Tidal effects on pulsation modes in close binaries

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Abstract. Light curve data from the *Kepler* satellite on pulsating eclipsing Algol-type binary systems display a peculiar feature: the primary shows preferential excitation of pulsation modes with frequencies resonant with the orbital frequency of the binary system. A proposed explanation of this phenomenon is tidal driving of pulsations by the secondary. This paper presents a preliminary calculation of the effects of linear representations of tides on the pulsation frequencies of a polytropic primary.

1. Introduction

The *Kepler* space telescope has produced a large number of time-domain observations of unprecedented regularity and precision. A few hundred close eclipsing binary systems have emerged from the *Kepler* database and many of these have been studied intensively since the first *Kepler* data became available in 2009. Due to the precision (noise levels around a few micromagnitudes in white light) of the data, weak pulsation signatures have been detected in many close (semi-detached) systems with circularised orbits. These pulsation signatures frequently point to a resonant coupling of pulsation frequencies with orbital motion. Initial expectations are that resonance should not occur in a circularised orbit, since there is no substantial difference between the periastron and apastron separations of the binary components that could drive such a resonance.

However, given that these are close systems, tidal effects might be able to produce the apparent observed resonances. Since mass transfer between binary components is not considered, attention is given specifically to detached systems where negligible mass transfer occurs [1]. We present a simple model of the effects of tides on a polytropic star and indicate how a numerical model may be used to test this conjecture. We use the formula for the tidal potential as presented in Reynier's papers [2, 3]. We then Fourier transform the tidal potential with respect to the time variable. This yields an equation containing three delta functions that select a triplet of frequencies indicating a splitting of the eigenstates into three levels. The eigenstates are evenly spaced in the frequency domain. The spacing between them is determined by the angular velocity of rotation of the primary and the mean motion of the secondary. This splitting does not occur in a phase-locked circularised orbit, since no tidal dragging is present.

2. Equations of motion

We begin by considering a close binary system. Suppose the primary star has mass M, radius R and constant angular velocity Ω with respect to an inertial frame. The axis of rotation of the binary system is assumed to be perpendicular to its orbital plane. The companion star, with mass m, is modelled as a point mass. We will use a reference frame, with spherical coordinates (r, θ, φ), that co-rotates with the primary whose center is at its origin with the frames z-axis being parallel to the primary's axis of rotation. In this reference frame the equations of motion for an ideal fluid incorporating the effects of a tidal interaction are presented below. The conservation of momentum equation is given by:

$$\frac{\partial \vec{v}}{\partial t}(\vec{r}, t) + \left(\vec{v}(\vec{r}, t) \cdot \vec{\nabla}\right) \vec{v}(\vec{r}, t) = -\vec{\nabla}\phi(\vec{r}, t) - \frac{1}{\rho(\vec{r}, t)}\vec{\nabla}P(\vec{r}, t) - \epsilon_T \vec{\nabla}W(\vec{r}, t) \tag{2.1}$$

The quantity ϵ_T is a small, dimensionless parameter describing the magnitude of the tidal interaction and is defined as:

$$\epsilon_T = \left(\frac{R}{a}\right)^3 \frac{M}{m}$$

where a is the semi-major axis of the orbit of the companion. The tidal potential $W(\vec{r}, t)$ is given by [2] as:

$$W(\vec{r}, t) = \frac{GM}{R}\left(\frac{r}{R}\right)^2 \left\{\frac{1}{2}P_2(\cos\theta) - \frac{1}{4}P_2^2(\cos\theta)\cos\left[2\varphi + 2t(\Omega - n)\right]\right\} \tag{2.2}$$

where G, n, P_2 and P_2^2 represent respectively Newton's gravitational constant, the mean angular velocity of the orbit of the companion, the second order Legendre polynomial and the second order associative Legendre polynomial with azimuthal number 2. The first term in equation (2.2) represents the time-independent static tide, characteristic of a synchronous orbit, in which $\Omega = n$. The second term represents the time-dependent dynamic tide, characteristic of an asynchronous orbit.

