Modified Hybrid Inflation, Reheating and Stabilization of the Electroweak Vacuum

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([4] M. Ibrahim, M. Ashry, E. Elkhateeb, A.M. Awad,A. Moursy, Modified Hybrid Inflation, Reheating and Stabilization of the Electroweak Vacuum, Physical Review D 107, 035023 (2023) (arXiv: 2210.03247))

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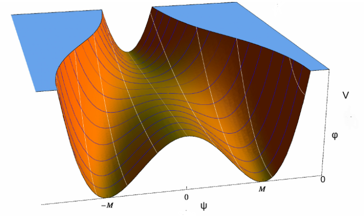
- Why Hybrid Inflation?
  1. Within the quest to connect Cosmology and Particle Physics, hybrid inflation is a perfect candidate for this.
  2. At its tree level, the HI potential is [5, 6]

$$V_{\mathrm{HI}}(\phi, \psi) = \kappa^2 \left(M^2 - \frac{\psi^2}{4} \right)^2 + \frac{m^2}{2}\phi^2 + \frac{\lambda^2}{4}\phi^2\psi^2 \tag{1}$$

  3. It looks as hybrid of chaotic inflation with $V(\phi) = \frac{m^2}{2}\phi^2$ and the usual spontaneous symmetry breaking potential $V(\psi) = \kappa^2(M^2 - \dfrac{\psi^2}{4})^2$.
  4. It is termed "hybrid" because the vacuum energy density $V_0 = \kappa^2 M^4$ is provided by the waterfall field ψ, while ϕ is the slowly rolling inflaton field.

- The effective mass squared of the waterfall field ψ field in the $\psi = 0$ direction is $m_\psi^2 = \kappa^2 M^2 + \lambda^2 \phi^2/2$. Thus, when $\phi > \phi_c = \sqrt{2}\kappa M/\lambda$ the inflation occurs.
- During the inflation, the inflaton field ϕ slowly rolls down the valley of ψ, on which ψ is frozen at zero.



- Upon reaching $\phi = \phi_c$, the waterfall phase is triggered, the minimum in the ψ direction becomes a maximum and the inflation ends.

- On that inflationary trajectory, by minimizing Eq.(1) w.r.t ψ, the (HI) effective potenial is

$$V_{\rm HI}^{\rm inf}(\tilde{\phi}) = V_0\big(1+\tilde{\phi}^2\big), \tag{2}$$

where $\tilde{\phi} = \sqrt{\frac{\eta_0}{2}}\phi$.

- The slow roll parameters of inflation are given by

$$\epsilon = \frac{\eta_0}{4}\left(\frac{V_{\tilde{\phi}}^{\rm inf}}{V^{\rm inf}}\right)^2, \quad \eta = \frac{\eta_0}{2}\left(\frac{V_{\tilde{\phi}\tilde{\phi}}^{\rm inf}}{V^{\rm inf}}\right), \tag{3}$$

where $\eta_0 = \frac{m^2 M_{\rm P}^2}{V_0}$ and $M_{\rm P}$ is the reduced Planck mass.

- The number of e-foldings N_e is given by

$$N_e = \frac{1}{\sqrt{\eta_0}} \int_{\tilde{\phi}_e}^{\tilde{\phi}_*} \frac{d\tilde{\phi}}{\sqrt{\epsilon(\tilde{\phi})}} \tag{4}$$

- The spectral index n_s, the tensor-to-scalar ratio r and the amplitude of scalar perturbations A_s are given respectively as (* means at horizon exit)

$$n_s = 1 - 6\epsilon_* + 2\eta_* = 1 - 4\eta_0 \frac{\tilde{\phi}^2 - 1/2}{(\tilde{\phi}^2 + 1)^2}, \tag{5}$$

$$r = 16\epsilon_* = \frac{16\eta_0\tilde{\phi}^2}{(\tilde{\phi}^2 + 1)^2}, \tag{6}$$

$$A_s = \frac{V_*^{\text{inf}}}{24\pi^2\epsilon_*} \tag{7}$$

- For sub-Planckian values of the field $ns \sim 1$ and r is very small. While for trans-Planckian values, we have $r > 0.1$.

- One of the solutions to the HI model drawbacks is to consider the one-loop corrections as following

$$V_{\text{loop}} = -A\phi^4 \log(\frac{y\phi}{\mu}) \tag{8}$$

where $A = \frac{y^4}{16\pi^2}$.

