_{cn}(x)\frac{x^2}{k_c^2 - x^2 + i\epsilon}\hat{\chi}_{cn'}(x)\,dx \right. \\ &\qquad \left. - \sum_{c=1}^{\text{closed}} \int_0^\infty \hat{\chi}_{cn}(x)\frac{x^2}{h_c^2 + x^2}\hat{\chi}_{cn'}(x)\,dx\right] \end{aligned}$$

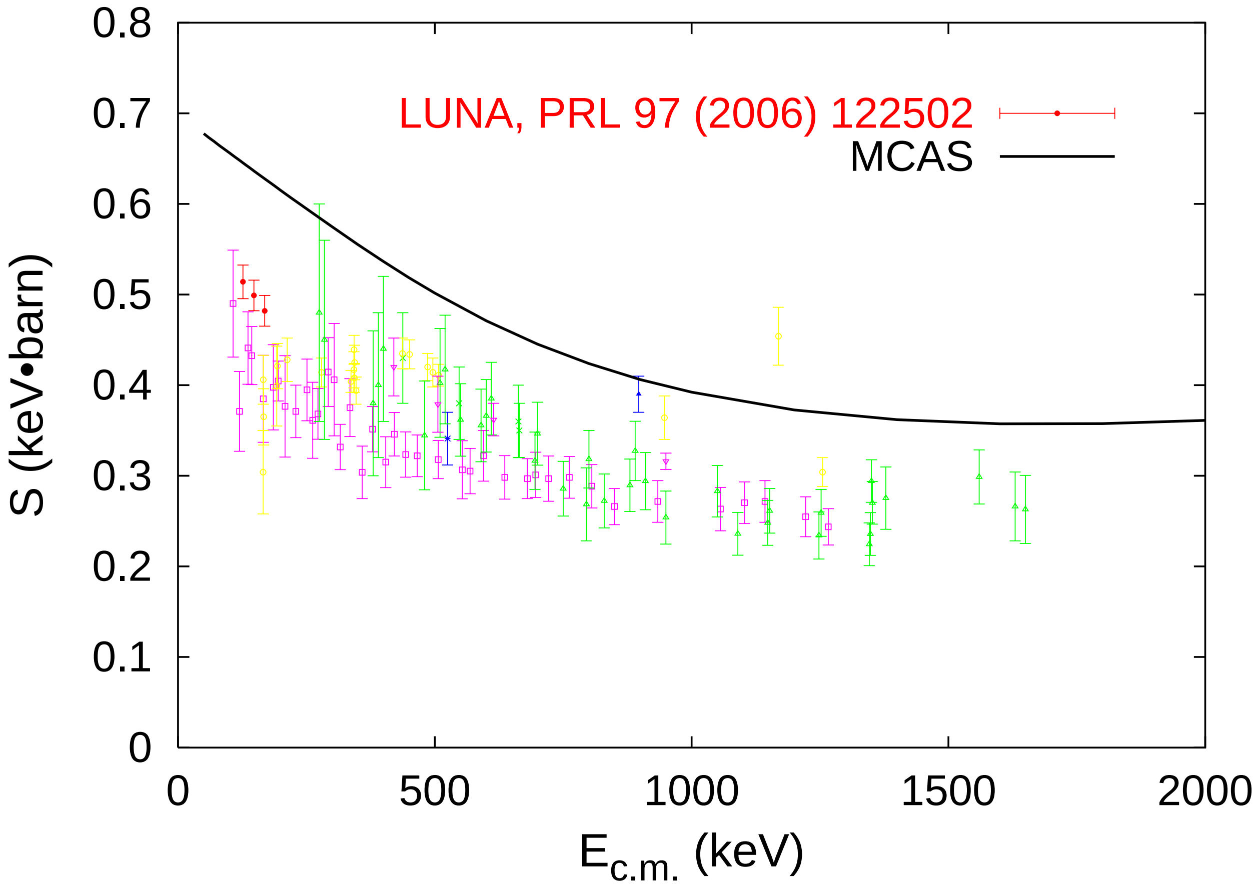
- Resonances and sub-threshold states of the compound from zeroes of $[\boldsymbol{\eta} - \mathbf{G}_0]$

MCAS results: The n+^{12}C system



^{3}He+^{4}He capture S-factor

$$S(E) = \sigma(E)\, E\, \exp(2\pi\eta)\ ; \qquad \eta = \text{Sommerfeld parameter}$$



Higher energy nucleon scattering (typically ≥ 30 MeV)