In addition to the above equations we have three more equations completing the set, they are:

$$\frac{\partial \rho}{\partial t}(\vec{r}, t) + \vec{\nabla} \cdot \left[\rho(\vec{r}, t)\, \vec{v}(\vec{r}, t)\right] = 0 \tag{2.3}$$

which is the continuity equation,

$$\nabla^2 \phi(\vec{r}, t) = 4\pi G \rho(\vec{r}, t) \tag{2.4}$$

which is Poisson's equation, and lastly

$$P(\vec{r}, t) = \kappa\left(\rho(\vec{r}, t)\right)^\gamma \tag{2.5}$$

which is the polytropic equation of state. Collectively, equations (2.1) - (2.5) constitute a complete set of equations for the fluid variables.

3. Perturbation equations

To determine the perturbation equations we first consider an "unperturbed" equilibrium configuration, which we then perturb and linearise. The initial configuration is assumed to be an hydrostatic, spherically symmetric configuration. Linearisation is achieved by assuming that all perturbations are small so that all products of perturbations can be neglected. In pulsation theory, two types of variation are considered, called Eulerian and Lagrangian respectively. The first defined as the perturbation of physical fields, denoted by a prime, and the second the perturbation suffered by a given material fluid element, denoted by a δ [4, 5]. Given an arbitrary physical quantity, the Eulerian and Lagrangian perturbations of Q are defined respectively by:

$$Q' = Q\left(\vec{r_0}, t\right) - Q_0\left(\vec{r_0}\right) \tag{3.1}$$

$$\delta Q = Q\left(\vec{r}, t\right) - Q_0\left(\vec{r_0}\right) \tag{3.2}$$

where $\vec{r_0}$ and $\vec{r}$ represent the current position of the material fluid element in the equilibrium configuration and the perturbed configuration respectively. The vector quantity relating these two positions is called the Lagrangian displacement and is defined as:

$$\vec{\xi} = \vec{r} - \vec{r_0} \tag{3.3}$$

Using equations (3.1) - (3.3) it can be shown that the relationship between the Eulerian and Lagrangian perturbations is given, to first order in $\vec{\xi}$, by:

$$\delta Q\left(\vec{r}, t\right) = Q'\left(\vec{r}, t\right) + \vec{\xi} \cdot \vec{\nabla} Q_0\left(\vec{r}\right) \tag{3.4}$$

If the unperturbed system is assumed to be in hydrostatic equilibrium, the velocity field $\vec{v}$ is related to $\vec{\xi}$ by:

$$\vec{v}\left(\vec{r}, t\right) = \frac{\partial \vec{\xi}}{\partial t} \tag{3.5}$$

and

$$\vec{v}\left(\vec{r}, t\right) = \delta \vec{v}\left(\vec{r}, t\right) = \vec{v}\,'\left(\vec{r}, t\right) = \frac{\partial \vec{\xi}}{\partial t} \tag{3.6}$$

Using equations (3.1) - (3.6) with equations (2.1) and (2.3) - (2.5) the perturbation equations can be shown to be:

$$\rho'\left(\vec{r}, t\right) + \vec{\nabla} \cdot \left(\rho_0\left(r\right)\ \vec{\xi}\right) = 0 \tag{3.7}$$

$$\frac{\partial^2 \vec{\xi}}{\partial t^2}\left(\vec{r}, t\right) + \frac{1}{\rho_0\left(r\right)} \vec{\nabla} P'\left(\vec{r}, t\right) + \vec{\nabla} \phi'\left(\vec{r}, t\right) + \frac{\rho'\left(\vec{r}, t\right)}{\rho_0\left(r\right)} \frac{\partial \phi_0}{\partial r}\left(r\right) + \frac{\epsilon_T}{\rho_0\left(r\right)} \vec{\nabla} W\left(\vec{r}, t\right) = 0 \tag{3.8}$$

$$\nabla^2 \phi'\left(\vec{r}, t\right) - 4\pi G \rho'\left(\vec{r}, t\right) = 0 \tag{3.9}$$

$$P'\left(\vec{r}, t\right) - \gamma \left(\frac{P_0\left(r\right)}{\rho_0\left(r\right)}\right) \rho'\left(\vec{r}, t\right) = 0 \tag{3.10}$$

where the primed quantities represent the Eulerian perturbation of that physical quantity. The tidal term is not perturbed as it is considered to be an applied external force which does not undergo perturbation.