- The effective potential in this case is

$$V_{\text{HI}}^{\text{inf}}(\tilde{\phi}) = V_0\big(1 + \tilde{\phi}^2 - \tilde{A}_\phi \phi^4\big), \tag{9}$$

where $\tilde{A}_\phi = \frac{4A\log(\phi/\phi_c)}{\eta_0^2(V_0/\text{M}_\text{P}^4)}$

- This solution improves the n_s and r values but they are ruled out by Planck/BICEP recent observations. Also, it spoils the EW vacuum stability.
- Another solution,on the the tree level, is to add an extra scalar field.
- We followed this approach as it's consistent with Planck/BICEP results and helps in stabilizing the EW vacuum.

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- A proposed MHI potential is [4]:

$$V_{\rm MHI}(\phi,\psi,\chi) = \lambda_\psi\Big(\psi^2 - \frac{v_\psi^2}{2}\Big)^2 + \frac{m^2}{2}\phi^2 + 2\lambda_{\phi\psi}\phi^2\psi^2 - 2\lambda_{\phi\chi}\phi^2\chi^2 + \lambda_\chi\Big(\chi^2 - \frac{v_\chi^2}{2}\Big)^2 + 2\lambda_{\psi\chi}\Big(\psi^2 - \frac{v_\psi^2}{2}\Big)\Big(\chi^2 - \frac{v_\chi^2}{2}\Big), \tag{10}$$

- The MHI effective potential is

$$V_{\rm MHI}^{\rm inf}(\tilde\phi) = V_0\big(1 + \tilde\phi^2 - \gamma\tilde\phi^4\big) \tag{11}$$

$$V_0 = \frac{v_\psi^4}{4}\Big(\lambda_\psi - \frac{\lambda_{\psi\chi}^2}{\lambda_\chi}\Big), \quad \eta_0 = \frac{1}{V_0}\Big[m^2 - 2\lambda_{\phi\chi}\Big(v_\chi^2 + \frac{\lambda_{\psi\chi}}{\lambda_\chi}v_\psi^2\Big)\Big], \quad \gamma = \frac{4\lambda_{\phi\chi}^2}{\lambda_\chi\eta_0^2 V_0}. \tag{12}$$

- The spectral index n_s and tensor-to-scalar ratio r are

$$n_s = 1 - 2\eta_0\frac{6\gamma^2\tilde\phi^6 - 5\gamma\tilde\phi^4 + (2+6\gamma)\tilde\phi^2 - 1}{(1+\tilde{\phi}^2 + \gamma\tilde{\phi}^4)^2} \tag{13}$$

$$r = \frac{16\eta_0(\tilde\phi - 2\gamma\tilde{\phi}^3)^2}{(1+\tilde{\phi}^2 - \gamma\phi^4)^2} \tag{14}$$

- The quartic term with γ in eq. (11) enables a hilltop type inflation in which the inflaton field ϕ can slowly rolls towards the origin.

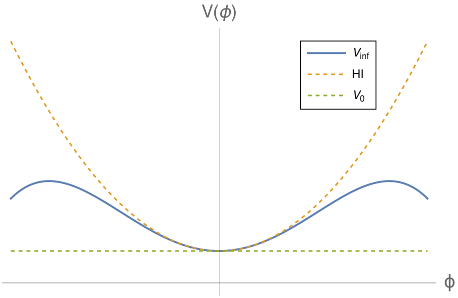

Figure 1: The solid (blue) curve reprsents the MHI inflation potential (10), while the dashed (orange) curve represents the standard hybrid inflation potential.

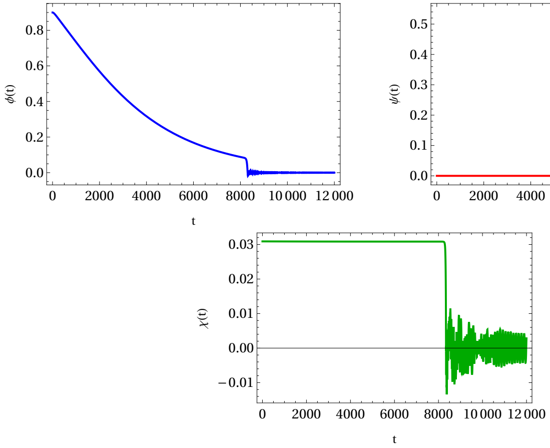

Figure 2: Simulation of the dynamics of the three scalar fields ϕ, ψ and χ as a function of time (t). The blue curve in the upper left represents $\phi(t)$, the red curve in the upper right represents $\psi(t)$ while the green curve in the lower middle represents $\chi(t)$.

