

Heavy Baryons with Strangeness

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Abstract. We apply the soliton description of baryons with a single heavy quark (charm or bottom). In this approach such baryons emerge as bound composites of a soliton of meson fields built from light quarks (up, down, strange) and a meson field that contains a heavy quark. We show that in this case the soliton must then be quantized as a diquark while the fermionic character arises from binding the heavy meson field. We are particularly interested in heavy baryons that have non-zero strangeness; in the quark model that corresponds to, say, up-strange-bottom (usb). Thus the flavor symmetry breaking among the light quarks must be fully incorporated when constructing diquark states. In the soliton model that symmetry breaking is parameterized by differences between the masses and decay constants of kaons and pions. Here we present computations of the diquark eigen-energies and eigen-functions that incorporates all orders of the light flavor symmetry breaking. We also compare these results to a leading order treatment of flavor symmetry breaking. The heavy meson couples according to the heavy spin-flavor symmetry to the chiral field that carries the soliton. In the background of the soliton the heavy meson field develop bound states. We compute the associated binding energies. These are the second major ingredient for our prediction of heavy quark baryons.

1. Introduction

There is some interest in the spectrum of baryons with a heavy quark. The light quark part of the model is formulated in terms of the chiral field U , which is a 3×3 matrix in the flavor space of *up*, *down* and *strange* quarks (q). The interactions of this field are dictated by chiral symmetry and its spontaneous breaking. In particular we consider the Skyrme model which has a soliton solution that describes baryons [1]. The heavy quark part is represented by a heavy meson field that has valence quark content *bottom* (or *charm*)– \bar{q} . The interactions of its heavy degrees of freedom are governed by the heavy quark effective theory (HQET) [2, 3] while the light degrees of freedom couple to U in a chirally invariant manner. Thus the model captures the major symmetries of QCD, the fundamental theory of strong interactions according to which hadrons form.

2. Skyrme Model

The model is defined by the action which is the sum of three contributions

$$\Gamma = \int d^4x [\mathcal{L}_{\text{SK}} + \mathcal{L}_{\text{SB}}] + \Gamma_{\text{WZ}}. \quad (1)$$

The field variable is the chiral field U , a 3×3 matrix that is the non-linear representation of the pseudoscalar pions, kaons and η . (Mixing between η and η' is an interesting issue, but not important in the present context). The Skyrme model Lagrangian also contains chiral symmetry breaking

$$\mathcal{L}_{\text{SK}} = \frac{f_\pi^2}{4} \text{tr} \left(\partial_\mu U \partial^\mu U^\dagger \right) + \frac{1}{32e^2} \text{tr} \left(\left[U^\dagger \partial_\mu U, U^\dagger \partial_\nu U \right]^2 \right) + \frac{f_\pi^2 m_\pi^2 + 2f_K^2 m_K^2}{12} \text{tr} \left(U + U^\dagger - 2 \right). \quad (2)$$

Flavor symmetry breaking is incorporated via

$$\mathcal{L}_{\text{SB}} = \frac{f_\pi^2 m_\pi^2 - f_K^2 m_K^2}{2\sqrt{3}} \text{tr} (\lambda_8 U) + \frac{f_K^2 - f_\pi^2}{12} \text{tr} \left([1 - \sqrt{3}\lambda_8] U (\partial_\mu U)^\dagger \partial^\mu U \right) + \text{h.c.}, \quad (3)$$

where h.c. stands for Hermitian conjugation and λ_a ($a = 1, \dots, 8$) are the eight Gell-Mann matrices. Finally, the anomaly is included as the Wess-Zumino term. It is an integral

$$\Gamma_{\text{WZ}} = -\frac{iN_c}{240\pi^2} \int d^5x \epsilon^{\mu\nu\rho\sigma\tau} \text{Tr} [\alpha_\mu \alpha_\nu \alpha_\rho \alpha_\sigma \alpha_\tau] \quad (\alpha_\mu = U^\dagger \partial_\mu U) \quad (4)$$

over a five dimensional manifold whose boundary is Minkowski space. Here N_C is the number of colors in QCD. The Skyrme model is expected to be valid (at least) in the limit $N_C \rightarrow \infty$. For actual calculations we use, of course, $N_C = 3$.

