# Aspects of the structure of heavy carbon isotopes

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**Abstract.** A multi-channel algebraic scattering (MCAS) method has been used to obtain spectra of a number of light-mass nuclei, which are treated as a two-cluster system, in these cases a nucleon plus nucleus. The MCAS method gives both sub-threshold and resonance states of the nuclei in question. To date, collective models have been used to specify the interactions between the nucleon and low-lying states of the nucleus that form the compound. For the case of the carbon isotopes, these studies have been complemented by sufficiently complex and complete shell-model calculations. Comparisons with the shell model results provide new insights into the validity of those from MCAS.

## 1. Introduction

In a recent article [1] we presented results from a study of the structures of $^{17}$C and $^{17}$Na that used a coupled-channel method with collective-model interactions between a neutron/proton and a select set of states of the core nucleus, $^{16}$C/$^{16}$Ne. A multi-channel algebraic scattering (MCAS) method was used, as has previously been applied to a number of other systems. For example, in the cases of $^{15}$C and $^{15}$F, modelled as $n$+$^{16}$C and $p$+$^{16}$O respectively [2], it was shown to have predictive power. In [2], the two resonances in $^{15}$F known at the time were well-reproduced, and further narrow resonances at higher energy were predicted. Resonances in the relevant energy regime were later observed in experiments by Mukha *et al.* [3, 4]. Fortune [5] criticised our mass-17 results based upon results of a simplified $0\hbar\omega$ shell model in which three neutrons are considered in the *sd* shell outside of an inert closed core of $^{14}$C. Comparisons of results of that structure model with those found using MCAS were used as presumptive proof of "flaws" with the MCAS approach. Basically, those were the number and spin-parities found using MCAS and of the resonance widths so determined. However, those problems ascribed to MCAS were refuted in a recent article [6].

By way of addressing these issues in a broader context, we present a study of the spectra of the isotopes $^{12-19}$C using no-core, complete (as far as possible) multi-$\hbar\omega$ shell models with a fixed set of two-nucleon potentials. The objective is not to attain spectral energies that best agree with experimentally known ones; it is already known that are discrepancies such as inversion of some low-lying states [7]. Rather we seek to find what are the orbit occupancies in the states and what connections there are to one and two neutron removal from each state of an isotope with

the low-lying states of a core system. For this objective, the consistent shell model approach serves. In that regard, this extends our earlier work which concentrated on $^{17}$C and $^{19}$C [8].

## 2. MCAS

While the MCAS method has been published elsewhere [9], we present a brief summary as a guide. The method is a means of solving the coupled-channel Lippmann-Schwinger (LS) equations describing a two-cluster system, which, for the present purpose, is given by a nucleon coupled to an even-mass nucleus. It is based on a collective-model description of the target (core) nucleus to specify the matrix of interaction potentials defining the coupled-channel problem. Nothing is assumed of the spectrum of target states. Sturmians are used to specify the matrix of potentials; a finite set of $\sim 30$ are needed to ensure convergence. In order to treat the problem of Pauli-exclusion, whereby the coupling of a nucleon to a filled orbit in the target is prohibited, we use orthogonalising pseudo-potentials (OPP's) with weights of $10^6$ MeV to ensure those sturmians are orthogonal to any nucleon orbit that is full. Smaller weights are used to account for partially-filled orbits. (A summary of both may be found in Refs. [6, 2].)

With those sturmians and OPPs set, the matrix of potentials is re-expressed as a sum of separable potentials in momentum space as input to the LS equations. Solutions of the coupled equations may be found for both bound (negative energy, with respect to the particle emission threshold in the compound nucleus) and scattering (positive energy) states. Energies and widths are found of the states in the compound nucleus, assuming that the channels specified are only those of a nucleon coupled to a nucleus, in which case the widths so obtained correspond to nucleon-emission only.

## 3. Shell-model considerations

The shell model we consider for the carbon isotopes is that specified in the $(0+2)\hbar\omega$ model specified within the *spsdpf* basis. That model space is complete in $\hbar\omega$ for $^{12,14}$C while for $^{16-18}$C the model space is truncated by necessity. The truncation, for the $(0+2)\hbar\omega$ calculations, is the exclusion of the $0g1d2s$ shell; a shell required for a complete evaluation of $1p-1h$ excitations by $2\hbar\omega$ from the *sd* shell. For $^{15}$C, the presence of the single nucleon outside the $0p$-shell requires that the ground state and positive parity excited states be calculated in a $(1+3)\hbar\omega$ model space, for which the truncation on the basis is more severe with regards to the $3\hbar\omega$ components. All calculations have been made using the OXBASH code [10] with two interactions: the WBP interaction of Warburton and Brown [11] and the MK3W interaction [12] for comparison.

