# Calculating solar irradiance to determine optimal angles of solar cells for De Aar

**Graham Webber and Hartmut Winkler**
Department of Physics, University of Johannesburg, PO Box 524, 2006 Auckland Park, Johannesburg, South Africa

E-mail: 201337952@student.uj.ac.za

**Abstract**. Solar energy harvesting is a growing industry in South Africa. De Aar is a favoured location for solar power stations, as it has high potential yield and is close to some of South Africa's largest power lines. This paper uses standard methods of solar irradiance estimation to calculate the potential yield with respect to wavelength for De Aar. It is necessary to take wavelength into account as light is not extinguished uniformly with respect to wavelength. De Aar was chosen for this paper because there are many years of total surface irradiance data available and some data for irradiance in specific wavelength bands. Comparison of these values with actual data collected in De Aar was done to determine the accuracy of these models for the conditions in De Aar. These estimations were done for a typical midsummer's day and a typical winter's day, employing four different photovoltaic device spectral response functions. We use these calculations to determine the optimum panel alignments.

## 1. Introduction

Solar irradiance research is necessary to provide information and knowledge regarding the optimum and expected energy yields to developers of photovoltaic solar power systems, whether these are on a domestic, medium or full solar power station scale. The specific location and meteorological conditions play a major role in the calculation of a photovoltaic system's yield. The ability to evaluate the impact of these factors is vital in determining the economic viability of a proposed installation and the optimal design, and prediction of the system performance. The best way of determining the amount of solar irradiance at a site is through long-term data monitoring, but because of practical and cost considerations these data are measured only at a few stations, and mostly only as integrated flux rather than as wavelength specific intensity.

In this paper we focus on the direct solar beam irradiance, which is under clear, moderately turbid skies by far the largest contributor to the total solar irradiance collected at ground level. Photovoltaic devices in addition also respond to the so-called diffuse light component, i.e. scattered sunlight reaching the ground from directions other than the solar position. This component is also a function of azimuth and zenith angle, and is particularly difficult to predict accurately. A full study of the solar irradiance at a site requires the full consideration of diffuse irradiance as well, and is in progress. Furthermore, while this paper restricts itself to photovoltaic devices, the end results of this study can also be applied to concentrated solar power (which only utilises the direct beam).

For this study we chose to apply irradiance models to the estimation of direct solar radiation at De Aar in the Northern Cape, a town near an important electric power line node that has, in view of its ample sunshine, been chosen as the site of several major solar power station developments. De Aar has the added advantage that it hosts a well-developed weather station which includes a node of the Baseline Surface Radiation Network (BSRN) [1]. Furthermore, there has in the past been an extensive study of the solar irradiance as a function of wavelength in De Aar [2]. Theoretical data from these calculations can therefore be compared to the recorded data for each wavelength-band.

## 2. Theory

The direct beam spectral irradiance $E_b$ is the solar light (in units of power per unit area per unit wavelength) that passes through the atmosphere and reaches the panel without being scattered away or absorbed by gases or aerosols. Direct beam irradiance is thus that part of the extra-terrestrial spectral irradiance $E_0$ incident on a ground level panel normal to the beam. We adopted for $E_0$ the synthetic/composite extra-terrestrial spectrum of Gueymard [3]. The factors that determine the magnitude of $E_b$ are the atmospheric transmittances for the different extinction processes including Rayleigh and Mie (from aerosol) scattering, ozone, uniformly mixed gases and water vapour absorption. The direct beam irradiance for a specific wavelength is calculated as [4]:

$$E_b = E_0 T_R T_a T_o T_g T_w \quad (1)$$

where $T_R$, $T_a$, $T_o$, $T_g$ and $T_w$ are the transmittance factors for Rayleigh scattering, aerosol, ozone, mixed gasses and water vapour respectively, and are all functions of the wavelength $\lambda$. The transmittance factors are described by Bouguer's Law [4]:

$$T = \exp(-m\,\tau) \quad (2)$$

where $\tau$ is the optical thickness, and $m$ is the optical airmass.