The hedgehog is embedded

$$U(\vec{x}, t) = A(t) \exp \left[i\hat{x} \cdot \vec{\lambda} F(r) \right] A^\dagger(t) \quad (5)$$

in $SU_F(3)$ with $\vec{\lambda} = (\lambda_1, \lambda_2, \lambda_3)$. Upon substitution this field configuration into the above defined action yields the Lagrange function (for $N_C = 3$)

$$L_{\text{SK}} = -E_{\text{cl}} + \frac{1}{2} \alpha^2 \sum_{i=1}^3 \Omega_i^2 + \frac{1}{2} \beta^2 \sum_{\alpha^4}^7 \Omega_\alpha^2 - \frac{\sqrt{3}}{2} \Omega_8 - \frac{1}{2} \gamma_S [1 - D_{88}]. \quad (6)$$

The coefficients $E_{\text{cl}}, \dots, \gamma$ are functionals of the chiral angle $F(r)$. In particular application of the variational principle onto the classical energy E_{cl} determines $F(r)$ together with the boundary conditions $F(0) = \pi$ and $\lim_{r \rightarrow \infty} F(r) = 0$, suitable for baryon number one. The information on the collective coordinates $A \in SU(3)$ is parameterized via the angular velocities $A^\dagger \dot{A} = \frac{i}{2} \sum_{a=1}^8 \Omega_a \lambda_a$ and the adjoint representation $D_{ab} = \frac{1}{2} \text{tr} [\lambda_a A \lambda_b A^\dagger]$.

3. Coupling of Heavy Mesons

As indicated in the introduction, the heavy mesons couple to the light mesons (parameterized via the chiral field U) according to chiral symmetry while the interactions of the heavy degrees of freedom are governed by HQET. The latter requires to treat the pseudoscalar (P) and vector components (Q_μ) on equal footing. Note that these fields are three component arrays in $SU_F(3)$:

$$\begin{aligned} \mathcal{L}_H = & (D^\mu P)^\dagger D_\mu P - \frac{1}{2} (Q^{\mu\nu})^\dagger Q_{\mu\nu} - P^\dagger M^2 P + Q^{\mu\dagger} M^{*2} Q_\mu \\ & + 2iMd \left[P^\dagger p_\mu Q^\mu - Q_\mu^\dagger p^\mu P \right] - \frac{d}{2} \epsilon^{\alpha\beta\mu\nu} \left[Q_{\nu\alpha}^\dagger p_\mu Q_\beta + Q_\beta^\dagger p_\mu Q_{\nu\alpha} \right], \end{aligned} \quad (7)$$

with $Q_{\mu\nu} = D_\mu Q_\nu - D_\nu Q_\mu$ and the covariant derivative is $D_\mu = \partial_\mu - iv_\mu$. Furthermore $M^2 = \text{diag}(M^2, M^2, M_s^2)$ is the matrix containing the square heavy pseudoscalar meson masses. Likewise, M^{*2} stands for the heavy vector meson masses. HEQFT enforces the coefficients of the two last terms in Eq. (7) in the specific way they are presented. Lorentz and chiral invariance alone do not relate these two terms. The light (pseudo)vector currents are given by

$$p_\mu = \frac{i}{2} (\xi \partial_\mu \xi^\dagger - \xi^\dagger \partial_\mu \xi) \quad \text{and} \quad v_\mu = \frac{i}{2} (\xi \partial_\mu \xi^\dagger + \xi^\dagger \partial_\mu \xi), \quad (8)$$

where ξ is the root of the chiral field, *i.e.* $U = \xi \cdot \xi$. Substituting the soliton solution into these currents generates an attractive potential for the heavy mesons fields. The resulting binding energies will be central to the model prediction for the spectrum of the heavy baryons. The most strongly bound states are expected in the P -wave channel. The corresponding *ansätze* introduce four profile functions

