Klein-Gordon energy states of SCPunder Plasma medium

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Contents

Part01:

Introduction

Obtaining precise solutions for the energy levels and wavefunctions of the Schrödinger equation across a range of quantum systems with different potentials is a major challenge in quantum mechanics. Among these significant potentials are the screened Coulomb potentials $[1]$, which are used to describe confined systems in various environments. Due to the unavailability of exact solutions for these potentials, several numerical $[2, 3]$ and analytical methods $[4, 5]$ have been employed. This work aims to apply a novel theoretical method to derive accurate solutions for the Schrödinger equation with screened Coulomb potentials, which are especially important in plasma physics and particle physics.

The objective of our study

We study the effect of plasma screening on the relativistic dynamics of an atomic system represented by a hydrogen-like atom by screened Coulomb potentials in spherical coordinate space. By using suitable approximations , the energy spectrum and its corresponding wave functions are determined with high accuracy using the conventional precise method, considering only one studied cases$(n = 0)$ of our atomic system.

What are the screened Coulomb potentials?

SCP represent a simple case of the more generalized exponential screened Coulomb potentials (**MGESC**) [6]. These are short-range potentials that effectively describe interactions in many-body environments and are also known as Yukawa and Debye potentials[7 − 8]. They are widely used to describe quantum systems, including hadron interactions in gauge theory[9 − 10], impurity-electron interactions in semiconductors[11 − 12], quantum dots, and the effects of charged particles in plasma.This is the focus of our analysis, given by the equation:

$$V\left(r\right) = -\frac{Ze^2}{r}e^{-kr} \tag{1}$$

Where Z is the atomic number, $e^2 = \frac{q^2}{4\pi\varepsilon_0}$is the coloumb constant and $k = \frac{1}{\lambda}$ corresponding to the screening parameter.

part02:Theoratical method

Confined Hydrogen-like atoms:

In spherical coordinates the general Klein-Gordon equation with the scalar and vector potential is gven as::

$$\left(E-V\left(r\right)\right)^{2}\psi\left(r,\theta,\varphi\right)=\left(\hat{p}^{2}c^{2}+\left(S\left(r\right)+Mc\right)^{2}\right)\psi\left(r,\theta,\varphi\right) \tag{2}$$

If $V\left(r\right)=S\left(r\right)$ rewrite the Eq (2) as follows:

$$\hbar^{2}c^{2}R''\left(r\right)+\left(\left(E^{2}-M^{2}c^{2}\right)^{2}-2\left(E+Mc\right)S\left(r\right)-\frac{l\left(l+1\right)\hbar^{2}c^{2}}{r^{2}}\right)R\left(r\right)=0 \tag{3}$$

By Taylor's series expansion up to order three in order to make the form of killingbeck potential:

$$-\frac{Ze^{2}}{r}e^{-kr}=-\frac{a}{r}+br+cr^{2}+d$$

Where introduce the screening strength parameters:

$$a=Ze^{2},b=-\frac{1}{2}\frac{\left(Ze^{2}\right)}{\lambda^{2}},c=\frac{1}{6}\frac{\left(Ze^{2}\right)}{\lambda^{3}}\text{ and }d=\frac{Ze^{2}}{\lambda} \tag{4}$$

After taking the expansion forms of the confining potential,rewrite the Eq (3) in the form:

$$\left[\frac{d^2}{dr^2}+C_0+C_1r+C_2r^2+\frac{C_3}{r}+\frac{C_4}{r^2}\right]R(r)=0 \quad (5)$$

With $C_0=\frac{\left(E^2-M^2c^2\right)-\frac{2(E+Mc)Ze^2}{\lambda}}{\hbar^2c^2}$, $C_1=\frac{\frac{(E+Mc)}{\hbar^2c^2}\left(Ze^2\right)}{\lambda^2}$, $C_2=-\frac{\mu}{3\hbar^2}\frac{\left(Ze^2\right)}{\lambda^3}$, $C_2=-\frac{\frac{(E+Mc)}{3\hbar^2c^2}\left(Ze^2\right)}{\lambda^3}$ and $C_4=-l\left(l+1\right)$

