Entanglement of two distant nitrogen-vacancy-center ensembles under the action of squeezed microwave field

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Abstract. We consider a circuit consisting of two distant nitrogen-vacancy-center ensembles coupled to separate transmission line resonators, which interact by means of a current biased Josephson junction. Our investigation is focused on transitions and dissipation in the Josephson junction leading to entanglement. In our approach the Josephson junction is regarded as a reservoir, whose variables are eliminated from the system dynamics. We include in this scheme also superconducting quantum interference devices, flux-driven Josephson parametric amplifiers, which are the sources of a squeezed microwave field. The entanglement was studied in terms of the logarithmic negativity. The logarithmic negativity was considered for different regimes: weak coupling and strong coupling of transitions of the Josephson junction, and under action of squeezed microwave fields. We show that different degrees, time and duration of entanglement can be reached for various parameters choices.

1. Introduction

We propose a new approach for the physical realization of thermal entanglement of a continuous variables system using spin ensembles. Nitrogen vacancy (NV) centers in diamond attract especial interest because the manipulation, storage, and readout of the quantum information encoded in the different sublevels can be implemented by means of laser and microwave fields [1, 2, 3, 4, 5]. Experimental confirmation of these properties led to engineering of various hybrid circuits, containing nitrogen-vacancy-center ensembles (NVEs), separate transmission line resonators (TRLs) and current biased Josephson junction (CBJJ) [6, 7, 8, 9, 10]. Application of NVE with N spins allows enhancement of the coupling strength by a factor $\sqrt{N}$, that is especially important for performing of measurement-based quantum computing. For the description of interaction of spin ensembles with external fields collective variables are used. In the low-excitation limit, collective variables of NVE can be described as bosonic modes or harmonic oscillators.

Recently, conditions of squeezing were investigated in the system of two distant NVEs coupled to separate TLRs, which are interconnected by a CBJJ [11]. Our investigation is focused on two-mode thermal entanglement, which can occur due to transitions and dissipation in the CBJJ and TLRs. Also we include in the scheme superconducting quantum interference devices, flux-driven Josephson parametric amplifiers (JPAs), which are the sources of a squeezed microwave field. Recently JPA was described theoretically [12] and also realized experimentally [13]. Squeezed

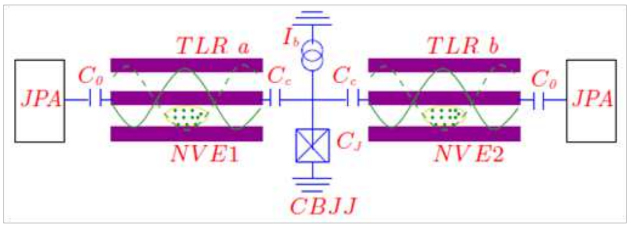

Figure 1. The coupled system of a current biased Josephson junction (CBJJ), two transmission line resonators (TLRs) and Josephson parametric amplifiers (JPAs). Current biased Josephson junction interconnects transmission line resonator a (TLR a) and transmission line resonator b (TLR b) by means of coupling capacitors C_c. I_b is bias current. C_J is the junction capacitor. Josephson parametric amplifiers enhance scheme on left and right sides via coupling capacitors C_0 . Nitrogen-vacancy-center ensemble 1 (NVE 1) and nitrogen-vacancy-center ensemble 2 (NVE 2) are shown inside transmission line resonator a and transmission line resonator b.

microwave field of JPA provides self-squeezing of each bosonic mode in separate TLR that can be important for applications of the continuous-variables approach. Logarithmic negativity was chosen as a measure of entanglement because the continuous-variables approach provides a simple way to find the covariance matrix, which is the basis for the calculation of the logarithmic negativity. In Section 2 we present the scheme, including NVEs, TLRs, CBJJ and JPA, and the model, which is proposed for the description of the interaction. In the section 3 we illustrate the results obtained for the entanglement investigation by means of a calculation of the logarithmic negativity for various parameters. In conclusion, we formulate the necessary conditions for the formation of entangled steady state.

2. Model description

We consider a circuit consisting of two distant nitrogen-vacancy-center ensembles coupled to separate transmission line resonators, which interact by means of a current biased Josephson junction, enhanced by JPAs symmetrically in both parts of system (figure 1).