We describe the solar position in terms of the zenith angle $\zeta$ and the azimuth $\gamma$. Unless the Sun is close to the horizon, the airmass is then well approximated by the expression $m = \sec\zeta$.

The Rayleigh optical thickness has been calculated from the equation below, which was developed from a least-squares curve fit to the theoretical expression and deviates from this by 0.01% or less throughout the spectrum [4]:

$$\tau_{R\lambda} = (\mathrm{P}/1013.25\ \mathrm{mbar})(117.2594\,\lambda^4 - 1.3215\,\lambda^2 + 3.2073\times10^{-4} - 7.6842\times10^{-5}\,\lambda^{-2})^{-1} \quad (3)$$

To determine the ozone, mixed gas and water vapour optical thickness, we adopted the expressions and constants of the SMARTS2 model of Gueymard [4]. Water vapour has minimal effect on the transmission outside the near infrared. There however it is by far the most important absorber. The total precipitable water in the atmosphere may vary rapidly with time and thus so does the water vapour optical thickness.

Detailed complete spectral determinations of the aerosol optical thickness are difficult and hence only rarely available. Hence the aerosol transmissivity can often only be estimated with the aid of climatological information. Alternatively, estimates of turbidity are sometimes based on meteorological visibility data. The lack of detailed knowledge of the aerosol properties justifies the use of a simplified methodology, the modified Angström approach, which considers two different spectral regions, below and above $\lambda_0 = 0.5$ µm. The aerosol transmittance is obtained from [4]:

$$\tau_{a\lambda} = (\beta_i \lambda)^{-\alpha_i} \quad (4)$$

where $\alpha_i = \alpha_1$ if $\lambda < \lambda_0$ and $\alpha_2$ otherwise, and $\beta_i = 2^{\alpha_2-\alpha_1}\beta$ if $\lambda < \lambda_0$ and $\beta_i = \beta$ otherwise. $\beta$ is the Angström coefficient and expresses the turbidity. $\alpha$ is the wavelength exponent it is related to the optical characteristic of the aerosols. The values of the wavelength exponents $\alpha_1$ and $\alpha_2$ were obtained by fitting the spectral optical coefficients of different reference aerosol models [4].

## 3. Procedure

We determined the solar zenith and azimuth angles through the PSA algorithm, which has average deviation in the solar vector of 0.147 minutes of arc [5].

We then examined data obtained from multifilter rotating shadowband radiometer measurements in De Aar for 4 August 2000 and 25 December 2000, previously presented by Winkler et al. [2]. These data contain measurement at 60 second intervals and for each of ~10 nm wide equivalent wavelength bands centred at 416 nm, 501 nm, 613 nm, 670 nm, 867 nm and 939 nm. We consequently also calculated the model fits at one minute intervals for each band.

The multifilter rotating shadowband radiometer tracks the Sun so the direct beam irradiance is calculated as if the panel in the model is normal to the solar vector. Thus the surface in the model was set to be normal to the solar vector.

The total precipitable water, which also depends on temperature and atmospheric pressure, only has an effect on one of the wavelength bands we are interested in: the one centred at 939 nm. Average temperatures for the days in question were determined from archived weather station data [6]. Air pressure was estimated from the altitude at the site. The parameters determining $T_a$ and $T_w$ were adjusted to find the best fit for the calculated data against each of the measured wavelength bands. The $\alpha_1$ and $\alpha_2$ values are under most conditions such that the resulting aerosol transmissivity is at its minimum at the shorter wavelengths.

The total daily ground level direct beam irradiance incident on a tilted panel was calculated for the two days examined by summing the amounts for each wavelength step and each time step. If the panel tilt angle is $\eta$ and the panel azimuth is $\gamma_P$, the panel inclination relative to the solar beam $\theta$ is then given by [7]

$$\cos\theta = \sin\zeta \cos(\gamma - \gamma_P) \sin\eta + \cos\zeta \cos\eta \tag{5}$$

The response curve for some typical solar cells was applied to the function of the incident light and the optimal stationary panel orientation was then calculated [8]. The azimuth angle of the panel was taken as zero, as this model is symmetrical around due north. The optimal stationary panel tilt was calculated by gradually varying the zenith angle of the panel and redoing the yield calculation until the maximum daily yield is reached.