After considering the solution $R(r)=r^{\alpha}e^{-\left(\beta r+\gamma r^2\right)}g\left(r\right)$ and let us use a new transformation $\rho=\sqrt[4]{-C_2}r$ can be obtain the canonical form of BHE:

$$\begin{array}{c}(\rho\frac{d^2}{d\rho^2}+\left(1+\alpha^{\backprime}-\beta^{\backprime}\rho-2\rho^2\right)\frac{d}{d\rho}+\left(\left(\gamma^{\backprime}-\alpha^{\backprime}-2\right)\rho\right.\\ -\frac{1}{2}\left(\delta^{\backprime}+\beta^{\backprime}\left(1+\alpha^{\backprime}\right)\right))g(\rho)=0\end{array} \quad (6)$$

Subsequently,by introducing the following arbitrary parameters: $\alpha^{\backprime}=2l+1$; $\beta^{\backprime}=\frac{-C_1}{\sqrt{-C_2}}\left(-C_2\right)^{\frac{-1}{4}}$; $\gamma\backprime=\frac{1}{\sqrt{-C_2}}\left(C_0+\frac{C_1^2}{4(-C_2)}\right)$and $\delta\backprime=\frac{-2C_3}{\sqrt[4]{-C_2}}$ In this way the heun's wave function is given by the regular solution:

$$g\left(\rho\right)=H_b\left(\alpha^{\backprime},\beta^{\backprime},\gamma\backprime,\delta^{\backprime},\rho\right)=\sum_{n\geq0}a_n\frac{\Gamma\left(1+\alpha^{\backprime}\right)}{\Gamma\left(1+\alpha^{\backprime}+n\right)}\frac{\rho^n}{n!} \quad (7)$$

After remplacing the Eq (12) into Eq (11) ,we take a polynomial of degreen n break with condition if and only:

$$\gamma^{\backprime}-\alpha^{\backprime}-2=2n, n=0,1,2,3,................$$

From$(n=0)$ We obtain

$$E^2=\hbar^2c^2\sqrt{\frac{\frac{(E+Mc)}{3\hbar^2c^2}\left(Ze^2\right)}{\lambda^3}}(2l+3)-\frac{4\left(E+Mc\right)^2}{\hbar^2c^2\left(l+1\right)^2}\left(Ze^2\right)^2+\frac{2\left(E+Mc\right)Ze^2}{\lambda}$$

$+\mathrm{M}^2c^2$(8)

So:

$$E=\pm\sqrt{\begin{array}{c}\hbar^2c^2\sqrt{\frac{\frac{(E+Mc)}{3\hbar^2c^2}\left(Ze^2\right)}{\lambda^3}}(2l+3)-\frac{3(E+Mc)\left(Ze^2\right)}{4\lambda}\\+\frac{2(E+Mc)Ze^2}{\lambda}+M^2c^2\end{array}} \quad (9)$$

With considering the radial wave function:

$$R_{0,l}(r) = A_{0,l} r^l e^{-\left(\left(\frac{-\left(\frac{\mu}{\hbar^2}\frac{(Ze^2)}{\lambda^2}\right)}{2\sqrt{\frac{\mu}{3\hbar^2}\frac{(Ze^2)}{\lambda^3}}}\right)r + \left(\frac{1}{2}\sqrt{\frac{\mu}{3\hbar^2}\frac{(Ze^2)}{\lambda^3}}\right)r^2\right)}(a_0) \qquad (10)$$

Where $a_0 = 1$ and $A_{0,l}$ is normalized constant.

The result and discussion:

The precise approach has been used to solve the Klein-Gordon equation with equal scalar and vector SCP potential. For our system, biconfluent heun polynomials were used to describe the eigenvalues of the energy levels and normalized radial wave functions. In some situations, thermodynamic properties may be predicted using the relativistic results.

Conclusion:

The main motivation for these studies and theoretical research is a number of calculations that have been carried out on each of the weak atomic systems. And the strong effects of confinement resulting from the environment by mastering existing models using numerical and analytical methods.

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Thank you!