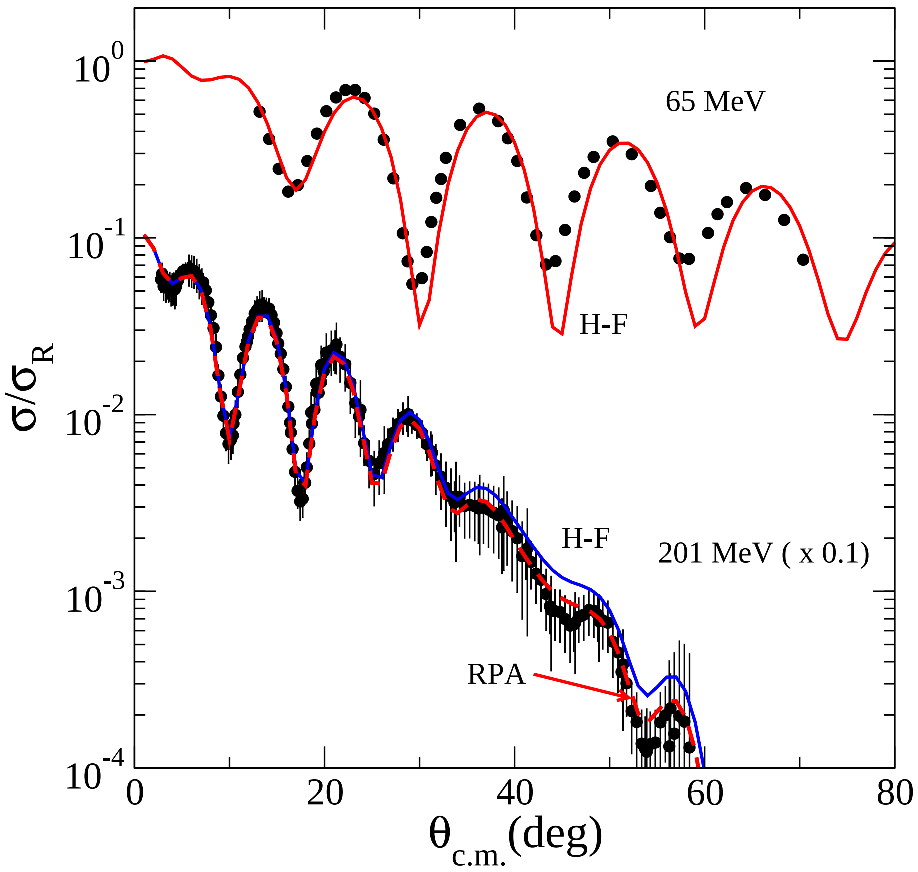
use the g-folding approach

- Optical potentials

$$
\begin{aligned}
U_{opt}(\mathbf{r_1}, \mathbf{r_2}; E) \quad &= \quad \delta(\mathbf{r_1} - \mathbf{r_2}) \sum_n \zeta_n \int \varphi_n^*(\mathbf{s})\, v_D(\mathbf{r_{1s}})\, \varphi_n(\mathbf{s})\, d\mathbf{s} \\
&\quad + \sum_n \zeta_n\, \varphi_n^*(\mathbf{r_1})\, v_{Ex}(\mathbf{r_{12}})\, \varphi_n(\mathbf{r_2}) \\
&\Longrightarrow U_D(\mathbf{r_1}; E) + U_{Ex}(\mathbf{r_1}, \mathbf{r_2}; E)
\end{aligned}
$$

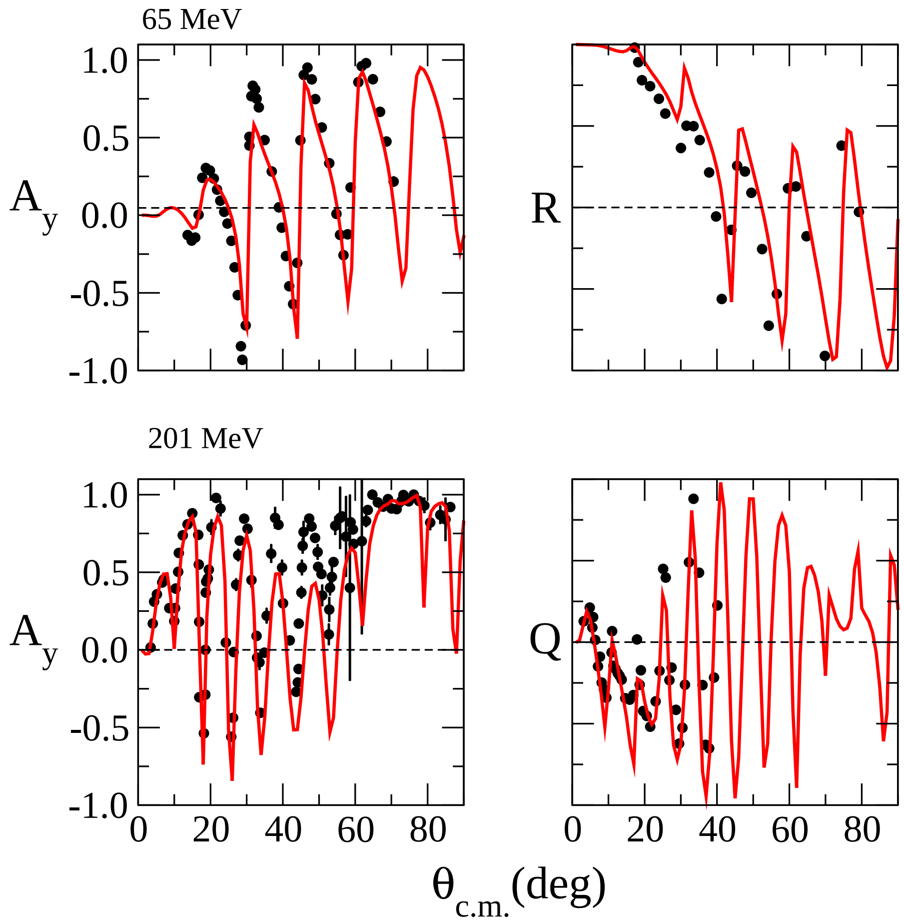
1. v_D ; v_{Ex}:– are combinations of $g_{eff}^{ST}(|\mathbf{r_{1s}}|; E, \rho[\mathbf{k_f}(\mathbf{s})])$
   (e.g. built from INM g-matrices of an NN force)
2. ζ_n:– are (bound state) occupancies — (more realistically – OBDME)
3. $\varphi_n(\mathbf{x})$:– single nucleon bound state wave functions — (H.O., WS, SHF)

