Extraction of surface impedance from magnetotelluric data

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Abstract. This paper presents the analysis of South African magnetotelluric (MT) data in the time domain. And the time domain is converted to frequency-domain for the purpose of extracting representative values of surface impedance. The surface impedance is used in the derivation of geo-electric fields produced by rapid variations in the geomagnetic field, as occurs during geomagnetic storms. The magnetotelluric method uses the spectra of associated time varying horizontal electric and magnetic fields at the Earth's surface to determine the frequency-dependent impedance tensor and equivalent surface impedance. The theory of operation of MT devices will be presented, as well as the typical MT data obtained from Hermanus, Vaalputs and Middelpos sites. The various steps in the analysis are aimed at reducing noise and outliers. In the time domain, a Hanning window is used to select data from successive periods during a day, while reducing the end effect (Gibbs' phenomenon) by tapering the series towards the start and ends of each selected time period. The spectral transformation is performed by means of a fast Fourier transformation (FFT). Spectral bands are selected by binning in the frequency-domain. Typical results and challenges in performing this analysis will be presented.

1. Introduction

The magnetotelluric (MT) method is based on measuring time variations of horizontal components of the electric and magnetic fields at the surface of the Earth [8]. The time invariant quantity called MT impedance tensor is the response of the Earth to electromagnetic induction and carries information about the conductivity distribution of the subsurface [7] & [9]. The electromagnetic fields can be produced by ionospheric, magnetospheric or atmospheric events [8]. At the surface, the plane waves induce current flows in the Earth which give rise to a secondary field [3].

The horizontal electric and magnetic fields are measured at Earth's surface, and they vary with time. In the frequency domain, the electromagnetic fields are assumed to be linearly related by the impedance tensor as given in equation 1[3]; [4] & [7],

$$\begin{pmatrix} E_x(\omega) \\ E_y(\omega) \end{pmatrix} = \frac{1}{\mu} \begin{pmatrix} Z_{xx}(\omega) Z_{xy}(\omega) \\ Z_{yx}(\omega) Z_{yy}(\omega) \end{pmatrix} \begin{pmatrix} B_x(\omega) \\ B_y(\omega) \end{pmatrix} \tag{1}$$

where B_x and B_y are the magnetic fields in nT, E_y and E_x are the electric fields in $mVkm^{-1}$ and $Z_{ij}(i, j = x, y)$ the components of the impedance tensor Z in Ω. The components of the impedance tensor are called polarizations, the E - B polarizations refer to the xy - yx tensor components [3].

The procedure that will be employed in this paper is called the spectral analysis method, which is similar to the Fourier transform. The reason why the Fourier transform is chosen is that the energy at any interval of the Fourier power spectrum is directly related to the energy in the same frequency interval for the signal source.

2. Processing algorithm

In this paper, the MT data were collected using the LEMI 417 instruments from three recording stations (Hermanus, Middelpos & Vaalputs sites). This instrument records five horizontal magnetotelluric components $(b_x, b_y, b_z, e_x \,\&\, e_y)$ as time series of which the spectra lie in the range between 1 Hz and 1 mHz. Data from three above sites are analysed in this paper with their geographic and geomagnetic locations of the stations as listed in table 1.

Table 1. Geographic and geomagnetic co-ordinates of the stations

Stations	Latitude ($°S$)	Longitude ($°E$)	Altitude (m)
Hermanus	34°.25'45.01''	19°.13'32.94''	26
Middelpos	31°.54'36.14''	20°.14'05.25''	1135
Vaalputs	30°.09'14.01''	18°.31'73.27''	1023

To reduce the bias of spectral estimation from the time series, a Hanning window was introduced and applied to the MT data. A Fourier transform of all the MT horizontal components was carried out and the Fourier coefficients were obtained. Likewise, Fourier transforms were combined into non smoothed auto and cross spectral. For further reading or details on the cross spectral see the paper by [6]. In order to analyse MT data, the robust estimate was adopted. Equation 1 was rewritten in terms of auto- and cross spectral densities:

$$\mu E_x B_x^* = Z_{xx} B_x B_x^* + Z_{xy} B_y B_x^* \quad (2)$$

$$\mu E_x B_y^* = Z_{xx} B_x B_y^* + Z_{xy} B_y B_y^* \quad (3)$$

$$\mu E_y B_x^* = Z_{yx} B_x B_x^* + Z_{yy} B_y B_x^* \quad (4)$$

$$\mu E_y B_y^* = Z_{yx} B_x B_y^* + Z_{yy} B_y B_y^* \quad (5)$$

where $E_x B_x^*$ denotes smoothed spectral densities. It is known that the smoothing procedure leads to the least squares solution that is biased by the uncorrelated noise in input channels B_x and B_y, the statistics used allow the data to be contaminated by such noise without bias to the estimator e.g. the $Z_{xy}(\omega)$ component of the impedance tensor is estimated as

$$Z_{xy}(\omega) = \frac{\mu E_x B_y^* B_x B_x^* - \mu E_x B_x^* B_x B_y^*}{B_x B_x^* B_y B_y^* - B_x B_y^* B_y B_x^*} \quad (6)$$