$$\begin{aligned} P &= \frac{e^{i\epsilon t}}{\sqrt{4\pi}} \Phi(r) A(t) \hat{x} \cdot \vec{\lambda} \chi, & Q_0 &= \frac{e^{i\epsilon t}}{\sqrt{4\pi}} \Psi_0(r) A(t) \chi, \\ Q_i &= \frac{e^{i\epsilon t}}{\sqrt{4\pi}} A(t) \left[i \Psi_1(r) \hat{x}_i + \frac{1}{2} \Psi_2(r) \epsilon_{ijk} \hat{x}_j \lambda_k \right] \chi, \end{aligned} \quad (9)$$

where $\chi = \begin{pmatrix} a_1 \\ a_2 \\ 0 \end{pmatrix}$ is a space independent spinor in flavor space. In the adiabatic approximation ($A = \text{const.}$) the field equations then turn into coupled ordinary differential equations for $\Phi(r)$ and $\Psi_{0,1,2}(r)$. We construct solutions with $|\epsilon| \leq M$ that decay exponentially as $r \rightarrow \infty$. This only occurs for discrete values ϵ_i which are the searched-for bound state energies. Finally the computed profile functions are normalized such that heavy meson field carries unit charge of the associated flavor[4]. The so constructed bound state contribute

$$L_H = -\frac{1}{2} \gamma_B [1 - D_{88}] + \frac{\sqrt{3}}{6} |Q_H| \Omega_8 + \frac{1}{2} \chi_P \chi^\dagger \left[\sum_{i=1}^3 \Omega_i \lambda_i \right] \chi + \dots \quad (10)$$

to the collective coordinate Lagrangian. The symmetry breaking term is the integral

$$\gamma_B = \int dr r^2 \left[(M_{B_s}^2 - M_B) \Phi(r)^2 + (M_{B_s}^{*2} - M_B^*) \left(\Psi_1(r)^2 + \frac{1}{2} \Psi_2(r)^2 - \Psi_0(r)^2 \right) \right] \quad (11)$$

and Q_H is the charge of the heavy flavor.¹ Furthermore χ_P is the hyperfine splitting coefficient, which is also an integral over all profile functions [4]. The ellipsis indicate terms that are subleading in the combined $1/N_C$ and heavy flavor expansion.

The ansatz, Eq. (9) corresponds to ordinary positive parity baryons. Negative parity baryons can be described by

$$\begin{aligned} P^{(-)} &= \frac{e^{i\epsilon t}}{\sqrt{4\pi}} \Phi(r) A(t) \chi, & Q_0^{(-)} &= \frac{e^{i\epsilon t}}{\sqrt{4\pi}} \Psi_0(r) A(t) \hat{x} \cdot \vec{\lambda} \chi, \\ Q_i^{(-)} &= \frac{i e^{i\epsilon t}}{\sqrt{4\pi}} A(t) \left[\Psi_1(r) \hat{x} \cdot \vec{\lambda} \hat{x}_i + \frac{1}{2} \Psi_2(r) r \vec{\lambda} \cdot (\partial_i \hat{r}) \right] \chi. \end{aligned} \quad (12)$$

Though in this case the profile functions obey different differential equations leading to different energy eigenvalues ϵ_i and integral² γ_B , the form of the Lagrangian, Eq. (10) remains unchanged.

¹ That is, $Q_H = 1$ if the bound state is occupied but vanishes otherwise.

² Formally only the factor of Ψ_2^2 changes from $\frac{1}{2}$ to 2, but the radial functions differ significantly.