g-folding results: p-^{208}Pb elastic scattering – cross sections

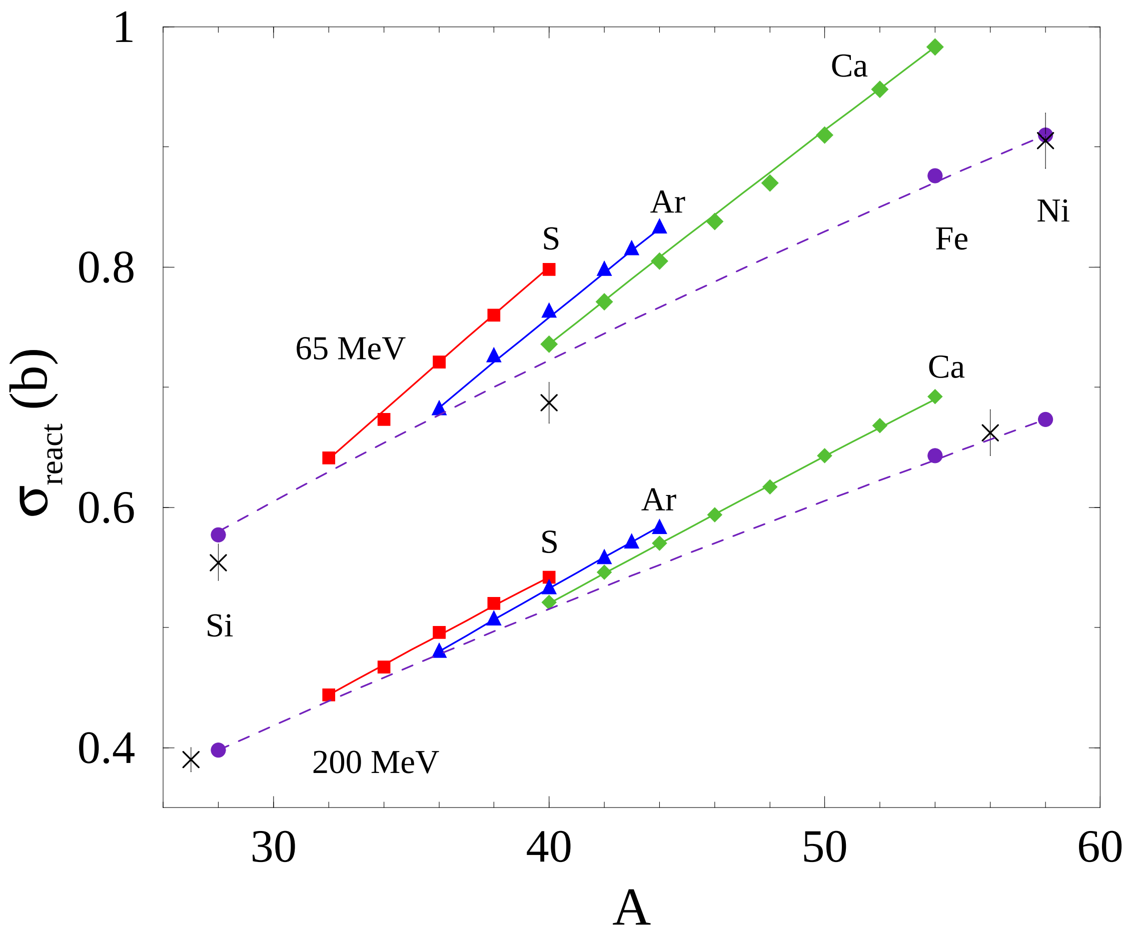


M. Dupuis *et al.* Phys. Rev. C **73**, 014605 (2006)

p-^{208}Pb elastic scattering – spin observables



Total proton reaction cross sections



--- Carlson model: $\sigma_R = \pi \left(R_p + r_0 A^{\frac{1}{3}} \right)^2$

BUT there is a concern!

- Elastic scattering cross sections:– (grossly simplified) are given by

$$\frac{d\sigma}{d\Omega} = \frac{1}{4k^2} \left| \sum_{\ell} (2\ell + 1) \left[e^{2i\delta_\ell} - 1 \right] \; P_l(\cos\theta) \right|^2$$

- Phase shifts:– are defined from asymptotic values of wave functions
- Uniqueness:– of phase shifts is NOT guaranteed, nor is the optical potential
- How the wave functions develops through the nucleus is NOT tested
- Physics principles: $\longrightarrow$ N-A OMPs are strongly non-local and E-dependent
- Wave functions of non-local and equivalent local potentials DIFFER markedly

So, fitting elastic scattering is a necessary but it is not a sufficient condition

for the validity of those distorted wave functions

Inelastic scattering — The distorted wave approximation (DWA)

