Modelling the light curves of *Fermi* LAT millisecond pulsars

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Abstract. There are 40 millisecond pulsars (MSPs) in the second pulsar catalogue (2PC) of the *Fermi* Large Area Telescope (LAT). This pulsar subpopulation is unique owing to the relatively small magnetospheres of MSPs. This may result in radio emission originating at higher altitudes and covering larger solid angles than in the case of their younger counterparts. MSP magnetic fields may also be more complex and their masses larger. Furthermore, these MSPs exhibit some profile patterns that are not seen in younger pulsars. We modelled the MSP radio and γ -ray light curves (LCs) using outer gap (OG), two-pole caustic (TPC), low-altitude slot gap (laSG), and pair-starved polar cap (PSPC) geometric models, combined with a semiempirical conal radio model. We find that no model fits all cases, with the OG and TPC models providing best fits for comparable numbers of MSP LCs. We find a broad distribution of best-fit inclination angles α as well as a clustering at large observer angles ζ . The OG model furthermore seems to require relatively larger α , while the TPC model hints at an inverse trend between α and pulsar spin-down luminosity \dot{E} . We expect these tentative trends to strengthen with the accumulation of more data, and we will discuss their significance in the context of our geometric models. Future work will include the implementation of new magnetic field geometries as well as higher-altitude and more complex radio emission patterns.

1. Introduction: why millisecond pulsars are special

There are 40 millisecond pulsars (MSPs) in the second pulsar catalogue [1] of the *Fermi* Large Area Telescope (LAT) [2], representing roughly a third of the current γ -ray pulsar population. This subpopulation is unique owing to the relatively small magnetospheres of MSPs. This is related to their small periods P, which determine their so-called light cylinder radius ($R_{\rm LC} = Pc/2\pi$) where the corotation speed equals the speed of light, acting as a boundary characterizing the magnetospheric size. Such smaller magnetospheres may result in radio emission originating at higher altitudes (and at larger corotation speeds, where relativistic aberration is enhanced), covering larger solid angles than in the case of their younger counterparts [3]. In turn, this may lead to radio beams that are detectable at relatively larger impact angles $\beta = \zeta - \alpha$, with α the inclination angle and ζ the observer angle, both measured with respect to the rotation axis. MSPs therefore offer unique opportunities to study pulsar emission geometries over a larger region in phase space than non-recycled, radio-loud γ -ray pulsars. In this context, it is interesting to note that all observed γ -ray MSPs are radio-loud, in contrast to non-recycled pulsars. Given their much longer and more violent evolutionary history, including spin-up via accretion [4], MSP magnetic fields may be more complex than those of younger pulsars, which may in part explain why their pulse profiles are more intricate and diverse. For example, MSPs were found to exhibit three classes of light curves (LCs) – those where the radio leads, is aligned with, or trails the γ -ray profile (termed 'Class I', 'Class II', and 'Class III'; see [5]). Conversely, the radio leads the γ -ray profiles in the bulk of young pulsars (making them 'Class I'), and only the Crab shows phase alignment of radio and γ -ray profiles ('Class II').

Lastly, MSPs generally seem to be more massive than their younger counterparts [6] due to accretion of matter from their companion stars, which may enhance General Relativistic effects such as frame dragging [7]. This may lead to increased electric fields which accelerate primary charges originating from the polar cap, especially close to the stellar surface.

2. Geometric modelling and light curve fitting

We modelled the MSP radio and γ -ray LCs using standard emission geometries, including the outer gap (OG) [8], two-pole caustic (TPC) [9], altitude-limited OG and TPC (alOG, alTPC) [5], low-altitude slot gap (laSG) [5], and pair-starved polar cap (PSPC) [10] geometric models. All these geometric models have physical counterparts¹ based on different electrodynamical assumptions. The latter lead to different locations and extents of acceleration regions (or 'gaps') within the magnetosphere, associated with high-energy emitting regions. We combined these γ -ray geometries with a semi-empirical conal radio model [11, 12], while using a core component when indicated by polarimetry (e.g., when sense changing of circular polarization is observed).

Our models assume the retarded dipole magnetic field [13] as basic magnetospheric geometry, and correct for aberration and time-of-flight delays of emitted photons [14], while assuming a constant emissivity in the corotating frame (for all but the laSG and PSPC models). After collecting the emission (per solid angle) of a particular simulated pulsar in a 2D skymap (ζ vs. phase) for a fixed value of α , we constructed LCs on a grid of model parameters, and then used a maximum likelihood approach to obtain best joint radio / γ -ray LC fits for each 2PC MSP using *Fermi* LAT and radio LC data. We lastly estimated errors on the model parameters using either one- or two-dimensional likelihood profiles. More details may be found elsewhere [5, 15, 16].