Par.	$V_0\ (M_{\mathsf{P}}^4)$	$\eta_0\ (M_{\mathsf{P}}^{-2})$	γ	$\tilde{\phi}_*$	$\tilde{\phi}_c$
BP1	3.00×10^{-11}	1.65×10^{-1}	1.54×10^{-2}	4.90	3.682
BP2	1.40×10^{-10}	4.70×10^{-2}	10.0	1.49×10^{-1}	1.24×10^{-2}

Table 1: BPs of the MHI effective inflation potential (11) which produce the observables in Table 2.

Obs.	N_e	n_s	r	A_s
BP1	59.6	0.9688	0.0165	1.98×10^{-9}
BP2	59.3	0.9674	0.0049	1.93×10^{-9}

Table 2: Inflation observables corresponding to the MHI.

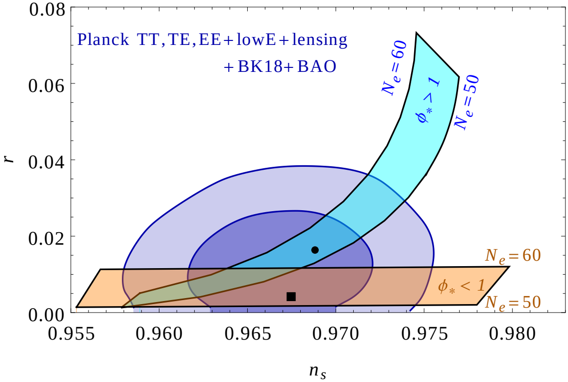

Figure 3: Predictions of the MHI model in the (n_s, r) plane given by the cyan patch (trans-Planckian) and the orange patch (sub-Planckian). The blue contours are the observed constraints extracted from Planck 2018, and they correspond to the observed 68% and 95% C.L. constraints in (n_s, r) plane when adding BICEP/Keck and BAO data [2, 1]. The two BPs indicated by the solid dot and square are BP1 and BP2 presented in Table 2.

Outline

- Inflation deletes the initial conditions which existed on small scale and hence cancels the *thermal phase*.
- However, the reheating mechanism solves this. The coupling of the inflaton field to other particles leads to a decay of the inflaton energy and the inflationary energy density is converted into SM particles.
- We consider the reheating phase where the inflaton field decays into *RH neutrinos*.
- The complete Lagrangian that is responsibe for neutrino masses and reheating, contains the SM higgs h and left handed neutrinos ν_L and has the form

$$\mathcal{L}_\nu = Y_\nu \, h \, \bar{\nu}_L \, N + Y_\phi \, \phi \, \bar{N} \, N + Y_\psi \, \psi \, \bar{N} \, N + Y_\chi \, \chi \, \bar{N} \, N + m_N \, \bar{N} \, N \tag{15}$$

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- The SM Higgs doublet couples to the singlet scalar fields of the MHI potential (10) to give the full scalar potential

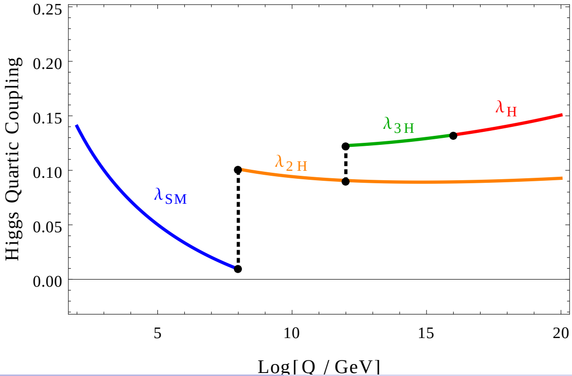
$$V(H,\phi,\psi,\chi) = V_{\mathrm{MHI}}(\phi,\psi,\chi) + \lambda_H\Big(H^2 - \frac{v^2}{2}\Big)^2 + 2\Big(H^2 - \frac{v^2}{2}\Big)\Big[\lambda_{H\phi}\phi^2 + \lambda_{H\psi}\Big(\psi^2 - \frac{v_\psi^2}{2}\Big) + \lambda_{H\chi}\Big(\chi^2 - \frac{v_\chi^2}{2}\Big)\Big]. \tag{16}$$