Elements in a DWA analyses:–

1. Theory:–

   ⋆ Fully antisymmetrized involving nonlocal optical potentials to get
   the distorted wave functions

2. An effective (NN) interaction as transition operator:–

   ⋆ those found from mapping of NN g-matrices
   (as used in g-folding potentials)

3. Nuclear spectroscopy:–

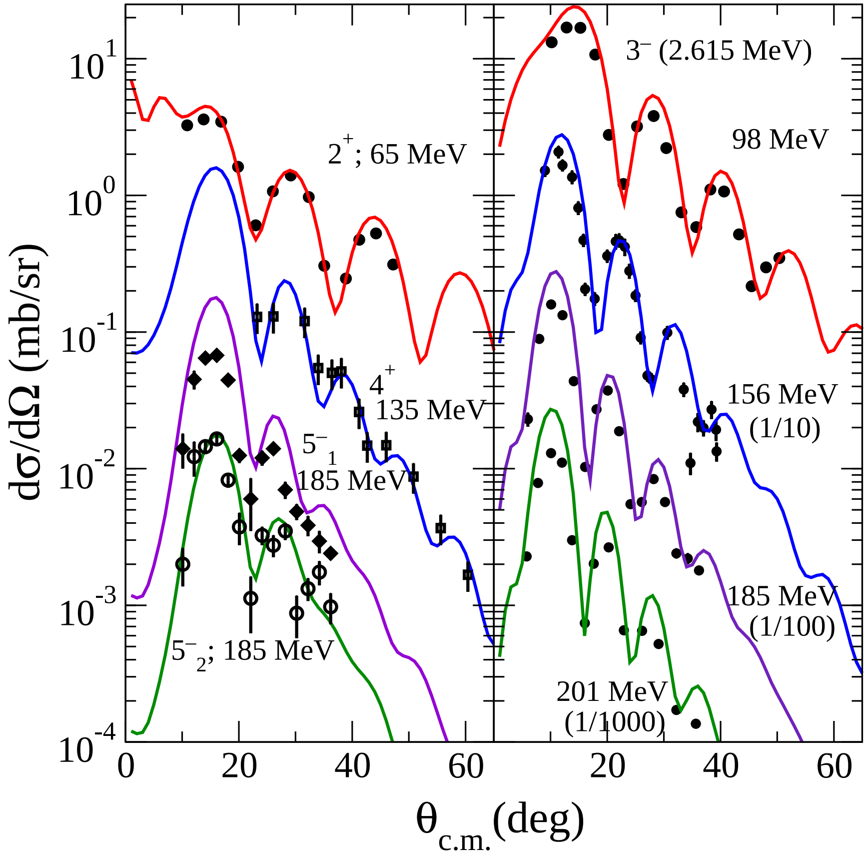
   ⋆ nucleon based model to give

   a) One-Body Density Matrix Elements (OBDME)

   b) Single nucleon bound state functions

Thus all details are preset – calculations are predictions and so tests of structure

Proton inelastic scattering from ^{208}Pb



M. Dupuis, *et al*. Phys. Letts. **B665**, 152 (2008)

The third age, a rebirth of Nuclear Physics

... the journey from stability

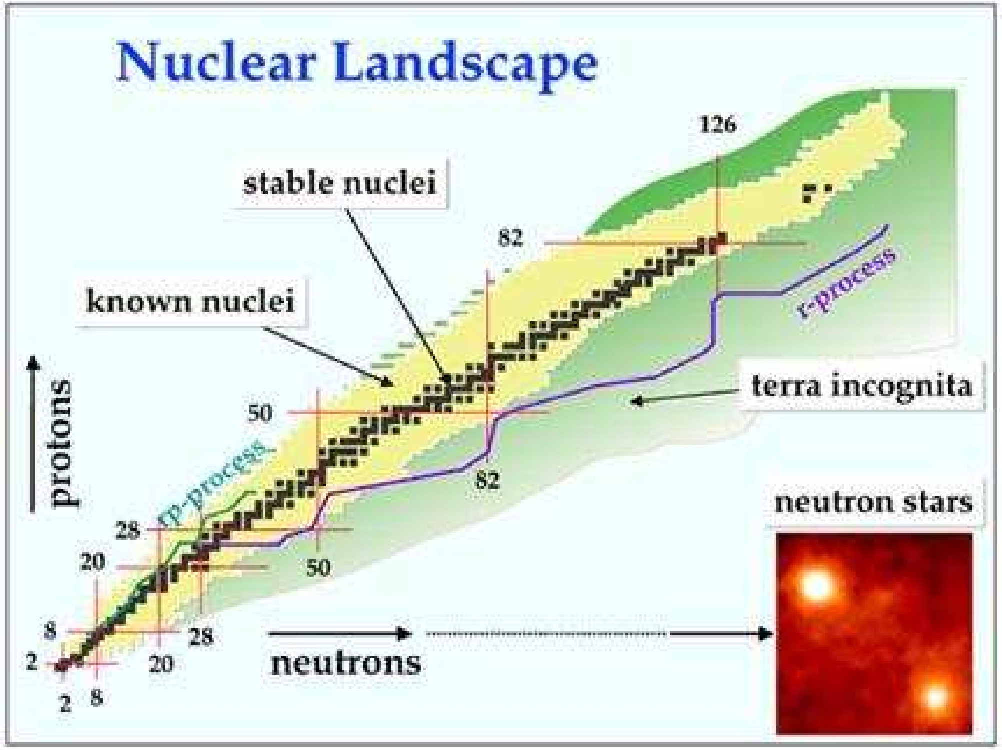
- The nuclear landscape, exotic nuclei, Borromean systems
- The CNO cycle: an example of a role of exotic nuclei
- Exotic systems: properties found using MCAS
- Exotic systems: properties found using g-folding