The ground states of $^{12,14}$C so obtained illustrate the importance of the $2\hbar\omega$ components. Using the WBP interaction, we find

$$\begin{aligned} \left|^{12}\mathrm{C}\right\rangle &= 87.0\%\,|0\hbar\omega\rangle + 13.0\%\,|2\hbar\omega\rangle \\ \left|^{14}\mathrm{C}\right\rangle &= 84.9\%\,|0\hbar\omega\rangle + 15.1\%\,|2\hbar\omega\rangle\,. \end{aligned} \tag{1}$$

(The results using the MK3W interaction show $\sim 20\%$ admixture of the $2\hbar\omega$ components in both nuclei.) The ground state of $^{15}$C is

$$\begin{aligned} \left|\tfrac{1}{2}^+;\tfrac{3}{2}\right\rangle_1 &= 78.1\%\,|1\hbar\omega\rangle + 21.9\%\,|3\hbar\omega\rangle \ \text{(MK3W)}, \\ &= 84.5\%\,|1\hbar\omega\rangle + 15.5\%\,|3\hbar\omega\rangle \ \text{(WBP)}. \end{aligned} \tag{2}$$

The negative parity states in $^{15}$C come from the $(2+4)\hbar\omega$ model space, with the dominant contributions ($\sim 85\%$) being $2\hbar\omega$ in character, consistent with the heavier isotopes, which are purely $2\hbar\omega$ in character.

An assumption often made in using a minimal (packed) model of the isotopes, particularly for mass-15 and higher, which have neutron occupation in the *sd* shell, is that the six proton

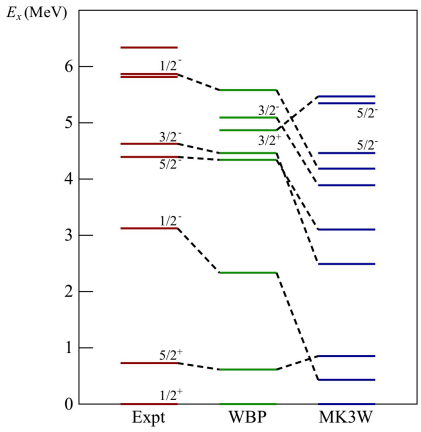


**Figure 1.** Spectrum of $^{15}$C as obtained from our shell model. The data are from [13].

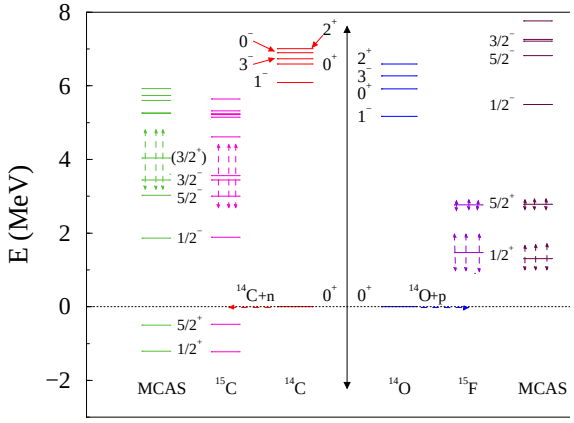


**Figure 2.** The mass-15 spectrum obtained from MCAS [2].

**Table 1.** $B(E2\uparrow)$ values, in units of $e^2\text{fm}^4$, for even-mass isotopes. Energies are in MeV, and the polarisation charge is in units of $e$.

| Nucleus | $E_x$ (exp.) | $B(E2)$ (exp.) | $B(E2)$ | | $e_{\text{pol}}$ | |
|---|---|---|---|---|---|---|
| | | | MK3W | WBP | MK3W | WBP |
| $^{12}$C | 4.43 | 40(3) | 30.54 | 16.03 | 0.07 | 0.28 |
| $^{14}$C | 7.01 | 19(3) | 25.30 | 17.66 | – | 0.04 |
| $^{16}$C | 1.77 | 3.1(6) | 4.44 | 1.76 | – | 0.05 |
| $^{18}$C | 1.59 | 4.3(1.0) | 3.33 | 2.52 | 0.02 | 0.05 |

play little or no role in defining the low-excitation spectrum. However, the six protons in these models are not passive: components of states, even the ground states, have admixtures of protons divided at least between the two $0p$ orbits. Then, as two protons in the $0p_{3/2}$ orbit can couple giving angular momentum values of 0 and 2, adding the latter to the angular momenta allowable with, for example, three $0d_{5/2}$ neutrons results in combinations with angular momenta ranging from $1/2^+$ to $13/2^+$ inclusive.