3. Results: tentative trends

3.1. Towards a hybrid geometry?

We find that no model can universally fit all MSP LCs. The OG and TPC models perform the best, providing best fits for 80% of the modelled MSP LCs (40% each). We find that the TPC geometry better fits LCs with significant off-peak γ -ray emission, while OG models prefer LCs with little or no off-peak emission. There are 27 Class I MSPs, 15 being best fit by the TPC, and 12 by OG geometry. There are 6 Class II MSPs, 1 of which is best fit by the alTPC model, 4 by the alOG model, and 1 by the laSG model. Lastly, the 7 Class III MSPs are exclusively best fit by the PSPC model. The above implies that a "mix" of models may be needed to fit all observed profiles, pointing to some hybrid model that incorporates the different models studied so far. Alternatively, a new geometry may be needed which may provide more richness in terms of potential LCs it can produce, e.g., mimicking TPC LCs in some cases and OG LCs in other.

3.2. Pulsar geometry and visibility constraints

From our best fits of MSP LCs, we find a broad distribution of α (Figure 1), which may argue against the idea [17] of spin-axis alignment with age, as this would imply a preference for smaller

 1 Technically, the PSPC model *is* the actual physical model in this case.



Figure 1. Best-fit α vs. ζ . Different symbols distinguish different emission geometries.

values of α for these old stars. This is in contrast to modelling results [18] obtained for nonrecycled γ -ray pulsars, where the best-fit $\alpha < 40^{\circ}$. The large range in α for MSPs partly reflects their wider radio beams and smaller magnetospheres, which allow both the radio and γ -ray beams to be visible over a larger region of phase space than for longer-period pulsars. While there may be younger pulsars with large α , their narrow radio beams may be missed due to an unfavourable observing geometry (i.e., large β), and they will be radio-quiet γ -ray pulsars, if indeed γ -ray pulsations may be identified via, e.g., a blind periodic search [19] (see Figure 7 of [18]). The α of such pulsars may therefore be very uncertain, if pulsations are indeed found; otherwise, these sources will be deemed unidentified. The smaller range in α may thus be indicative of this selection effect. For the MSPs, on the other hand, a relatively larger range of β is allowed, restricting the number of radio-quiet MSPs to very few. Indeed, there are no radio-quiet MSPs in the 2PC, while half of the young γ -ray pulsars is radio-quiet.

There seems to be a 'zone of avoidance' along $\alpha = \zeta$, although this is not a priori expected in terms of the possible α and ζ that MSPs may assume. While it would be interesting to see if this region is filled as more MSPs are discovered, this effect may point to some necessary refinement of our assumed radio emission geometry (i.e., one where profiles with lower multiplicities may be produced, even at $\alpha \approx \zeta$).

Figure 1 also indicates a clustering at large ζ , corresponding to the fact that it is generally the bright caustic emission (at large ζ , near the spin equator) that is sampled by the observer to form the γ -ray profile peaks. A preference of ζ close to 90° is also expected if pulsar spin axes are distributed randomly with respect to the Earth line of sight (in which case ζ will follow a sin ζ distribution).

The relatively larger α for the OG (vs. the TPC model) is connected to visibility – the OG model is simply not visible when α is too small, due to the fact that it does not assume any emission below the null charge surface² at lower altitudes, as in the case of the TPC model. In the PSPC case, the best-fit α values in the range $40^{\circ} - 60^{\circ}$ give optimal off-peak emission levels and radio peak multiplicities. Interestingly, all currently modelled Class III MSPs (using the PSPC model) have $\zeta > \alpha$.

² Defined by the condition $\rho_{GJ} = 0$, with ρ_{GJ} the Goldreich-Julian charge density [20].



Figure 2. Plot of best-fit α vs. \dot{E} .

3.3. An evolutionary trend?

Figure 2 indicates the best-fit α vs. spin-down power \dot{E} . The TPC model best-fit results hint at an inverse trend between α and \dot{E} for Class I MSPs. At larger $\dot{E} \propto \dot{P}P^{-3}$, we have smaller P, so that smaller α seem to be associated with more powerful pulsars in this case. If we accept that ζ is generally large (Section 3.2), this implies a larger impact angle β for these pulsars. This corresponds to the fact that the TPC is indeed visible for larger values of β compared to the OG model, since it includes low-altitude emission not present in the OG geometry. Furthermore, since all MSPs in this plot are radio-loud, we expect that the radio beams of high- \dot{E} MSPs will generally be at higher altitudes, and therefore wider, so that one may probe smaller α . Lower- \dot{E} MSPs may not be visible in radio if their narrower radio beams are pointing away from the observer, and so may not be easily identified as γ -ray pulsars. This 'inverse trend' may therefore derive from visibility rather than evolutionary considerations. However, this trend must be seen as preliminary, noting the presence of outliers and points with large uncertainties.