4. Eigenvalue problem

To determine the pulsation frequencies of the system, we search for normal mode solutions to equation (3.8). Because the perturbation equations are linear, the general solution can be constructed from a linear superposition of the normal mode solutions. Taking the Fourier transform of equations (3.7) - (3.10), we obtain the following equations:

$$\rho'(\vec{r},\omega) + \vec{\nabla}\cdot\left(\rho_0(r)\ \vec{\xi}(\vec{r},\omega)\right) = 0 \quad (4.1)$$

$$-\omega^2\vec{\xi}(\vec{r},\omega) + \frac{1}{\rho_0(r)}\vec{\nabla}P'(\vec{r},\omega) + \vec{\nabla}\phi'(\vec{r},\omega) + \frac{\rho'(\vec{r},\omega)}{\rho_0(r)}\frac{\partial\phi_0}{\partial r}(r) + \frac{\epsilon_T}{\rho_0(r)}\vec{\nabla}W(\vec{r},\omega) = 0 \quad (4.2)$$

$$\nabla^2\phi'(\vec{r},\omega) - 4\pi G\rho'(\vec{r},\omega) = 0 \quad (4.3)$$

$$P'(\vec{r},\omega) - \gamma\left(\frac{P_0(r)}{\rho_0(r)}\right)\rho'(\vec{r},\omega) = 0 \quad (4.4)$$

where $W(\vec{r},\omega)$ can be calculated using:

$$\begin{aligned} W(\vec{r},\omega) &= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} W(\vec{r},t)\,e^{-i\omega t}\,\mathrm{d}t \\ &= r^2\left(\frac{GM}{R^3}\right)\Bigg(\sqrt{\frac{\pi}{8}}P_2(\cos\theta)\,\delta(\omega) - \sqrt{\frac{\pi}{32}}P_2^2(\cos\theta) \\ &\quad \left\{e^{2i\varphi}\delta\left(2(\Omega-n)-\omega\right) - e^{-2i\varphi}\delta\left(2(\Omega-n)+\omega\right)\right\}\Bigg) \end{aligned} \quad (4.5)$$

Here δ represents the Dirac delta function. Given a set of boundary conditions, the values of ω satisfying these boundary conditions can be found numerically.

If we remove the tidal term, assume the Cowling approximation ($\Phi(\vec{r},t) \approx \Phi_0(r)$) and consider the perturbation of a spherically symmetric equilibrium configuration that admits only radial oscillations we obtain the eigenvalue equation for the radial oscillations of a polytropic star:

$$\alpha\frac{\partial^2\rho'}{\partial r^2}(r,\omega) + \beta\frac{\partial\rho'}{\partial r}(r,\omega) + \lambda\rho'(r,\omega) = 0 \quad (4.6)$$

with

$$\begin{aligned} \alpha &= \gamma\sigma \\ \beta &= 2\gamma\frac{\partial\sigma}{\partial r} + \frac{\partial\phi_0}{\partial r} + \frac{2\gamma\sigma}{r} \\ \lambda &= \omega^2 + \gamma\frac{\partial^2\sigma}{\partial r^2} + \frac{\partial^2\phi_0}{\partial r^2} + \frac{2\gamma}{r}\frac{\partial\sigma}{\partial r} \end{aligned}$$

where $\sigma = \frac{P_o(r)}{\rho_o(r)}$. All quantities contained in the coefficients α, β and λ are known, except for ω. By implementing finite difference expressions for the derivatives and introducing the boundary conditions for ρ' we can search for values of ω numerically.

5. Concluding remarks

The naive model described in this paper can be extended to a more realistic model, which takes into account in a better way the several effects that need to be incorporated. We have considered a polytrope, which does not cover all stellar types and configurations. Using a more realistic equation of state will allow for the final results to be coded in such a way that they can be included in well developed models of stellar structure such as MESA and ASTEC. Non-radial oscillations have not been considered. These can be included in the calculation by not requiring spherical symmetry in the perturbed configuration. In standard pulsation theory the pulsation equations can be separated into radial and angular parts. This separation may not be possible when we include the tidal term, leading to a coupling of radial and angular modes. Assuming that the oscillations are radial is very unrealistic, especially since the the external forces, resulting from the inclusion of tides, have non-radial as well as radial components. We are currently coding our preliminary model and will be extended to include non-radial pulsations.

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