The spectrum of $^{15}$C, as obtained from our shell-model calculation, is shown in Fig. 1, while Fig. 2 shows the spectrum obtained for mass-15 from MCAS [2]. It is clear that both the shell model and MCAS reproduce the experimental spectrum for $^{15}$C quite well; the latter assuming a neutron coupled to low lying states in $^{14}$C.

Another observable that is quite sensitive to details of the nuclear structure are the $B(E2)$ values. Table 1 shows the $B(E2)$ values for the even isotopes between $^{12}$C and $^{18}$C. Most of the data were taken from [14] and supplemented with data on $^{16}$C from [15, 16] and on $^{18}$C from [17]. Apart from the WBP result for $^{12}$C, the required polarisation charges are all less than $0.1e$ reflecting that, for this observable, the wave functions from both models are quite good. For $^{14}$C and $^{16}$C the $B(E2)$ values from the MK3W are higher than the measured values. A polarisation charge cannot correct for that model; a smaller oscillator length may lead to agreement with data.

Figs. 3 and 4 show the spectrum of $^{17}$C as obtained by the shell model and MCAS, respectively.

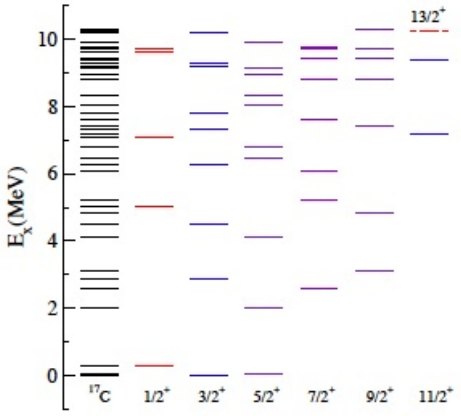


**Figure 3.** Low-excitation spectrum of $^{17}$C as obtained from the shell model.

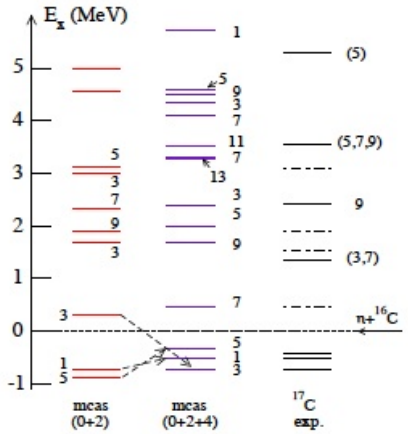


**Figure 4.** As for Fig. 3 but as obtained from MCAS using coupling of the neutron to the $0^+$ and $2^+$ as well as coupling to $0^+$, $2^+$, and $4^+$ states in $^{16}$C.

(The spins shown therein are $2J$. The MCAS calculation was obtained by assuming the coupling of a neutron to the low-lying states $(0^+, 2^+, 4^+)$ states in $^{16}$C. The two approaches reproduce reasonably well the experimental spectrum. Fortune [5] finds that the underlying structures of the states obtained in MCAS are different to those obtained in the shell model. Yet the shell model studies therein [5] are simple $0\hbar\omega$ models which indicate a large $1s_{1/2}$ component, which is missing in MCAS. That is true, but not obvious: the mixing of the $1s_{1/2}$ orbit is there, but its contribution is part of the whole mixing due to the channel-coupling. This aspect of a small $1s_{1/2}$ component is also found by our shell model. The spectroscopic factors for the addition of a neutron to states in $^{16}$C forming the compound $^{17}$C ground state indicate a dominant $0d_{5/2}$ component when capture to the $0^+$, $2^+$, and $4^+$ states in $^{16}$C are taken collectively. A dominant $1s_{1/2}$ component is only present for coupling to the $0^+$ state in $^{16}$C.

## 4. Conclusions

The neutron-rich isotopes of carbon have been described using both a large-scale shell model and also MCAS. The results between the two schemes indicate that the collectivity inherent in MCAS may indicate admixtures of higher $\hbar\omega$ components are necessary in any shell-model description. This also negates any simplistic description of the carbon isotopes based on limited-basis $0\hbar\omega$ shell models.

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