Twisting photons: the role of photon orbital angular momentum in astrophysics

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Abstract. The total angular momentum of a photon field can be decomposed canonically into spin and an orbital parts. Methods are now available for measuring the orbital angular momentum of light. These allow measurement the orbital angular momentum of a single photon, as well as of a beam. Little use has been made of these techniques in astrophysics. If used, they might unlock an as yet untapped source of information about distant sources.

Orbital angular momentum can be produced in a beam of light of known polarisation by passing the beam through a spiral phase plate in the direction of its with optical axis. It has been suggested that inhomogeneous media may produce orbital angular momentum in an analogous way. A simple spiral phase plate model may thus be useful in studying plasma vortices in cosmic structures such as astrophysical jets, AGN's, turbulent plasma in galaxies and galaxy clusters, and the vorticity field of the CMB. This paper discusses some general features of POAM, gives a mathematical description of the action of a spiral phase plate and shows how it might be used as a model in astrophysical studies.

1. Introduction

Electromagnetic waves have both spin and orbital angular momentum. The spin is related to the polarisation, and the orbital angular momentum is related to the surfaces of constant phase of the electromagnetic wave.

Recognition in the 1990s that light beams with an helical wave front have orbital angular momentum has been exploited in applications ranging from optical manipulation to quantum information processing. Wang et al. (2012) who have demonstrated that four light beams with different values of orbital angular momentum are able to transmit data at a capacity of 2.56 Tbit/s.This suggests that orbital angular momentum could be a useful degree of freedom for increasing the capacity of free-space communications. It may also provide a new source of information for astronomy and astrophysics.

Various methods have been suggested to measure POAM in astrophysics (Leach et al 2002). These range from measurement of the orbital angular momentum of a single photon to measuring the orbital angular momentum of beams of photons. Figure 1 shows the surfaces of constant phase of a beam of light in different states of orbital angular momentum. Such a beams can be produced by a spiral phase plate, shown in Figure 2. Harwit (2003) conjectured that the effect of an inhomogeneous medium on light might be modelled in this way. In this paper we consider a model of the spiral phase plate constructed using a geometric method due to Guillemin and Sternberg (1984). We setup the matrix that describes the propagation of a beam of light through this optical system. The matrix provides a geometrical optics description of the

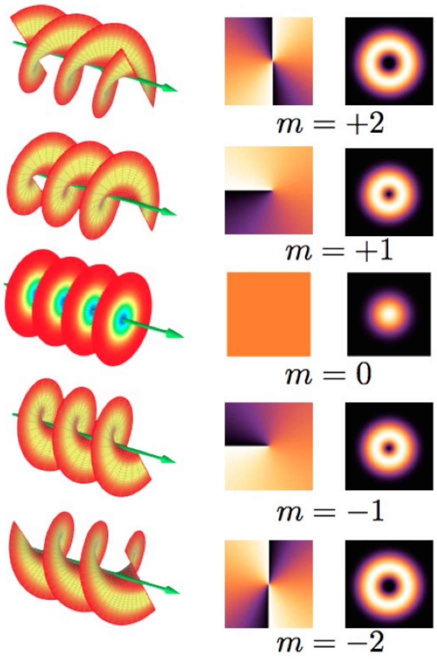

Figure 1. Different columns show the beam helical structures, phase fronts, and corresponding intensity distributions (Wikipedia).

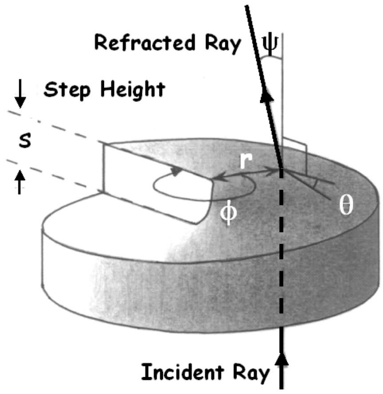

Figure 2. Photon orbital angular momentum (POAM) produced by a spiral phase plate (Harwit 2003).

beam. It therefore does not contain any phase information. To incorporate phase information we construct an associated integral, and reconstruct the wave using a method due to Haus (1984). The POAM information is extracted using a method due to Elias (2008). Some of these results are reported in the appendix to this paper.

2. Ways to calculate POAM

There are three main ways to calculate POAM. These are the paraxial approximation, the quantum mechanical treatment, and the Helmholtz approach. All three yield useful information about POAM. The paraxial approximation shows that the surfaces of constant phase are responsible for transferring orbital angular momentum from the photons to the medium through which they propagate. The quantum mechanical treatment shows that the orbital angular momentum is transferred in quanta. The Helmholtz treatment shows that when a beam propagates through empty space, the rotational part of the electric field of the beam carries information about the orbital angular momentum. We are currently implementing a geometrically based method, described by Guillemin and Sternberg (1984).

At each point on the spiral phase plate construct the matrix that describes the action of the spiral phase plate on the beam using the geometrical optics approximation. This matrix is an element of Sp(4, R), the group of 4x4 symplectic matrices with real coefficients. These matrices together completely determine the outgoing directions of the rays that constitute the beam. We compute the corresponding Fresnel integrals, which form a metaplectic group G on a Hilbert space. On integration, these produce the change in phase of the beam as it passes through the spiral phase plate. It can be shown that there is an homomorphism from G to Sp(4,R) which is 2 to 1. G thus provides a double cover of Sp(4,R).

The model of Guillemin and Sternberg (1984) uses a scalar field to model light. To model

POAM we will need to use a vector field model. Using a method by Haus (1984) we are able to reconstruct the electric field of the outgoing beam. Once we have the electric field of the outgoing beam we are finally ready to extract the POAM, for which we follow the method of Elias (2003). Some relevant calculations are displayed in the appendix to this paper.