4. Quantization in $SU_F(3)$

Quantization is performed canonically, *i.e.* canonical commutation relations are imposed for the collective coordinates and their conjugate momenta. This is equivalent to $SU(3)$ commutation relations for the (right) generators[1]

$$R_a = -\frac{\partial L}{\partial \Omega_a} \quad [R_a, R_b] = -if_{abc}R_c, \quad (13)$$

where $L = L_{Sk} + L_H$. The full wave-function of the heavy baryon is the product of the heavy meson bound state and the wave-function $\Psi(A)$. If it were not for flavor symmetry breaking (measured) by $\gamma = \gamma_S + \gamma_B$, these were $SU(3)$ D-functions. We include the effects of flavor symmetry breaking according to the Yabu-Ando method which goes beyond a perturbative expansion in γ [5]. Before doing so, we need to emphasize the most important consequence of the heavy meson bound state on the structure of $\Psi(A)$. The angular velocity Ω_8 only appears linearly in the Lagrangian. Hence

$$R_8 = \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{6}Q_H \quad (14)$$

is a constraint. Without the heavy meson bound state it requires $\Psi(A)$ to be a wave-function with half-integer spin. With the bound state $\Psi(A)$ must have integer spin, *i.e.* the soliton with spin J_s must be quantized as diquark in flavor space for $Q_H = \pm 1$ [6].

The Yabu-Ando method requires to find the eigenvalues ϵ_{SB} in

$$\left(\sum_{a=1}^8 R_a^2 + \gamma\beta^2 [1 - D_{88}] \right) \Psi_{IJ_s}(A) = \epsilon_{SB}(I, J_s) \Psi_{IJ_s}(A) \quad (15)$$

for a given set of soliton spin (J_s) and isospin (I) quantum numbers. Without flavor symmetry breaking the solutions to this eigenvalue equation are elements of a definite $SU(3)$ multiplet. When the strangeness zero element of such a multiplet is uniquely identified by its isospin projection I_3 , the soliton spin, J_s is uniquely given by the total isospin of that element.³ The spin of the baryon is related to the soliton spin as $J = J_s \pm \frac{1}{2}$.

For a pertinent parameterization of collective coordinates in terms of $SU(3)$ *Euler angles* the above eigenvalue equation reduces to ordinary differential equations for just a single angle, the so-called strangeness changing angle ν [5, 1]. Collecting pieces, the mass of a baryon is

$$M_B(I, J) = \text{const.} + \frac{J_s(J_s + 1)}{2} \left(\frac{1}{\alpha^2} - \frac{1}{\beta^2} \right) - \frac{R_8^2}{2\beta^2} + \frac{\epsilon_{SB}(I, J_s)}{2\beta^2} + |Q_H \epsilon_i|. \quad (16)$$

The constant includes all contributions that are identical for all baryons, such as the classical energy. Also, we have omitted the hyperfine contribution (related to χ_P) because its role in flavor $SU(3)$ is still under investigation. This will ultimately resolve the ambiguity between J and J_s . Without that, we observe a degeneracy between the $J = J_s \pm \frac{1}{2}$ baryons.

5. Results

In a first step we investigate the eigenvalue equation (15) in the relevant diquark channels. In the absence of flavor symmetry breaking ($\gamma = 0$) the eigenstates are pure $SU(3)$ representation and the lowest-lying diquarks are within the anti-triplet ($J_s = 0$) and the sextet ($J_s = 1$). Once symmetry breaking is included admixtures of higher dimensional representations occur. They can be perturbatively estimated as a power expansion in $\gamma\beta^2$ [6] or by directly solving Eq. (15). We compare the results of both procedures in figure 1 as a function of the perturbation

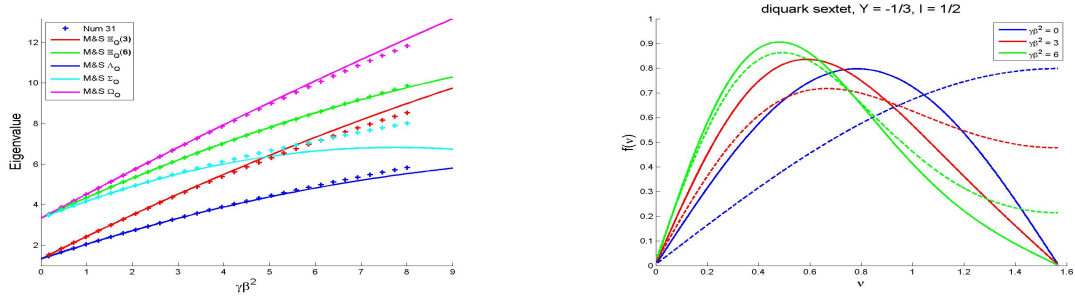


Figure 1. Comparison of second order perturbative (M & S[6]) and exact (numerical) solutions to Eq. (15).

