Extraction of surface impedance from magnetotelluric data

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Abstract. This paper presents the analysis of South African magnetotelluric (MT) data in the time and frequency domain for the purpose of extracting representative values of surface impedance. The surface impedance is used in the derivation of geo-electric fields produced by rapid variations in the geomagnetic field, as occurs during geomagnetic storms. The magnetotelluric method uses the spectra of associated time varying horizontal electric and magnetic fields at the Earth's surface to determine the frequency dependent impedance tensor and equivalent surface impedance. The theory of operation of MT devices will be presented, as well as typical data obtained from the MT installations in Hermanus, Vaalputs and Middelpos sites. The various steps in the analysis are aimed at reducing noise and outliers. In the time domain, a Hanning window is used to select data from successive periods during a day, while reducing the end effect (Gibbs' phenomenon) by tapering the series towards the start and ends of each selected time period. The spectral transformation is performed by means of a fast Fourier transformation (FFT). Spectral bands are selected by frequency domain filtering. Typical results and challenges in performing this analysis will be presented.

1. Introduction

The magnetotelluric (MT) method is based on measuring time variations of orthogonal components of the electric and magnetic fields at the surface of the earth [8]. The time invariant quantity called MT impedance tensor is the response of the Earth to electromagnetic induction and carries information about the conductivity distribution of the subsurface [7] & [9]. The electromagnetic fields can be produced by ionospheric, magnetospheric or atmospheric events [8]. At the surface, the plane waves induce current flows in the earth which give rise to secondary field [3].

The horizontal electric and magnetic fields are measured at Earth's surface, and they vary with time. In the frequency domain, the electromagnetic fields are assumed to be linearly related by the impedance tensor as given in equation 1[3]; [4] & [7],

$$\begin{pmatrix} E_x(\omega) \\ E_y(\omega) \end{pmatrix} = \frac{1}{\mu} \begin{pmatrix} Z_{xx}(\omega) Z_{xy}(\omega) \\ Z_{yx}(\omega) Z_{yy}(\omega) \end{pmatrix} \begin{pmatrix} B_x(\omega) \\ B_y(\omega) \end{pmatrix}$$
(1)

where B_x and B_y are the magnetic fields in nT, E_y and E_x are the electric fields in $mVkm^{-1}$ and $Z_{ij}(i, j = x, y)$ the components of the impedance tensor Z in Ω . The components of the impedance tensor are called polarizations, the E - B polarizations refer to the xy - yx tensor components [3]. The procedure that will be employed in this paper is called spectral analysis method like the Fourier transform, and it is used to estimate the spectrum. The reason, why the Fourier transform is chosen, is that the energy at any interval of the Fourier power spectrum is directly related to the energy in the same frequency interval for the signal source.

2. Processing algorithm

In this paper, the MT presented was collected using the LEMI 417 instruments from three recording stations (Hermanus, Middelpos & Vaalputs sites). At time series, this instrument records five horizontal magnetotelluric components $(b_x, b_y, b_z, e_x \& e_y)$, and in the frequency domain they range between 1 H_z and 1 mH_z . Data from three above sites are analysed in this paper with their geographic and geomagnetic locations of the stations as listed in table 1.

Table 1. Geographic and geomagnetic co-ordinates of the stations

Stations	Latitude (S)	Longitude (E)	Altitude (M)
Hermanus	3425.4501	1913.3294	26
Middelpos	3154.3614	2014.0525	1135
Vaalputs	3009.1401	1831.7327	1023

To reduce the bias of spectral estimation in the time series, Hanning window was introduced and applied in the MT data. A Fourier transform of all the MT horizontal components was carried out and the Fourier coefficients were obtained. Furthermore, Fourier transforms were combined into non smoothed auto and cross spectral. For further reading or details on the cross spectral see the paper by [6]. In order to analysis MT data, the robust estimate was adopted. An equation 1 was rewritten in terms of auto- and cross spectral densities:

$$E_{x}B_{x}^{*} = Z_{xx}B_{x}B_{x}^{*} + Z_{xy}B_{y}B_{x}^{*}$$
⁽²⁾

$$E_{x}B_{y}^{*} = Z_{xx}B_{x}B_{y}^{*} + Z_{xy}B_{y}B_{y}^{*}$$
(3)

$$E_{y}B_{x}^{*} = Z_{yx}B_{x}B_{x}^{*} + Z_{yy}B_{y}B_{x}^{*}$$
(4)

$$E_{y}B_{y}^{*} = Z_{yx}B_{x}B_{y}^{*} + Z_{yy}B_{y}B_{y}^{*}$$
(5)

where $E_x B_x^*$ denotes smoothed spectral densities. It is known that the smoothing procedure leads to the least square solution that is biased by the uncorrelated noise in input channels B_x and B_y , the statistics used allow the data to be contaminated by such noise without bias to the estimator e.g. the $Z_{yy}(\omega)$ component of the impedance tensor is estimated as

$$Z_{xy}(\omega) = \frac{E_{x}B_{y}^{*}B_{x}B_{x}^{*} - E_{x}B_{x}^{*}B_{x}B_{y}^{*}}{B_{x}B_{x}^{*}B_{y}B_{y}^{*} - B_{x}B_{y}^{*}B_{y}B_{x}^{*}}$$
(6)

The same procedure was used to derive other impedance tensor components $(Z_{yx}(\omega), Z_{xx}(\omega)) \& Z_{yy}(\omega))$.