Studies of exotic nuclei: Use RIBs from heavy ion facilities

Low Energies (e.g. ≤ 10 MeV/A) via ISOL

Higher Energies (e.g. > 10 MeV/A) via In-flight fragmentation

A stylised modern chart of nuclei



- *'terra incognita'* edges: are nucleon drip-lines
- Most nuclei $\beta^{\pm}$ decay
- Some particle emissive
- Near drip lines: nucleons weakly bound
- Astrophysics: r-, rp-processes shown

What are exotic nuclei?

- The exotics are – nuclei with an excess of either protons or neutrons

  usually much more than that with stable nuclei

- There are over 2000 known. Most β-decay (neutron-rich, β^- proton-rich β^+)

- Most have extended distributions of the excess nucleons

  forming skins or halos

- As one approaches the drip-lines, nuclei become weakly bound
  - the cause of extended nucleon distributions
  - some have Borromean character –

    removal of one excess nucleon leaves a particle unstable system

    ^{8}He – a neutron skin nucleus — ^{7}He is neutron unstable

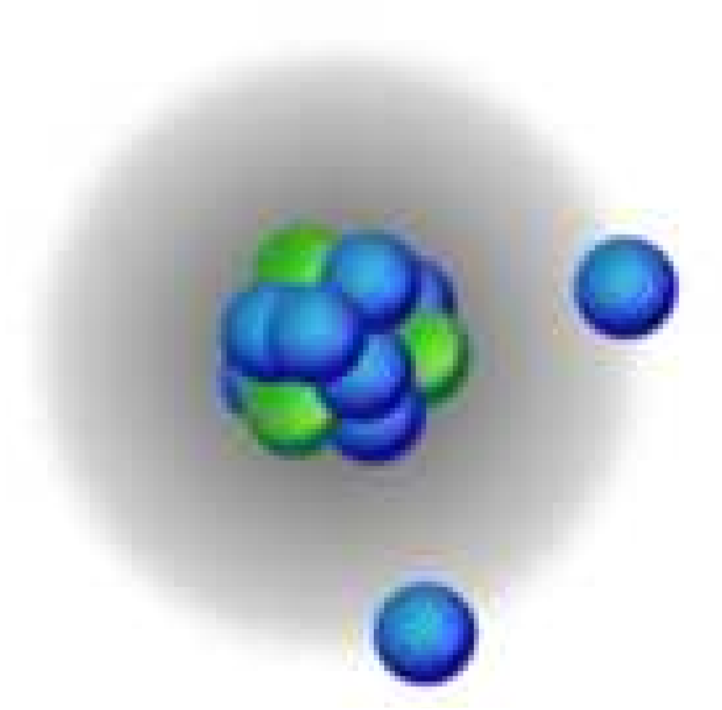
    ^{6}He – a neutron halo (extended distribution) — ^{5}He is neutron unstable

    ^{17}Ne – a proton halo — ^{16}F is proton unstable $\Longrightarrow$ ^{15}O + p ($\sim$ 0.5 MeV)

Borromean nucleus

from the three interlocking rings on the 15th century coat of arms of family Borromeo