The same procedure was used to derive other impedance tensor components ($Z_{yx}(\omega)$, $Z_{xx}(\omega)$ & $Z_{yy}(\omega)$).

Using equation 6 and assuming that the Earth is homogenous and isotropic [1] & [9], then $Z_{xx}(\omega) = Z_{yy}(\omega)$=0, and $Z_{xy}(\omega) = Z_{yx}(\omega)$, and the apparent resistivity and phase impedance of the Earth can be expressed in the frequency domain as

$$\rho_{xy}(\omega) = \frac{|Z_{xy}(\omega)|^2}{\omega\mu} \quad (7)$$

$$\phi_{xy} = \tan^{-1}\left(\frac{\mathrm{Im}\left(Z_{xy}(\omega)\right)}{\mathrm{Re}\left(Z_{xy}(\omega)\right)}\right) \tag{8}$$

where ω is the angular frequency and Im & Re indicate the imaginary and real parts, respectively. The same method was used to derive other resistivity and phase impedance components ($\rho_{yx}(\omega)$, $\rho_{xx}(\omega)$ & $\rho_{yy}(\omega)$ and $\phi_{yx}(\omega)$, $\phi_{xx}(\omega)$ & $\phi_{yy}(\omega)$), respectivily.

3. Results and discussion

The MT data were collected at three stations, Hermanus, Middelpos and Vaalputs. For each of the sites, the raw time series recorded at 1 s interval were reprocessed using the robust method [6] based on the least squares solution. It involves creating a time series by sampling a signal at equal intervals of time. These segments are cosine tapered prior to the Fourier transforms. The Fourier coefficients are corrected for the influence of the instrument response functions and subsequently they are divided into sub-bands centred around frequencies that are equally distributed on a logarithmic scale. For each of these sub-bands and electromagnetic field components, smoothed autospectra and cross-spectra are computed. The final response function estimates are derived by stacking the single event spectra from all frequency bands using the iterative robust algorithm described in [4]. For further understanding about the method read also a paper by [6]. In this study, remote reference processing was no considered. For details about the remote reference method refer to [4].

Figure 1 shows the apparent resistivity (a,b) and phase (c,d) of both off diagonal components of the impedance tensor at the Middelpos station plotted as a function of period (period=1/frequency). The different lines represent data on different days in all three stations. In Fig. 1, the apparent resistivity (a,b) and phase impedance (c,d) plots appear variable from day to day around periods less than 100 s. For good data a small day to day variability is expected such as documented in [10].

Figure 2 shows the apparent resistivity (a,b) and phase (c,d) of both off diagonal components at the Vaalputs station. In Fig. 2, the apparent resistivity (a,b) and phase impedance (c,d) plots have a high day-to-day variability; no conclusions can be made because of the noise at these sites.

Figure 3 shows the apparent resistivity (a,b) and phase (c,d) of both off diagonal components at the Hermanus station. In Fig. 3, the apparent resistivity (a,b) and phase impedance (c,d) plots have a high day-to-day variability . Of the three sites where MT data were collected, Middelpos seems to provide the lowest variability. According to [5] the robust processing method should reduce the variability in the data at any site. However, in Figs 2-3 no improvement is seen using the robust processing technique and averaging over a long period should be done to reduce what seems to be noise in these sites. An article by [2] indicated that a comparison of techniques for magnetotelluric response function estimation could be used to reduce noise in these sites.