## 4. Results

The model predictions closely match the available empirical data. We display two examples of direct beam irradiance vs. time plots, for 4 August 2000, in figure 1 and figure 2. The discrepancies at approximately 8h30 are also visible in the other bands, and we believe these to be caused by an inaccuracy in the provided calibration of the instrumental angular response function for a particular solar zenith angle.

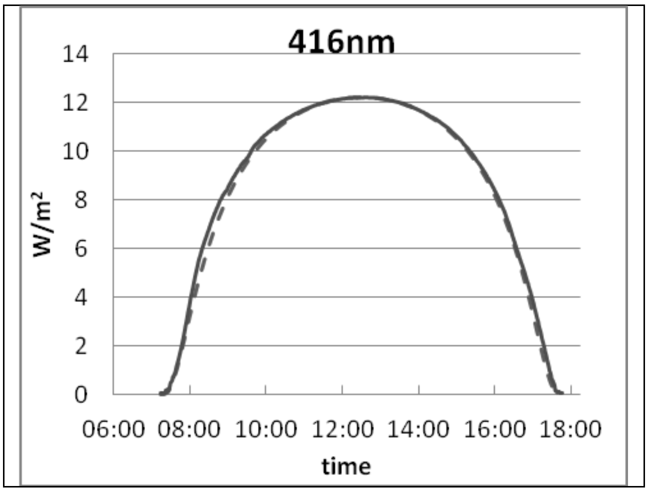


**Figure 1**: Empirical (solid line) and calculated data (dashed line) for 416 nm for 4 August 2000

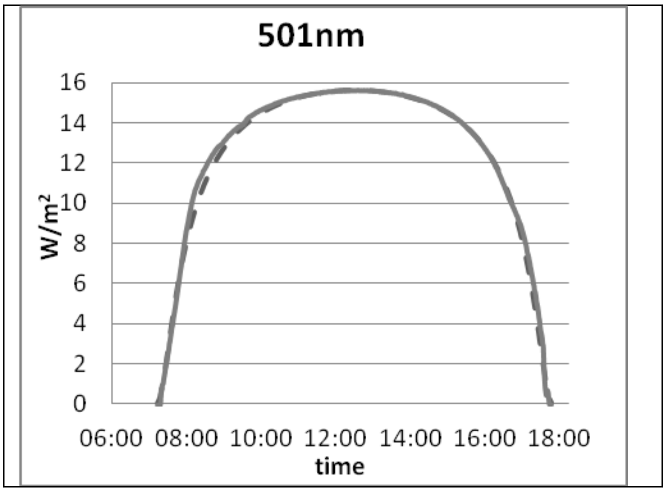


**Figure 2**: Empirical (solid line) and calculated data (dashed line) 501 nm for 4 August 2000

The optical thicknesses calculated were assumed not to vary in the course of the day. For ozone we adopted a seasonal average, which is justified in view of the ozone transparency being equal or close to 1 in all wave bands used. For this study we also ignore the mixed gas optical depth, as $CO_2$ and $O_2$ only absorb light at specific wavelengths, and these wavelengths do not correspond to the bands used in this study. The optical thickness for water vapour is zero for all the measured bands except at 939 nm where the values of 0.143 and 0.142 were determined for August and December respectively. The values for the Angström coefficient that yielded the best fits to the direct irradiance vs. time curves for 4 August 2000 and 25 December 2000 were 0.0122 and 0.0223 respectively.