Action of different types of fields, such as fields of two resonators, squeezed microwave field, classical microwave fields and constant magnetic field, is considered for the description of the dynamics of the scheme. A constant magnetic field removes the degeneracy between the levels of spin states, and induces energy splitting (figure 2). The experimental results [14] show that the resonant frequencies can be approximately equal for all NV centers under particular conditions. Sublevels of each NV center in the both ensembles are coupled by a corresponding mode of TLR with vacuum Rabi frequency g_a and g_b respectively. Simultaneously, NV centers of each ensemble are driven by two classical microwave fields, where Ω_1 and Ω_2 are their Rabi frequencies. In the scheme the logic states 0 and 1 are denoted by corresponding levels: $|0\rangle$ and $|1\rangle$, respectively. Due to the large detuning the coupling is considered perturbatively, using the second-order perturbation theory. The ground state is eliminated from the system dynamics. The interaction of the NV with two fields in such a system can be described by the corresponding Hamiltonian, where $\hbar = 1$

$$
\begin{aligned}
H_I \; = \; & \sum_{j=1}^{N_1}[\frac{\Omega_1^2}{4\Delta_1}|1\rangle_{1,}^{j}\,{}_{1}^{j}\langle 1| + \frac{g_a^2}{\Delta_1}a^\dagger\, a|0\rangle_{1,}^{j}\,{}_{1}^{j}\langle 0| + \left(\frac{\Omega_1 g_a a^\dagger}{2\Delta_1}|0\rangle_{1,}^{j}\,{}_{1}^{j}\langle 1| + H.c.\right)] \\
& + \sum_{j=1}^{N_2}[\frac{\Omega_2^2}{4\Delta_2}|1\rangle_{2,}^{j}\,{}_{2}^{j}\langle 1| + \frac{g_b^2}{\Delta_2}b^\dagger\, b|0\rangle_{2,}^{j}\,{}_{2}^{j}\langle 0| + \left(\frac{\Omega_2 g_b b^\dagger}{2\Delta_2}|0\rangle_{2,}^{j}\,{}_{2}^{j}\langle 1| + H.c.\right)], \qquad (1)
\end{aligned}
$$

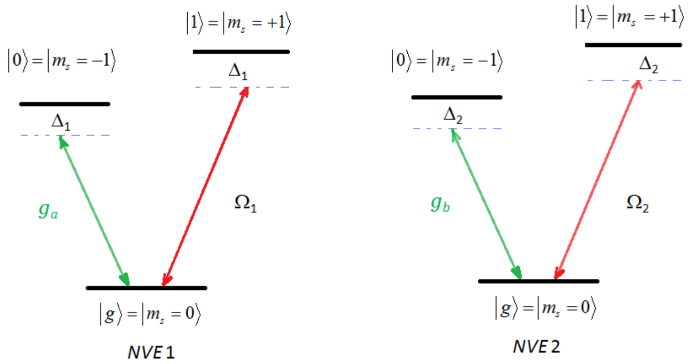

Figure 2. Level structure of single a NV center under the action of an external magnetic field.

where N_1, N_2 are the number of NV centers in the corresponding NVE. We use the collective spin operators [11] for each of two NVE ($i = 1, 2$ denotes ensemble)

$$S_i^- = \sum_{j=1}^{N_i} |0\rangle_{i,}^{j} {}_{i}^{j}\langle 1|, \quad S_i^+ = \sum_{j=1}^{N_i} |1\rangle_{i,}^{j} {}_{i}^{j}\langle 0|,$$

and map them into the boson operators $c_i(c_i^\dagger)$ by means of the Holstein-Primakoff transformation

$$\begin{aligned} S_i^- &= c_i\sqrt{N - c_i^\dagger c_i} = \sqrt{N_i}c_i, \\ S_i^+ &= c_i^\dagger\sqrt{N - c_i^\dagger c_i} = \sqrt{N_i}c_i^\dagger, \\ S_i^z &= \left(c_i^\dagger c_i - \tfrac{N_i}{2}\right). \end{aligned} \tag{2}$$

The effective Hamiltonian can be obtained by neglecting the constant energy terms in H_I

$$H_{\text{eff}} = \widetilde{\Omega_1}(a^\dagger c_1 + a c_1^\dagger) + \widetilde{\Omega_2}(b^\dagger c_2 + b c_2^\dagger), \tag{3}$$

where

$$\widetilde{\Omega_1} = \sqrt{N_1}\frac{\Omega_1 g_a}{2\Delta_1}, \widetilde{\Omega_2} = \sqrt{N_2}\frac{\Omega_2 g_b}{2\Delta_2}.$$