Using equation 6 and assuming that the earth is homogenous and isotropic [1] & [9], and then one can derive the true resistivity and phase impedance of the earth in the frequency domain as follows,

$$\rho_a(\omega) = \frac{\left|Z_{xy}(\omega)\right|^2}{\omega\mu} \tag{7}$$

$$\phi = \tan^{-1} \left(\frac{\operatorname{Im}(Z_{xy}(\omega))}{\operatorname{Re}(Z_{xy}(\omega))} \right)$$
(8)

where ω is the angular frequency and Im & Re indicate the imaginary and real parts, respectively.

3. Results and discussion

The MT data discussed was collected in three stations, Hermanus, Middelpos and Vaalputs. For each of the site, the raw time series recorded at 1 s interval were reprocessed using the robust method [6]. This method of robust is based on the least squares solution for the usual MT response functions relating the Fourier transforms of the horizontal electric and magnetic fields. For further understanding about the method read a paper by [6]. In this study, remote reference processing was no considered. Details about remote reference method read a paper by [4].

Figs. 1-3 show the apparent resistivity and phase of both off diagonal components at three stations. In Fig. 1, the *xy* and *yx* apparent resistivity and phase impedance plots appear disturbed at around low periods less than 100s. Figs. 2-3 are strongly disturbed; no conclusions can be made because of seen as the lot of noise in these sites. According to [5] the robust processing should reduce the biases in the data at any site. However, in Figs 2-3 no improvement is seen using the robust processing technique and more analysis should be done to reduce what seen as the appearance of the noise in these sites. An article by [2] called a comparison of techniques for magnetotelluric response function estimation could be used to try different methods to reduce noise in these sites.

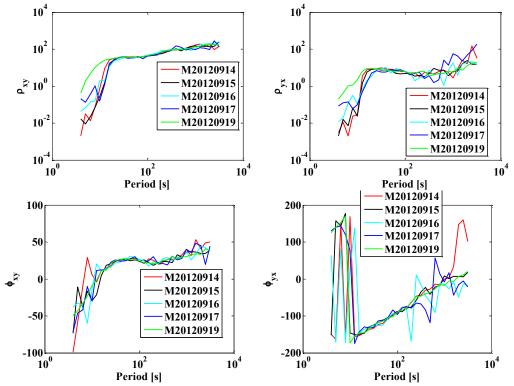


Figure 1. The solid line shows the estimate from robust processing of all the data set. Apparent resistivity (top) and impedance phase (bottom) as function of period for Middelpos site.

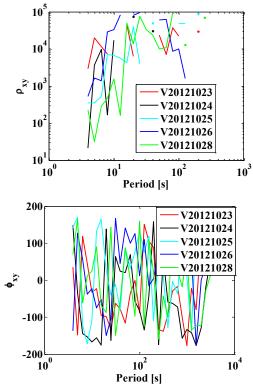


Figure 2. Same as Fig.1, but for Vaalputs site.

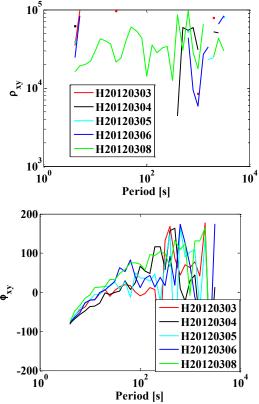
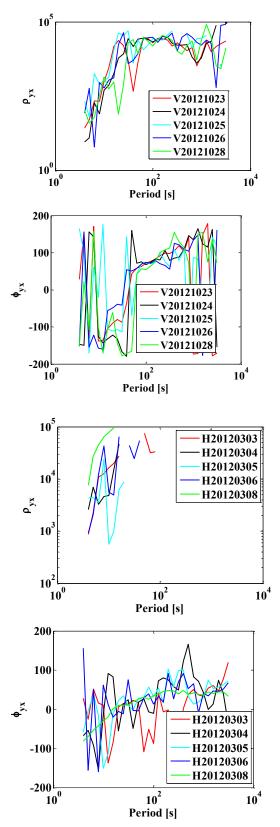


Figure 3. Same as Fig.1, but for Hermanus site.



4. Conclusions

In this paper emphasize the importance of high-quality horizontal magnetic field components for a meaningful estimation of geomagnetic and magnetotelluric response functions. Further analysis need to be done, especially statistical analysis to qualify the results.

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