3.4. Caustic radio emission

In the case of Class II MSPs, we find that the radio emission may be caustic in nature (i.e., emission originating at different altitudes in the magnetosphere being bunched in phase by relativistic effects; see [21]), since the radio and γ -ray profiles are phase-aligned, implying a common origin of the emission. This radio emission therefore originates at higher altitudes as opposed to the usual low-altitude conal emission found in the Class I and Class III MSPs [5], and should be associated with depolarization and rapid position angle swings, since the emission from a large range of altitudes and magnetic field orientation is compressed into a narrow phase interval when forming the bright peaks [14]. Although the radio emission altitudes cannot be well constrained within current statistics, we do find that the radio and γ -ray emission regions typically have significant overlap, while the radio is generally more limited in altitude and higher up compared to the γ -rays.

3.5. Tapping the power source - luminosity and beaming

The γ -ray luminosity L_{γ} is a very important intrinsic parameter characterizing how rotational energy is converted into γ -ray emission. This is estimated from the observed γ -ray energy flux G using (e.g., [22]) using $L_{\gamma} = 4\pi f_{\Omega}Gd^2$, with f_{Ω} the beaming correction factor accounting for the fact that emission is not beamed isotropically. We can estimate f_{Ω} from the emission



Figure 3. Plot of L_{γ} vs. \dot{E} .

pattern implied by the model by comparing the total emitted pattern to the one observed at a particular ζ , for a given α . Our fits indicate that $f_{\Omega} < 1$ in most cases, so we are typically sampling emission that is above the average emission level. Conversely, for PSPC best-fit profiles $f_{\Omega} > 1$ in most cases, since we are missing the brightest part of the emission concentrated at low altitudes near the polar caps. The evolution of L_{γ} with \dot{E} is very important, as this captures the regime in which energy conversion takes place. It is expected that younger pulsars find themselves in a screened-potential regime, characterized by a relation $L_{\gamma} \propto I_{\rm PC} \propto \dot{E}^{1/2}$ (with $I_{\rm PC}$ the PC current, and the polar cap potential $V_{\rm PC}$ being roughly constant in this case), while older pulsars operate in a rather more pair-starved regime where conversion of emitted γ -rays into electron-positron pairs is inefficient, and $L_{\gamma} \propto V_{\rm PC}I_{\rm PC} \propto \dot{E}$ [23]. We find that L_{γ} roughly follows a linear trend with \dot{E} for the MSPs, confirming this expectation to some extent. We lastly observe a clustering around a γ -ray efficiency of $L_{\gamma}/\dot{E} = 10\%$.

3.6. Discriminating between the different LC classes

Figure 4 shows the positions of the modelled MSPs on a period-period-derivative $(P\dot{P})$ diagram, with the different classes differentiated by different symbols, as described in the legend. Grey dots are radio MSPs with no detection in 2PC. Contours of constant magnetic field $B_{\rm LC}$ at the light cylinder are indicated by dashed lines, while constant- \dot{E} contours are indicated by dotdashed lines, assuming dipole spin-down and a canonical moment of inertia of $I = 10^{45}$ g cm². We see no clear differentiation of MSP LC classes according to the usual pulsar variables such as P, \dot{P}, \dot{E} , or $B_{\rm LC}$, although it seems that Class II MSPs favour lower values of P, and Class III MSPs scatter about low values of $B_{\rm LC}$.

4. Conclusion

We have described our effort to model the LCs of all MSPs that appear in *Fermi* LAT's 2PC. We noted some tentative trends (e.g., a broad distribution in α , clustering at large ζ , and a linear relation between L_{γ} vs. \dot{E}), which may strengthen with the accumulation of more data. A new hybrid model may be needed to unify the different older models and reasonably reproduce all existing LCs. The different LCs classes are not easily distinguished based on canonical pulsar variables alone, but may rather be a reflection of the complex electrodynamical environment of the pulsar. We will next study the effects of new magnetic field geometries (e.g. [24, 25, 26]) and higher-altitude, more complex radio emission patterns on the predicted MSP LCs.



Figure 4. The $P\dot{P}$ diagram indicating different MSP LC classes.

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