We model the CBJJ as a two-level artificial atom, considering two lowest levels with frequency of transition $\omega_{10} = 12$GHz. Such distribution of energy levels is provided by the choice of CBJJ parameters [15, 16, 17]. We denote these two states by $|0\rangle_{CBJJ}, |1\rangle_{CBJJ}$. So we consider the two-level system driven by the quantized fields of TLRs, using the rotating-wave approximation. The Hamiltonian describing the interaction of CBJJ with TLRs fields is [15]

$$H_{\text{int}}^{\text{eff}} = \widetilde{g_a}(a\Sigma^+ e^{i\phi} + a^\dagger \Sigma^- e^{-i\phi}) + \widetilde{g_b}(b\Sigma^+ e^{i\theta} + b^\dagger \Sigma^- e^{-i\theta}), \tag{4}$$

where $\widetilde{g_t} = [2C_t(C_J + 2C_c)]^{-1/2}\omega_t C_c \cos\delta$ is the coupling factor, $t = a, b$; δ is a small phase shift on coupling capacitance C_c, which connects TLR and CBJJ; $\Sigma^+ = |1\rangle_{CBJJ}\langle 0|$, $\Sigma^- = |0\rangle_{CBJJ}\langle 1|$ are the raising and lowering operators of the CBJJ; $\Sigma_z = |1\rangle_{CBJJ}\langle 1| - |0\rangle_{CBJJ}\langle 0|$ is the inversion operator of the CBJJ; ϕ and θ represent phases. Including the expressions for dissipation in the

TRLs and in the CBJJ [15], we describe the transfer processes in the system by means of the master equation

$$\begin{aligned} \frac{\partial\rho}{\partial t} &= -i[H_{\rm eff},\ \rho] - i[H_{\rm int}^{\rm eff},\ \rho] + k_a(a\rho a^\dagger - \frac{1}{2}a^\dagger a\rho - \frac{1}{2}\rho a^\dagger a) + \\ &\quad + k_b(b\rho b^\dagger - \frac{1}{2}b^\dagger b\rho - \frac{1}{2}\rho b^\dagger b) + \frac{\gamma_\varphi}{2}(\Sigma_z\rho\Sigma_z - \rho) + \\ &\quad + (\frac{\gamma_{10}+\Gamma_1}{4})(\Sigma^-\rho\Sigma^+ - \frac{1}{2}\Sigma^+\Sigma^-\rho - \frac{1}{2}\rho\Sigma^+\Sigma^-) - i[V_a,\ \rho] - i[V_b,\ \rho], \end{aligned} \tag{5}$$

where γ_φ is the dephasing factor for the considered transition, γ_{10} is the decay factor describing spontaneous emission, Γ_1 is the quantum tunnelling rate, that represents probability of transitions to continuum. The Γ_0 is not taken into account, because it is much smaller than other decay rates, ionization is mostly from the upper levels [17].

The terms $V_a = \beta(a^{\dagger 2}e^{-i\varphi} + a^2 e^{i\varphi})$, $V_b = \xi(b^{\dagger 2}e^{-i\varphi} + b^2 e^{i\varphi})$ describe squeezed fields of JPA; β, ξ are real amplitudes of squeezed fields, φ is phase of squeezed fields.

Based on the Master equation (5) set of differential equations for observables was obtained, using two different approaches: method of decorrelations and adiabatic elimination of fast variables of CBJJ and TLRs. Solution of this system leads to covariance matrix, which is necessary for the calculation of the logarithmic negativity [18] and for the analysis of the entanglement in our model.

3. Results

To analyse the entanglement between bosonic modes of two separate NVEs we use the logarithmic negativity [18]

$$E_N = -\sum_{i=1}^{2}\log_2(\min(1, |\gamma_i|)), \tag{6}$$

where γ_i are symplectic eigenvalues of the partially transposed covariance matrix γ^{T_1}. γ^{T_1} is obtained from the covariance matrix γ by time reversal of the momentum operator of the first system by means of the transformation $\hat{p_1} \to -\hat{p_1}$

$$\gamma^{T_1} = P\gamma P, \tag{7}$$

where

$$P = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \oplus \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

The elements of the covariance matrix γ are calculated, using the found values of NVEs observables $\left\langle c_1^\dagger c_1 \right\rangle$, $\left\langle c_2^\dagger c_2 \right\rangle$, $\langle c_1 c_2 \rangle$ and other. The symplectic eigenvalues γ_i are calculated as the positive square roots of the usual eigenvalues of $-\sigma\gamma^{T_1}\sigma\gamma^{T_1}$ [18], where

$$\sigma = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \oplus \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

If $\gamma_i \geq 1$ and $E_N = 0$, state is separable.

Figure 3 shows the dynamics of the entanglement for different values of the squeezed fields, which act on both sides of the circuit with the same amplitude. The phase of the squeezed fields and the phase in the term describing coupling transitions of the CBJJ are equal to zero. Thus we consider a symmetric system with equal parameters of dissipation, pumping and coupling for