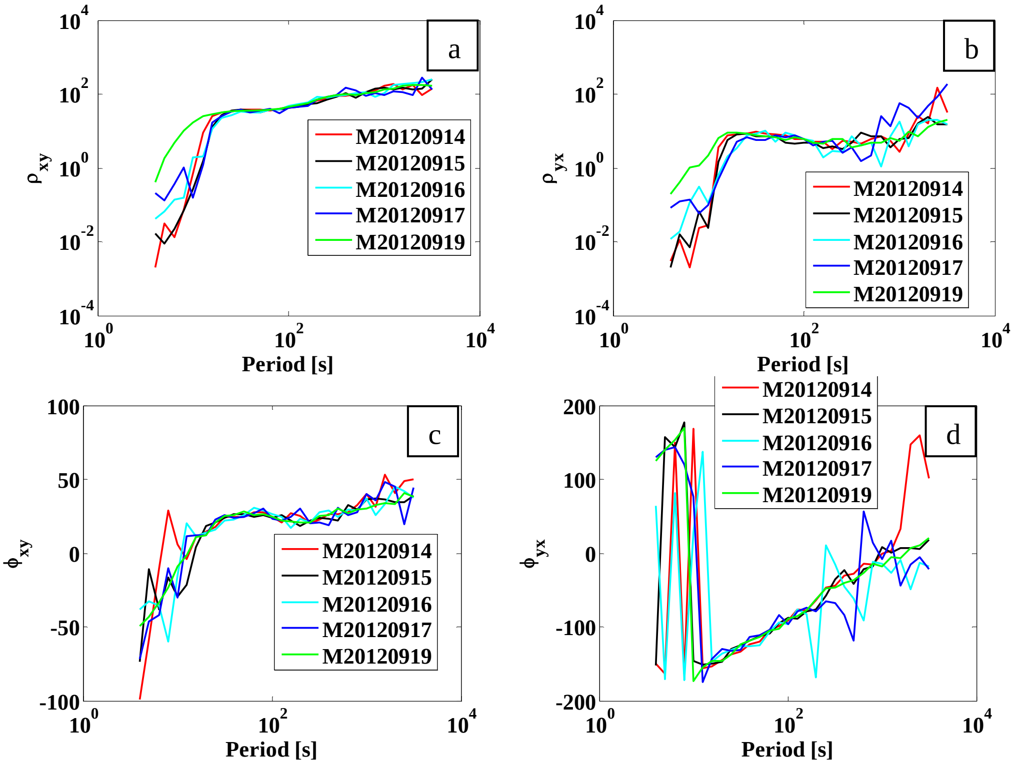

Figure 1. The solid lines show the estimates of the off-diagonal elements of the apparent resistivity using robust processing for five subsequent days. Apparent resistivity (a, b) and impedance phase (c, d) as function of periods at Middelpos site. The similarity of the ρ_{xy} and ρ_{yx} values indicate a fairly isotropic ground resistivity at Middelpos in the frequency range corresponding to periods of 10 to 100 s

