

First calculation of the full space-time evolution of jets

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Abstract. Particle physics has had remarkable success in describing collider data using usual Feynman diagram techniques, but there is still little knowledge regarding what happens to particles during the time of interaction. We use the Schwinger-Keldysh finite-time formalism applied to an interacting scalar field theory we lay down the foundation to derive a perturbative expression for the energy momentum tensor associated with the production of an off-shell jet, in an effort to analytically probe the untouched regime of finite-time physics. Possible applications include perturbative calculations of dispersion relations for interacting non-linear theories, insight into the flow of momentum for off-shell particles, and the creation of a hybrid early-time pQCD/late-time AdS/CFT energy loss model to describe high-momentum observables in heavy-ion collisions.

1. Introduction

Asymptotic freedom in QCD suggests that a perturbative approach with weak coupling will be well justified during the initial stages of a high energy collision.[4] We want to understand the behaviour of particles in a weakly-coupled interacting theory as a function of time. To discuss the flow of momentum in the interacting system, we find the expectation value of the energy momentum tensor given by $T_{\mu\nu} = \sum_i \partial_\mu \phi_i \partial_\nu \phi_i - g_{\mu\nu} \mathcal{L}$.

This is typically done by choosing an asymptotically free initial state, propagating this state in time to the full interacting theory, measuring the operator on this state, and propagating the result back to the initial state. The path given by this evolution is known as the Schwinger-Keldysh contour. We find

$$\langle \hat{O}_H \rangle(t) = \langle \underline{T} \exp \left(i \int_{-\infty}^t dz_1 \hat{H}_I^-(z_1) \right) \hat{O}_I(t) \overline{\underline{T}} \exp \left(-i \int_{-\infty}^t dz_1 \hat{H}_I^+(z_1) \right) \rangle \quad (1)$$

Here the + and - are only used to distinguish the Hamiltonians in the time ordered and anti-time ordered exponentials, but the same Hamiltonians are used. It can be shown that this object is equivalent to the contour ordered exponential

$$\langle T_C \left(e^{-i \int_{-\infty}^t dz_1 (\hat{H}_I^+(z_1) - \hat{H}_I^-(z_1))} \hat{O}_I(t) \right) \rangle, \quad (2)$$

where T_C is the contour ordering operator which orders the fields by their position along the Schwinger-Keldysh contour given below.¹ Causality allows us to send t limit in the integral to



Figure 1. Time evolution indicated by the Schwinger-Keldysh contour.

∞ to simplify the calculations without changing the result.

This system can be solved in a similar way to usual diagrammatic calculations, now with a different set of propagators. There will now be 4 possible propagators due to contractions between $+$ and $-$ field operators. The contractions $\langle 0|T_C\{\phi^i\phi^j\}|0\rangle$ will give propagator contributions in momentum space of the form

$$D^{ij}(p) = \begin{cases} \frac{i}{p^2 - m^2 + i\epsilon} & \text{if } i, j = +, + \\ 2\pi\theta(-p^0)\delta(p^2 - m^2) & \text{if } i, j = +, - \\ 2\pi\theta(p^0)\delta(p^2 - m^2) & \text{if } i, j = -, + \\ \frac{-i}{p^2 - m^2 - i\epsilon} & \text{if } i, j = -, - \end{cases}$$

External states taken to be on-shell at infinity are given by

$$|\phi\rangle = \int \frac{d^3q}{(2\pi)^3 \sqrt{2E_q}} \varphi(\vec{q}) e^{-i(q \cdot z_1)} \Big|_{q^0=E_q} |\vec{q}\rangle. \quad (3)$$

where $\varphi(\vec{q})$ is a smooth, complex function sharply peaked at some momentum P_0 . In this paper we will choose at most a single particle to be in the initial and final states.² A further analysis of these techniques found in [2] and [5].