- When the heavy degrees of freedom(ψ,ϕ,χ) are integrated out successively [3], the SM Higgs quartic coupling is modified at the instability scale thereshold Λ_I with the *matching condition*:

$$\lambda_{2H}\Big|_{\Lambda_I} = \Big[\lambda_{\mathsf{SM}} + \frac{\lambda_{2H\chi}^2}{\lambda_{2\chi}}\Big]\Big|_{\Lambda_I} \approx \frac{\lambda_{2H\chi}^2}{\lambda_{2\chi}}\Big|_{\Lambda_I}. \tag{17}$$

Mass	m_χ	m_ϕ	m_ψ
BP1	1.41×10^8	5.80×10^{12}	2.86×10^{15}
BP2	3.64×10^8	5.03×10^{14}	2.24×10^{15}

Table 3: Scalar masses (GeV) corresponding to the MHI.



- The relevant *one-loop renormalization group equations* (RGE's) of the Higgs quartic coupling takes the form(for $i = 2, 3, 4$)

$$16\pi^2 \frac{d\lambda_{iH}}{dt} = \beta_{iH} = \beta_H^{\mathsf{SM}} + \beta_H^{\mathsf{int}} \tag{18}$$

where

$$\beta_H^{\mathsf{SM}}(\lambda_{iH}) = \frac{27g_1^4}{200} + \frac{9g_1^2g_2^2}{20} + \frac{9g_2^4}{8} - 9\big(\frac{g_1^2}{5} + g_2^2\big)\lambda_{iH} + 24\lambda_{iH}^2 + 12\lambda_{iH}Y_t^2 - 6Y_t^4 \tag{19}$$

- The *beta functions* of the effective 2-field model for $t \sim [8, 20]$

$$\beta_{2H} = \beta_H^{\text{SM}}(\lambda_{2H}) + 4\lambda_{2H\chi}^2, \quad (20)$$

$$\beta_{2H\chi} = \beta_{H\chi}^{\text{SM}}(\lambda_{2H\chi}), \quad (21)$$

$$\beta_{2\chi} = 8{\lambda_{2H\chi}}^2 + 20{\lambda_{2\chi}}^2. \quad (22)$$

- As a very good approximation, we consider the effective 2-field model $V_{2\text{eff}}(H, \chi)$. In this case, the 2×2 mass matrix of H and χ is

$$\mathcal{M}_{H\chi}^2 = 2 \begin{pmatrix} \lambda_{2H} v^2 & \lambda_{2H\chi} v v_\chi \\ \lambda_{2H\chi} v v_\chi & \lambda_{2\chi} v_\chi^2 \end{pmatrix}. \quad (23)$$

- With approximated squared masses

$$m_h^2 \approx 2v^2 \left[\lambda_{2H} - \frac{\lambda_{2H\chi}^2}{\lambda_{2\chi}}\right]\Big|_{\mathrm{EW}} \sim (125.25)^2 \tag{24}$$

$$m_\chi^2 \approx 2v_\chi^2 \left[\lambda_{2\chi} + \frac{\lambda_{2H\chi}^2}{\lambda_{2\chi}} \frac{v^2}{v_\chi^2}\right]\Big|_{EW} \sim \mathcal{O}(10^8)^2 \tag{25}$$

- Accordingly, we have the following boundary constraint for the SM effective Higgs quartic coupling

$$\lambda_{\mathrm{eff}} = \left[\lambda_{2H} - \frac{\lambda_{2H\chi}^2}{\lambda_\chi}\right]\Big|_{\mathrm{EW}} \sim 0.12 \tag{26}$$

- Also, the mixing angle

$$\tan 2\theta_{H\chi} = \frac{2\lambda_{2H\chi} v v_\chi}{\lambda_{2\chi} v_\chi^2 - \lambda_{2H} v^2}\bigg|_{\text{EW}} \sim \mathcal{O}(10^{-7}), \tag{27}$$

and this preserves the SM Higgs physics and up to the Planck scale for the BPs in Table 2 and Table 3 as checked for the running of the mixing angle (27).

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Conclusions

- MHI:
  1. The modification results in an inflation potential in which ϕ *rolls down near a hilltop in the valley of the other hybrid fields*.
  2. n_s problem is then resolved since the inflation occurs mostly in the *negative curvature* part of the potential.
  3. The inflation observables in both trans-Planckian and sub-Planckian cases are *in consistency with the recent Planck/BICEP observations.*
- EWVS: Providing the couplings of the SM Higgs with the inflation singlets and hence *stabilizing the electroweak vacuum up to Planck scale.*
- Reheating: Inflaton field *decays into right handed neutrinos*, that allow for reheating the universe.

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