3. Application

Harwit (2003) suggests that a turbulent medium with discontinuities can be regarded as a screen of randomly distributed spiral phase plates. We would like to model this screen. Discontinuities in the immediate surroundings of an object, or in the medium between the source and the telescope, produce orbital angular momentum in the photon beam as it traverses the medium. The propagating electromagnetic wave consists of m intertwined helical wave fronts. m is called the winding number. The multi-pole axis of the detected radiation lies along the line of sight. This makes it possible in principle to measure the POAM of the beam. We wish to model plasma vortices in cosmic structures such as galaxies and galaxy clusters. Plasma Vortices are believed to exist in Astrophysical Jets. Sources of Astrophysical Jets include Extragalactic Radio Sources powered by AGNs, Bipolar Flows from Young Stellar Objects, Jets from Young Binary Neutron Stars and or Black Hole Candidates, and Jets inside Planetary Nebulae, illuminated by a young or forming White Dwarf (Kundt 1996). It is anticipated that they may be studied through POAM.

4. Conclusion

The matrix that describes the spiral phase plate in the geometrical optics approximation is equation (10) in the appendix. Since there is one at each point on the spiral phase plate, this system may be described in terms of a principal bundle. The corresponding Fresnel integrals that describe the beam passing through the spiral phase plate are of the form of equation (11) in the appendix. We have written a program that calculates the matrix and integrates the corresponding integral at each point on the phase plate. We have written another program that reconstructs the electric field of the out going beam. We are in the process of writing a program that calculates the photon orbital angular momentum of the outgoing beam. We will then be in a position to model the screen of spiral phase plates.

Appendix

1. Geometrical Optics

In linear optics we assume the index of refraction is constant across refracting surfaces. In geometrical optics we assume it is a smoothly varying function. In Gaussian optics we want to trace the path of a beam through an optical system. We can do this by studying rays that are coplanar. We then only need to compare the direction of the light after it exits the optical system with the direction of the incoming ray. Define the reference plane as a plane perpendicular to the optical axis (z axis). We can then completely specify the direction of the light with q, its height above the z axis, and θ, the angle it makes with the z axis. Then for a simple optical system we may choose two reference planes at z_1 and z_2, positioned respectively behind and in front of the system, we only need to find

$$u' = \begin{pmatrix} q_2 \\ \theta_2 \end{pmatrix} \tag{1}$$

as a function of

$$u = \begin{pmatrix} q_1 \\ \theta_1 \end{pmatrix} \tag{2}$$

Taking the index of refraction into account, put $p = n\theta$, then since we are only interested in the linear terms, we want

$$\begin{pmatrix} q_2 \\ p_2 \end{pmatrix} = M \begin{pmatrix} q_1 \\ p_1 \end{pmatrix} \tag{3}$$

In Gaussian optics, there are only three ways that an optical system can affect the beam, namely transmission, refraction or reflection. We do not consider reflection. In linear optics, we assume that the transformations are linear from Gaussian optics and drop the rotational symmetry. We thus need four variables, two to describe where the beam intersects the reference plane and another two to define the direction of propagation. Therefore, let z_1 and z_2 be two reference planes and then a ray will be completely defined by

$$u_1 = \begin{pmatrix} q_{x_1} \\ q_{y_1} \\ p_{x_1} \\ p_{y_1} \end{pmatrix} \tag{4}$$

and

$$u_2 = \begin{pmatrix} q_{x_2} \\ q_{y_2} \\ p_{x_2} \\ p_{y_2} \end{pmatrix} \tag{5}$$

where the symbols have the usual meaning. To describe the effect of the system on the ray, we need a 4×4 matrix M such that

$$u_2 = M u_1 \tag{6}$$

In this text we are not going to follow the convention established by Guillemin and Sternberg by combining the angle of incidence and the index of refraction under the symbol p. We are going to use the angle and the index of refraction explicitly.

1.1. 4×4 *Linear Optics calculation of the Spiral phase plate*

The progression of a beam passing through the spiral phase plate can be expressed in terms of three separate propagation matrices. The first describes incidence on the base of the spiral phase plate,

$$M_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{n_1}{n_2} & 0 \\ 0 & 0 & 0 & \frac{n_1}{n_2} \end{pmatrix} \tag{7}$$

The second describes propagation through the medium

$$M_2 = \begin{pmatrix} 1 & 0 & d + s(\varphi) & 0 \\ 0 & 1 & 0 & d + s(\varphi) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{8}$$

The final matrix describes the beam exiting the spiral phase plate,

$$M_3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{n_2}{n_1} & 0 \\ 0 & 0 & 0 & \frac{n_2}{n_1} \end{pmatrix} \tag{9}$$

Therefore the matrix that describes the action of the system in the linear optics approximation is the product $M = M_3 M_2 M_1$ or

$$M = \begin{pmatrix} 1 & 0 & (d + s(\varphi))\frac{n_2}{n_1} & 0 \\ 0 & 1 & 0 & (d + s(\varphi))\frac{n_2}{n_1} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{10}$$

1.2. Wave Optics

The prescription in Guillemin and Sternberg (1984) then yields the following integral that corresponds to our matrix,

$$(Fc_1)(q_2) = exp(-i\pi/4)|(dn_2/n_1)^2/\lambda|^{-1/2} \int c_1(q_1) exp[-i\pi(q_1 - q_2)^2/\lambda(dn_2/n_1)^2] dq_1 \tag{11}$$

2. References

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