2. Previous work

It can easily be shown that the energy momentum tensor associated with the free theory $\mathcal{L} = \frac{1}{2}(\partial_\mu\psi)^2 - \frac{1}{2}m_\psi^2\psi^2$ will be given by

$$\langle\psi|T_{\mu\nu}|\psi\rangle = \frac{P_{0\mu}P_{0\nu}}{E_{P_0}}, \quad (4)$$

where we have subtracted the vacuum expectation value $\langle 0|T_{\mu\nu}|0\rangle$. Similarly by considering a theory with coupling to a classical source $\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m_\phi^2\phi^2 - gj(x)\phi$ we find that

$$\langle 0|T_{\mu\nu}|0\rangle = \partial_\mu\varphi\partial_\nu\phi - g_{\mu\nu} \left(\frac{1}{2}\partial_\alpha\varphi\partial^\alpha\varphi - \frac{1}{2}m^2\varphi^2 + j(x)\varphi \right) \quad (5)$$

where $\varphi(x) = i \int d^4y j(y)D_R(x-y)$ and $D_R(x-y)$ is the retarded Green's function associated with the classical equations of motion.

These results are not profound, but serve as a comparison to check that this method is sensible and consistent. The vacuum expectation in the classically coupled system will presumably be comparable to more complicated scalar theories to the leading order.

¹ The superscript $+$ indicates a field on the top path of the contour, which will be evaluated before any field with index $-$ which is located on the bottom path.

² “Final” here is used in the sense that it appears at the end of the Schwinger-Keldysh contour, but at the same initial time as the initial state. Because this is an expectation value, the initial and final states will have to be the same.

3. Feynman Rules for Expectation values in the Schwinger Keldysh Approach

As with the standard Feynman approach, we are able to draw diagrams indicating possible contractions of the fields. Measuring a composite operator $\mathcal{O}(x)$ which is the product of n field operators will result in an insertion in each diagram connecting to n legs.³ We can take the fields of the composite operator to be evaluated on either the $+$ or the $-$ contour without changing the result. We therefore choose the operator to be evaluated entirely on the $+$ contour.

Typically we can pull out an overall factor of $e^{-ix(q_1-q_2)}$ in each diagram. One way of motivating this is by noting that our composite operators (and thus diagrams) will be x dependent, but that integration over d^4x should make them mathematically equivalent to diagrams associated with a scattering problem with an additional interaction term $\int d^4z \mathcal{O}(z)$. In such a problem, the diagram should be proportional to $\delta^4(q_1 - q_2)$, where q_1 and q_2 indicate this initial and final momenta. The only way this can be possible is if our original diagram is proportional to $e^{-ix(q_1-q_2)}$.

3.1. Feynman rules

Here we consider a scalar field theory with a single interaction term proportional to λ . We write down every possible diagram at relevant order, and add a single n -point insertion indicating the propagator (or vertex) on which the operator is measured. Every vertex given by an interaction is labeled by some index $i, j, k \dots \in \{+, -\}$ and all combinations of these are summed. This is done for every possible position of this insertion. All combinations of the insertion are then also summed.

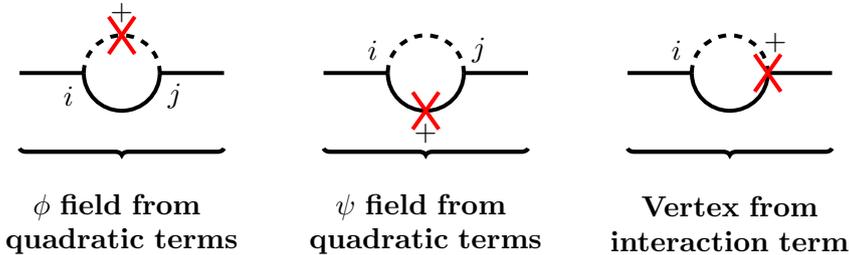
- Internal lines contribute a propagator term D^{ij} in momentum space, where i and j are given by the indices of the corresponding attached vertices.
- Vertices of type “ $+$ ” contribute $-i\lambda$, and vertices of type “ $-$ ” contribute a factor of $+i\lambda$. If a vertex is the result of an operator insertion, do not include this extra factor.
- External lines give a contribution of 1, but we multiply the overall result by $e^{-ix(q_1-q_2)}$ and integrate over the momenta of the initial and final states.

Lastly, for a physical result we subtract out contributions of $\langle 0|T_{\mu\nu}|0\rangle$ and $\langle 0|\phi T_{\mu\nu}|0\rangle$ (both momentum independent infinite constants) from our diagrams to exclude vacuum and tadpole contributions. We will also consider only amputated diagrams with no operator insertions on the external legs.