The total daily direct beam irradiance was calculated to be 13064 $Wh.m^{-2}$ for 25 December and 8798 $Wh.m^{-2}$ for 4 August. This would correspond to the yield of a ‘perfect panel’ (with 100% quantum efficiency) able to track and align itself to the Sun throughout the day. Because of cost considerations, solar panels are usually fixed rather than Sun tracking. They are then set up at a specific tilt angle and orientation that is considered optimal for maximum power generation, either annually or seasonally. This optimum configuration depends on site latitude, altitude, prevailing meteorological and atmospheric conditions as well as the panel’s efficiency (which is a function of wavelength).

The optimal tilt angle for a fixed, ‘perfect’, north-south aligned solar panel was calculated as 52.6° north for 4 August and 4.4° south for 25 December, yielding 6930 $Wh.m^{-2}$ and 9037 $Wh.m^{-2}$ respectively. The ratio between the yield, at their optimum tilts, for 25 December and that of 4 August is 1.30. The apparent contradiction of deriving an optimal south-facing tilt angle in midsummer for a latitude south of the Tropic of Capricorn is explained by the fact that the early morning and late afternoon solar azimuths are south-east and south-west respectively at that time of the year.

We repeated the calculations using relative spectral response functions typical for three photovoltaic technologies using CdS, CuInS and c-Si. The results of this are summarised in Table 1.

**Table 1.** Optimal tilt angles and relative energy yield ratios calculated for different solar panel technologies.

| | perfect panel | CdS | CuInS | c-Si |
|---|---|---|---|---|
| optimal tilt: 25 Dec 2000 | 4.4° S | 1.7° S | 3.8° S | 5.4° S |
| optimal tilt: 4 Aug 2000 | 52.6° N | 51.4° N | 52.3° N | 53.0° N |
| optimal tilt yield ratio: 25 Dec vs. 4 Aug | 1.30 | 1.45 | 1.30 | 1.25 |

## 5. Discussion

The work presented in this paper is the precursor of an ongoing wider study that seeks to determine optimal solar panel orientations and projected energy yields for photovoltaic technologies under South African conditions, through the construction of site-specific solar spectral irradiance models and the convolution thereof with instrumental spectral response characteristics.

The results show that daily spectral irradiance curves calculated for specific wavelengths for selected days through the adopted model correlate to a good approximation to empirical data. Although the wavelength bands adopted in this study only represent a small part of the solar spectrum, they are well distributed across that part of the spectrum where photovoltaic cells are most sensitive. The greatest uncertainty is in those parts of the spectral irradiance curve affected by water vapour, such as near 939 nm, especially because of the sometimes rapid humidity fluctuations. The spectral regions most influenced by water vapour are however deeper in the infrared, where most panels show little or no sensitivity.

The case study presented here is based on calculations for two days that represent different times of year and thus different atmospheric conditions. The model’s data fit both times of year well. This gives us confidence to continue developing this model and expanding the test sites and probing other time periods. Some error is expected as the model is using average aerosol characteristics of the atmosphere while in reality high turbidity episodes occur periodically [2].

The Angström coefficient monthly averages determined by Power and Willmott [9] for Grootfontein Agricultural College, 140 km from De Aar, are 0.021 for December and 0.013 for August. These are in good agreement with the Angström coefficients that gave us the best fits for the days investigated. If these are typical for the site and season, as we believe they are, the Angström coefficient at De Aar is generally very low by international standards, confirming De Aar as a good location for solar power stations.

The difference between the optimal tilt yield ratio of each photovoltaic technology and that of the 'perfect' solar panel highlights the accuracy that may be gained when taking wavelength into account for solar power estimations. For a typical c-Si solar cell there is a lower fluctuation in yield with time of year than a 'perfect' solar panel (which represents a wavelength independent solar cell).

For 25 December there is a difference of ~1° (and ~0.5° on 4 August) between the optimal fixed tilt angles of a 'perfect' solar panel and a typical c-Si solar cell. This may only be a small value, but it does highlight how detailed calculations that fully consider wavelength dependence can lead to different outcomes, both in terms of the optimum tilt angle of a panel as well as the total energy harvested.

**References**

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