| Flavor | $\Delta\epsilon_B(\text{MeV})$ | Baryon | $\Delta m(\text{MeV})(\text{exp})$ |
|--------|--------------------------------|-------------|------------------------------------|
| B | 257 | Λ_b | 293 |
| C | 285 | Λ_c | 306 |

Table 1. Mass differences between S and P-wave compared to mass differences between heavy baryons of opposite parity.

parameter. In case of the eigenvalues we present results for all interesting spin and isospin channels. For $\gamma\beta^2 \geq 6$ we observe sizeable deviations from the second order perturbation calculation. This requires to extend that calculation to third or even higher order. That would be cumbersome since the number of the required $SU(3)$ Glesch–Gordan coefficients is approximately proportional to the factorial of the order. On the other hand, the effort in numerically integrating Eq. (15) is independent of the value of $\gamma\beta^2$.⁴ For the wave–functions we restrain ourselves to display the variation with the strangeness changing angle for $J_s = 1$, $I = \frac{1}{2}$ channel that involves two functions and would be a member of the sextet at zero symmetry breaking. As $\gamma\beta^2$ increases, the wave–functions get more pronounced at smaller values of the strangeness changing angle. This is to be expected because the probability to rotate into strangeness direction decreases with the mass of the strangeness carrying fields.

Of course, $\gamma\beta^2$ is not a parameter to be chosen. Rather, once the model parameters are fixed, it is a prediction as both factors are functionals of the chiral angle $F(r)$. The parameters entering the Skyrme model action, Eq. (1) are taken from meson properties as far as possible: $f_\pi = 93\text{MeV}$, $f_K = 1.22f_\pi$, $m_\pi = 138\text{MeV}$ and $m_K = 495\text{MeV}$. Though the Skyrme constant e could be related to pion–pion scattering, this does not give a sufficiently definite value and also other terms from the chiral expansion would contribute. Therefore we use $e \approx 5$ that reasonably reproduces the Δ –nucleon mass difference[1]. For the masses of the heavy mesons we take $M = 1865\text{MeV}$, $M^* = 2007\text{MeV}$, $M = 1969\text{MeV}$ and $M = 2106\text{MeV}$ in the charm sector or $M = 5279\text{MeV}$, $M^* = 5325\text{MeV}$, $M = 5367\text{MeV}$ and $M = 5416\text{MeV}$ in the bottom sector[8]. The coupling constant for the interaction between heavy and light mesons is estimated from heavy meson radiative decays $d \approx 0.53$ [7].

The P and S channel bound states correspond to heavy baryons of opposite parities. Hence the differences should be compared to the mass difference of the observed $\Lambda_{c,b}$ with positive and negative parity. This comparison is shown in table 1 and yields satisfactory results.

Having not yet finalized the investigation of hyperfine splitting in the context of flavor $SU(3)$ we can only compare predictions for $J_s = 0$, as it unambiguously is associated with total spin

³ This feature persists when flavor symmetry breaking is included.

⁴ At very large values, the boundary solutions require some care.

| I | P | Model | PDG |
|---------------|---|-------|-----|
| 0 | - | 308 | 308 |
| $\frac{1}{2}$ | + | 88 | 151 |
| $\frac{1}{2}$ | - | 408 | 503 |

Table 2. Comparison of model predictions to experiment (PDG [8]) for mass differences of charmed baryons with respect to Λ_c . Note that the PDG has not determined the J^P quantum numbers but adopted them from the non-relativistic quark model.

$J = \frac{1}{2}$. For the charm sector this is shown in table 2. Our predictions are in the right ball park, though the mass difference between the Λ_c and the Ξ_c 's is somewhat underestimated.

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