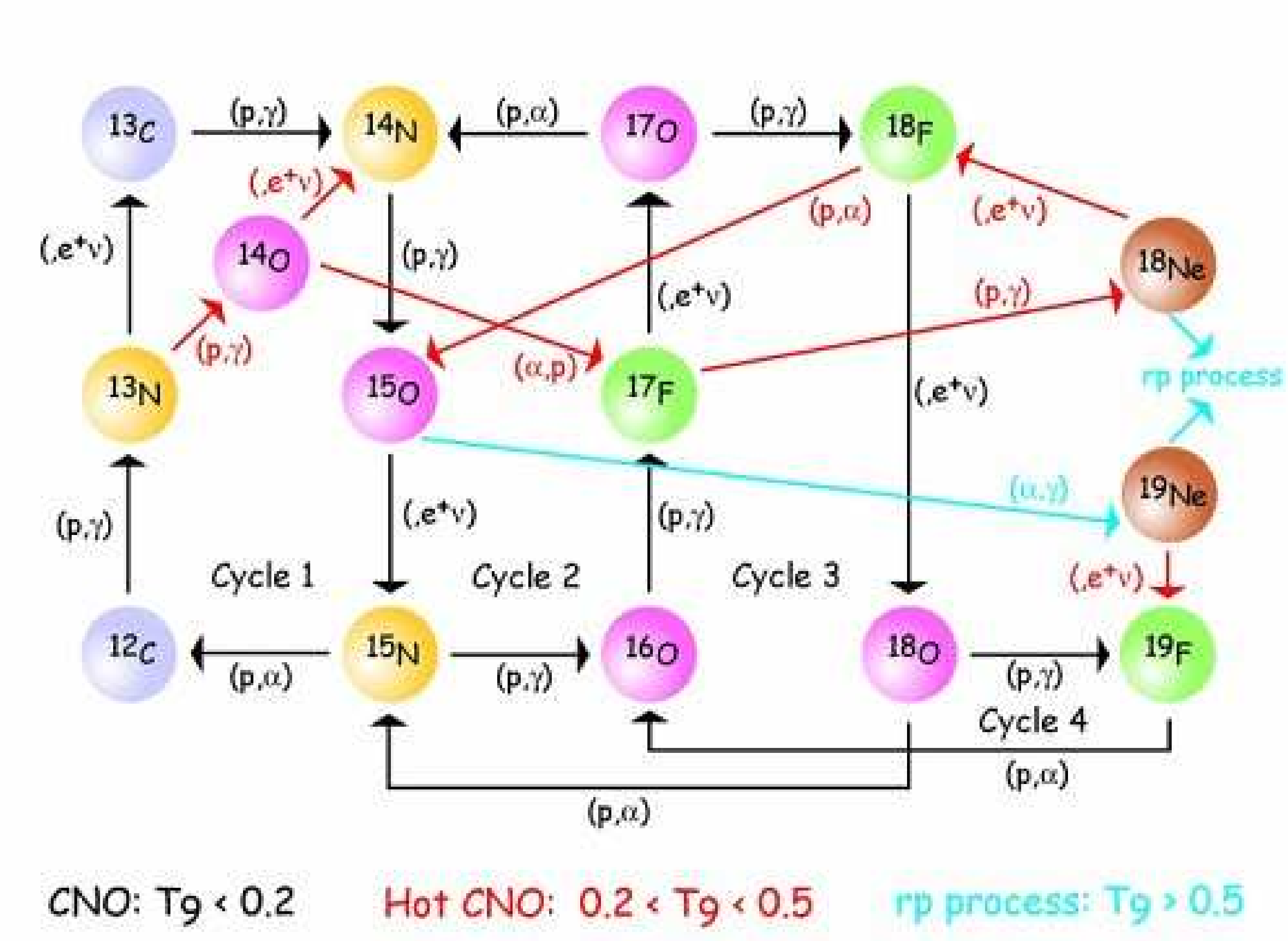
The family Borromeo coat of arms



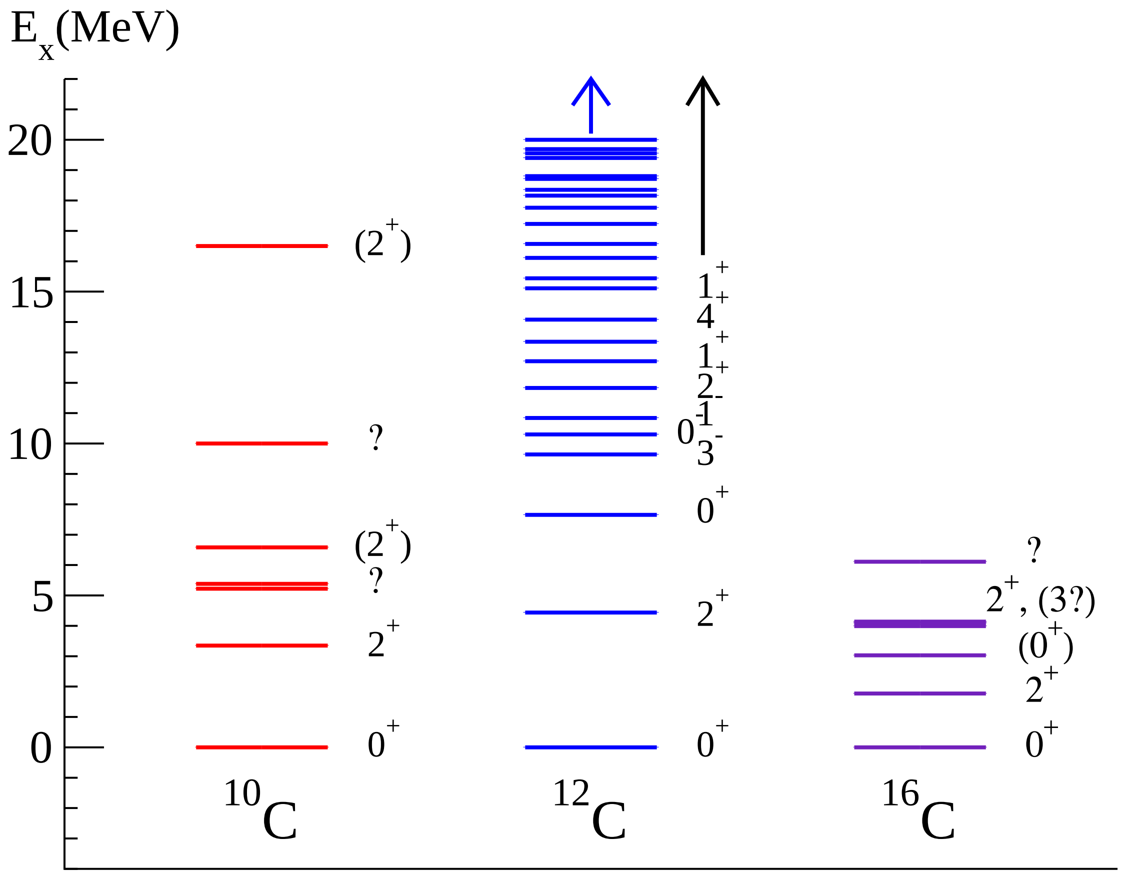
a poor (if not dangerous) pictorial