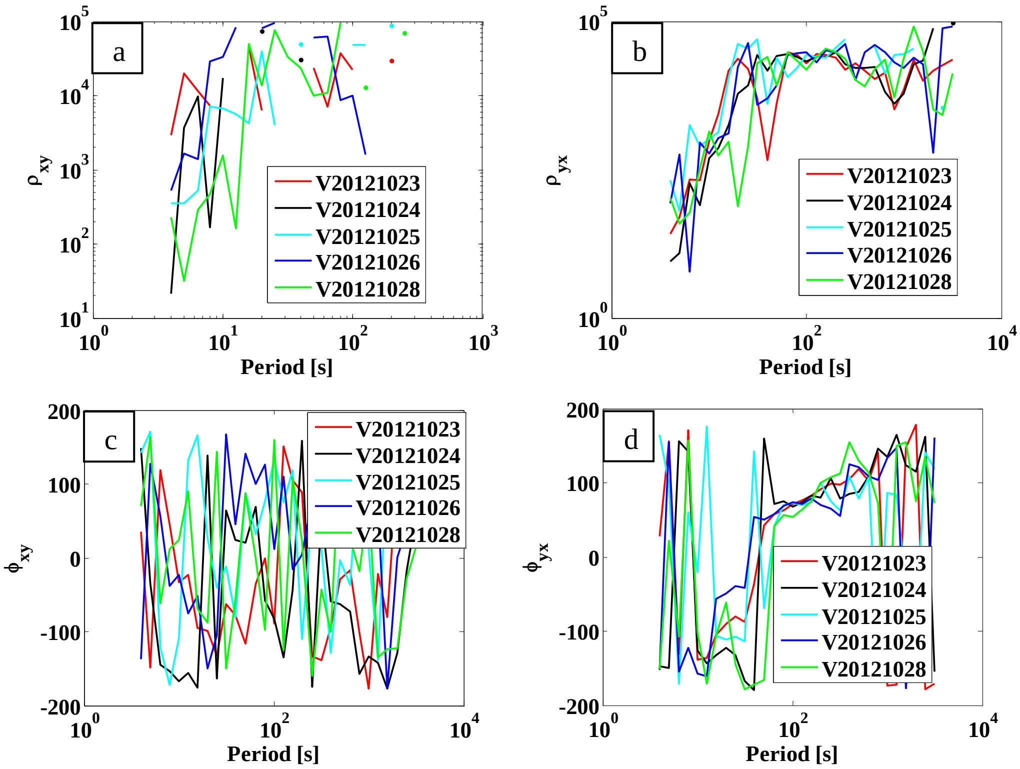

Figure 2. Same parameters as in Fig.1, but at Vaalputs site. The large day-to-day variability indicates that there is a source of noise at this site which reduces the consistency of the data.

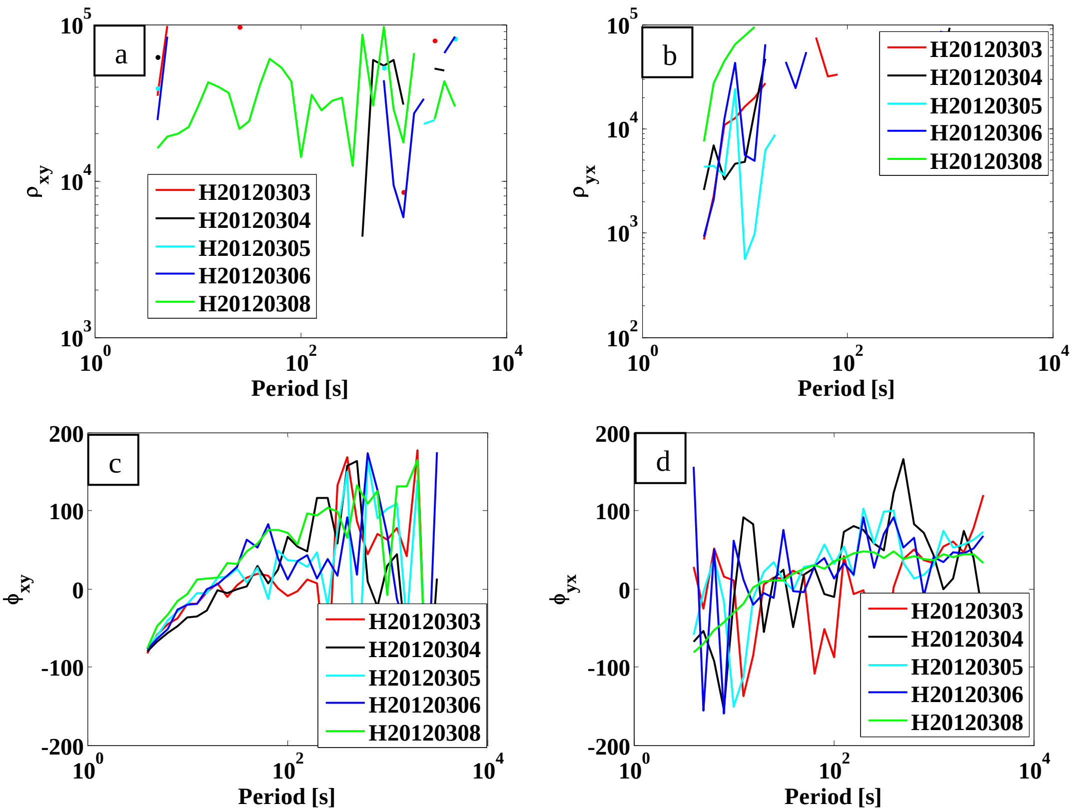

Figure 3. Same parameters as in Fig.1, but at Hermanus site. The variability of the Hermanus data is similar to that at Vaalputs, and much higher that at Middelpos.

4. Conclusions

In this paper, we present for the first time MT data from three recently deployed MT instruments in South Africa. The typical day-to-day variability of the off-diagonal components of the apparent resistivity is demonstrated. The results also show the variability from site to site, with the Middelpos site showing the least day-to-day variability and the closest correspondence to previous MT measurements in Southern Africa [10]. Typically, most observed electromagnetic field components are contaminated to some extent with noise. In order to retrieve unbiased estimates of the apparent resistivity and surface impedance, long recording times are required to ensure that sufficient data are available for processing through long-duration averaging. Further analysis of the data from these sites are required to determine their usefulness for the estimation of geomagnetically induced currents.

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