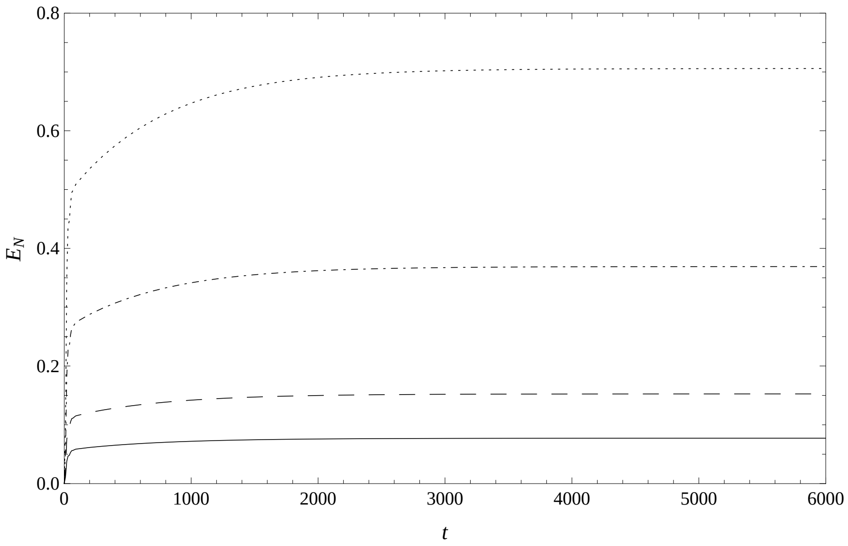

Figure 3. Logarithmic negativity as a function of dimentionless time for $\widetilde{g_a} = \widetilde{g_b} = 1$MHz, $\widetilde{\Omega_1} = \widetilde{\Omega_2} = 0.1$MHz, $\beta = \xi = 1$kHz (solid line), 2 kHz (dashed line), 5 kHz (chain line), 10 kHz (dotted line), $\Gamma_1 = 0.1$MHz, $\gamma_{10} = 0.2$MHz, $\gamma_\varphi = 0.1$MHz and $k_a = k_b = 0.1$MHz.

both sides of the scheme. One can see significant entanglement for various values of amplitude of the driving field. The growth of entanglement is finalised by stabilization after the steady state is reached. Such a steady state remains entangled. Increasing of the field amplitude leads to growth of entanglement, and direct dependence of the degree of entanglement from the field value is observed. But further amplification of pumping can cause loss of the stationary regime. In such a case the steady state is not formed.

Figure 4 confirms entanglement in the considered system for a range of parameters. One can see that the significant entanglement can be reached also for weak coupling of transitions in the CBJJ and NV centers. In this case dissipation in the CBJJ is the driving force for intermode entanglement. Weak coupling of transitions in the NV center causes an increase of entanglement time. Increasing of transitions coupling in CBJJ provides growth of entanglement, and very weak coupling induces delay in the growth of logarithmic negativity.

4. Conclusion

We investigated thermal entanglement in the system consisting of two distant nitrogen-vacancy-center ensembles coupled to separate transmission line resonators, which interact by means of a current biased Josephson junction. The case of equal parameters for both parts of the circuit was chosen. It was found, that without pumping of squeezed microwave field intermode entanglement is not observed. Entanglement in such a scheme is reached only under the action of the squeezed field. Increasing of field of JPAs leads to growth of the logarithmic negativity, but the amplitude of pumping must be much smaller than the decay rates for a steady state to be formed. If the fields of JPA are weak enough, the entangled steady state is formed. Entanglement can be observed for a large range of parameters. Values of the coupling factors for the transitions in the CBJJ play an important role for the entanglement, the increasing leads to growth of logarithmic negativity. But when these coupling factors are small, dissipation is a driving force, which provides intermode entanglement. The choice of coupling factors for transitions of NV centers allows to control the time of reaching the steady state.

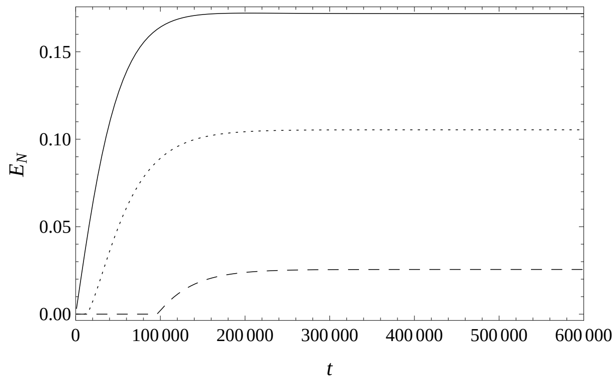

Figure 4. Logarithmic negativity as a function of dimensionless time for $\widetilde{\Omega_1} = \widetilde{\Omega_2} = 0.1$kHz, $\beta = \xi = 0.1$kHz, $\widetilde{g_a} = \widetilde{g_b} = 6$kHz (solid line), 5 kHz (dotted line), 4 kHz (dashed line), $\Gamma_1 = 0.1$MHz, $\gamma_{10} = 0.2$MHz, $\gamma_\varphi = 0.1$MHz and $k_a = k_b = 1$kHz.

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