of a ‘Borromean’ nucleus

Example of the need to study exotic systems: the CNO cycle

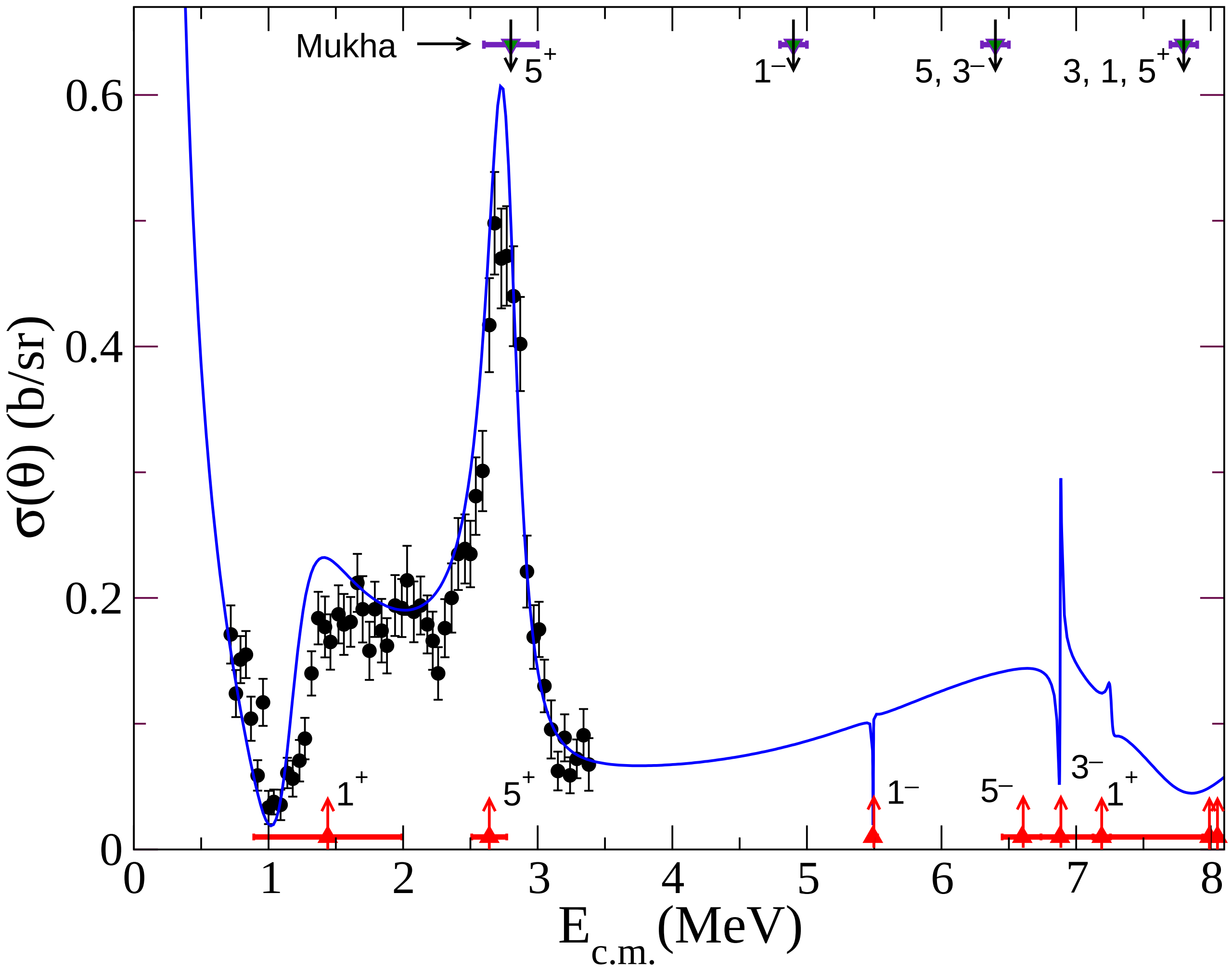


Example of known spectra (^{12}C is stable)