4. Scalar Yukawa Theory

We consider a theory with lagrangian $\mathcal{L} = \frac{1}{2}(\partial_\mu\psi)^2 - \frac{1}{2}m_\psi^2\psi^2 + \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m_\phi^2\phi^2 - \lambda\psi\phi\psi$.

The quadratic terms in the energy momentum tensor will add insertions to modify the internal propagators, the cubic term will modify the vertices between 3 propagators. The diagrams to consider for the λ^2 contributions are given by:



For example, we consider the diagram contribution for $\langle \psi|\partial_\mu\phi(x)\partial_\nu\phi(x)|\psi\rangle$. The terms that contain anything other than the D^{++} propagator turn out to vanish due to overconstraining by the δ functions. We are left with

$$-8\lambda^2 e^{-ix(q_1-q_2)} \int \frac{d^4p}{(2\pi)^4} \left(\frac{i}{p^2 - m_\psi^2 + i\epsilon} \frac{i(q_2 - p)_\mu}{(q_2 - p)^2 - m_\phi^2 + i\epsilon} \frac{i(q_1 - p)_\nu}{(q_1 - p)^2 - m_\phi^2 + i\epsilon} \right). \quad (6)$$

³ It can be shown that (at least for the cases we consider) the operator contracting with itself will yield no contribution to the total expectation value.

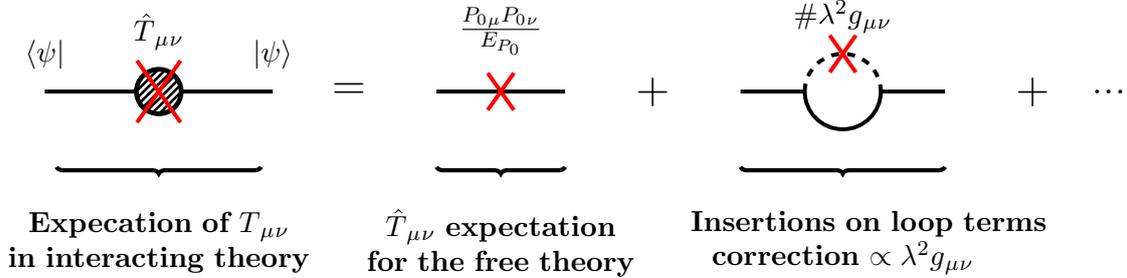
Introducing Feynman parameters and performing a Wick rotation on the p^0 co-ordinate, we obtain an equivalent expression,

$$8\lambda^2 e^{-ix(q_1 - q_2)} \int_0^1 dx \int_0^{1-x_1} \int \frac{d^4 p_E}{(2\pi)^4} \frac{(p_E + (q_2 - k))_\mu (p_E + (q_1 - k))_\nu}{(p_E^2 + \Delta)^3}, \quad (7)$$

where $\Delta = (x_1 q_1 + x_2 q_2)^2 + m_\psi^2 - (2m_\psi^2 - m_\phi^2)(x_1 + x_2)$. Multiplying out the numerator, the terms linear in p_μ will vanish over the integral. Using that $\frac{1}{d} g_{\mu\nu} p_E^2 = p_{E\mu} p_{E\nu}$, we see that there will be a term proportional to

$$\frac{1}{d} g_{\mu\nu} \int \frac{d^d p_E}{(2\pi)^d} \frac{p_E^2}{(p_E^2 + \Delta)^3} = \frac{1}{d} g_{\mu\nu} \frac{1}{(4\pi)^{d/2}} \frac{d \Gamma(2 - \frac{d}{2})}{2 \Gamma(3)} \left(\frac{1}{\Delta}\right)^{2 - \frac{d}{2}} \quad (8)$$

For $d \rightarrow 4$ this expression is logarithmically divergent. Methods exist to renormalize expectation values of operators which can be found in [3] but as a conserved quantity $T_{\mu\nu}$ may not require this treatment. Our expectation is that when we add the relevant terms together the divergences will cancel. The diagram below gives a description of the terms contribute to the full energy momentum tensor.



References

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