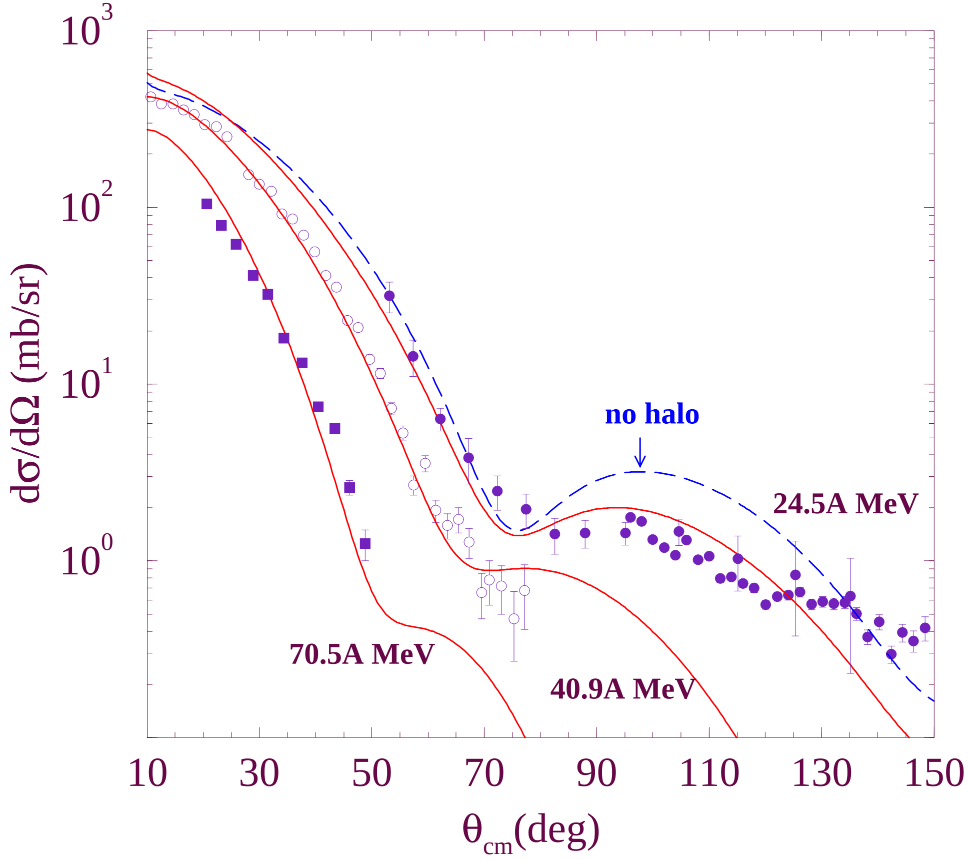


$$^{10}\text{C} \Longrightarrow_{e^+} {}^{10}\text{B} \qquad ; \qquad ^{16}\text{C} \Longrightarrow_{e^-} {}^{16}\text{N} \Longrightarrow_{e^-} {}^{16}\text{O}$$

Using MCAS: ^{14}O-hydrogen scattering (147°)



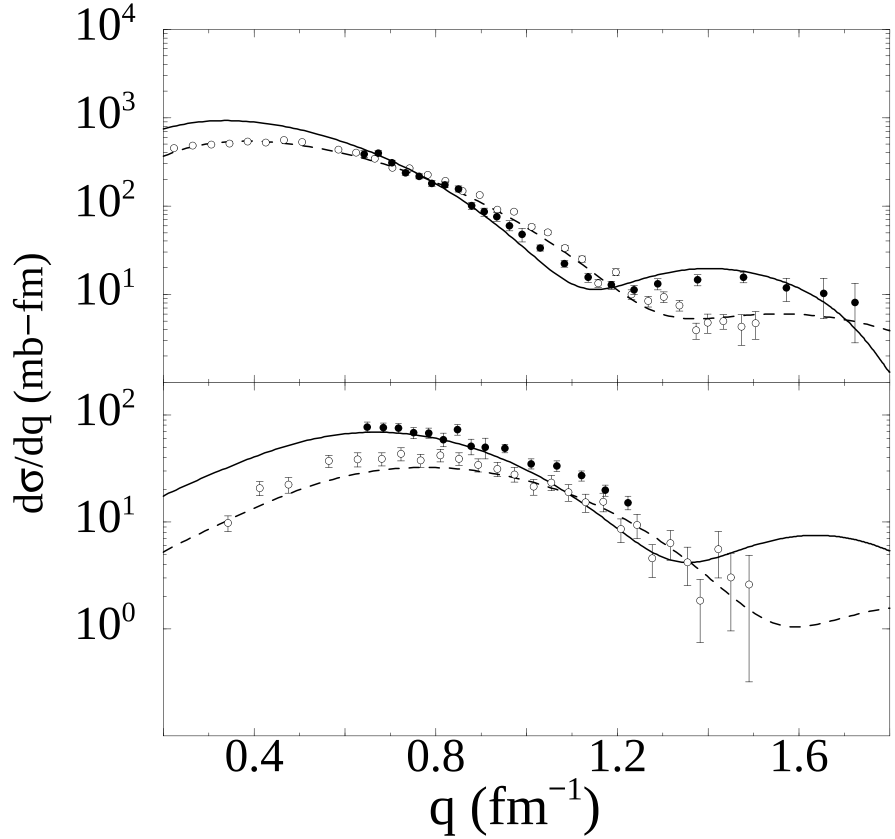
Using g-folding: ^{6}He-hydrogen elastic scattering



$\sigma^R_{expt.} = 420 \pm 5\%$ **mb ;** $\sigma^R_{calc} \sim 350/440$ **No halo/halo**

^{6}He-hydrogen scattering

Elastic (top) and inelastic to the 2_1^+ state (bottom)



24.5A MeV:– filled circles, solid lines ; 40.9A MeV:– open circles, dashed lines

To conclude:– A Few Adages

An Exhortation

Don't be afraid to try anything new!

Remember, amateurs built the ark,

professionals built the Titanic!

A Caution

Even if you are on the right track,

you get run over if you just sit there!

A Fear

Age doesn't always bring wisdom.

Sometimes age comes alone!