A Physics Approach to Ecological Networks

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Ecosystem

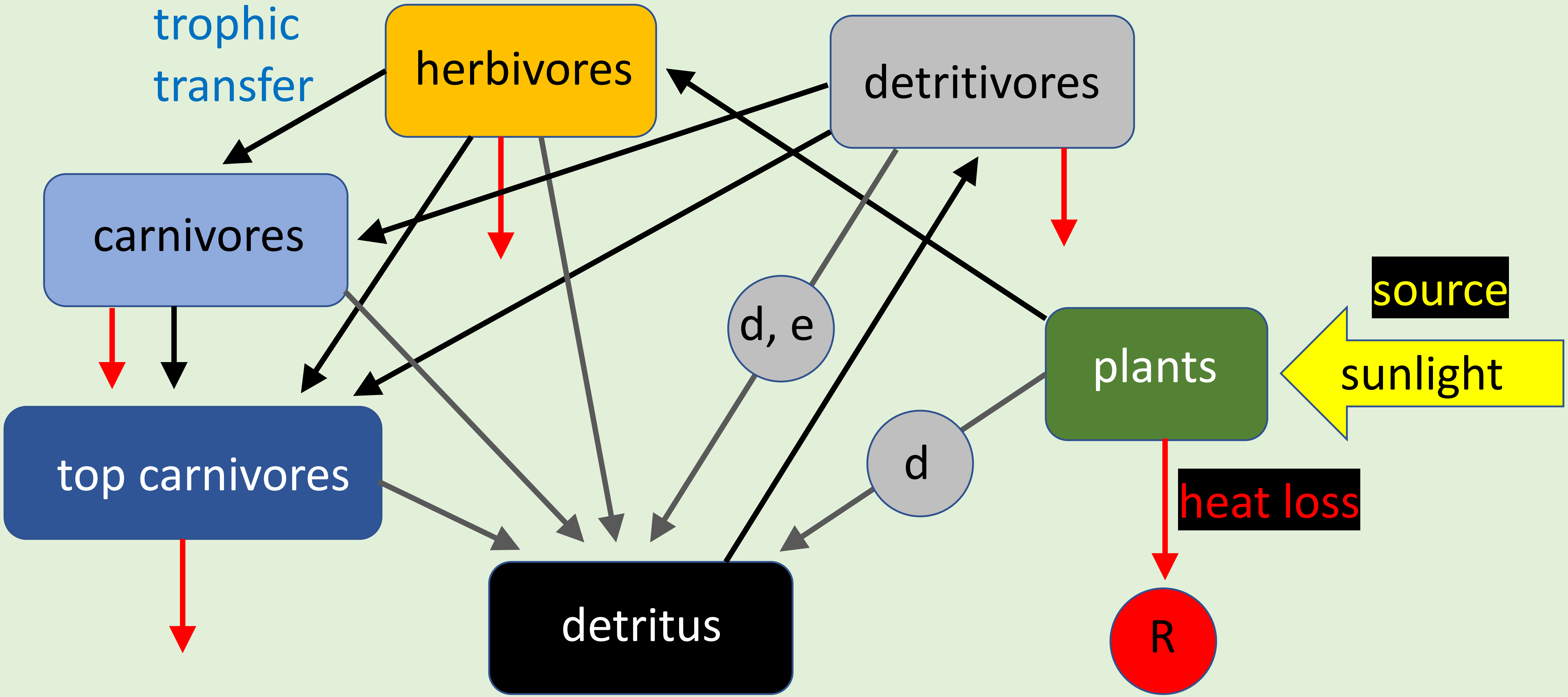
Environment +

Living organisms

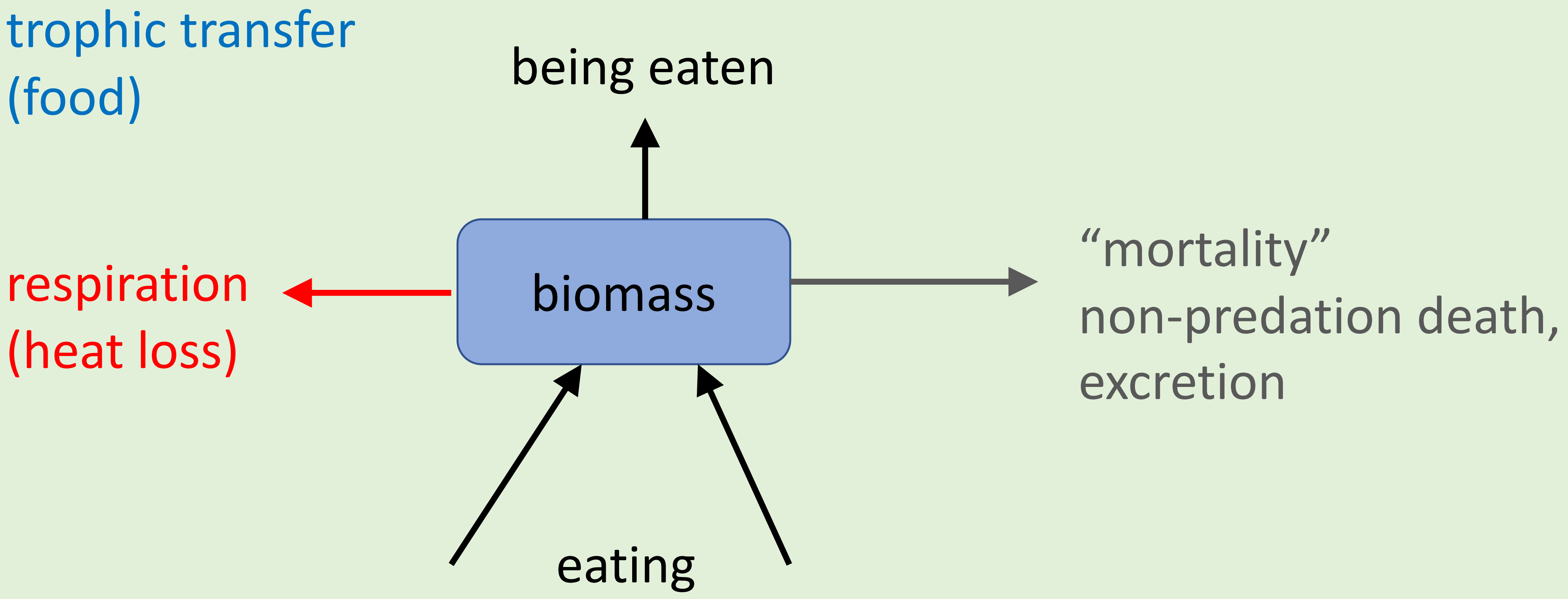
- Ecology
- Biology
- Chemistry
- Geology
- Geography
- Physics!

Ecological Network

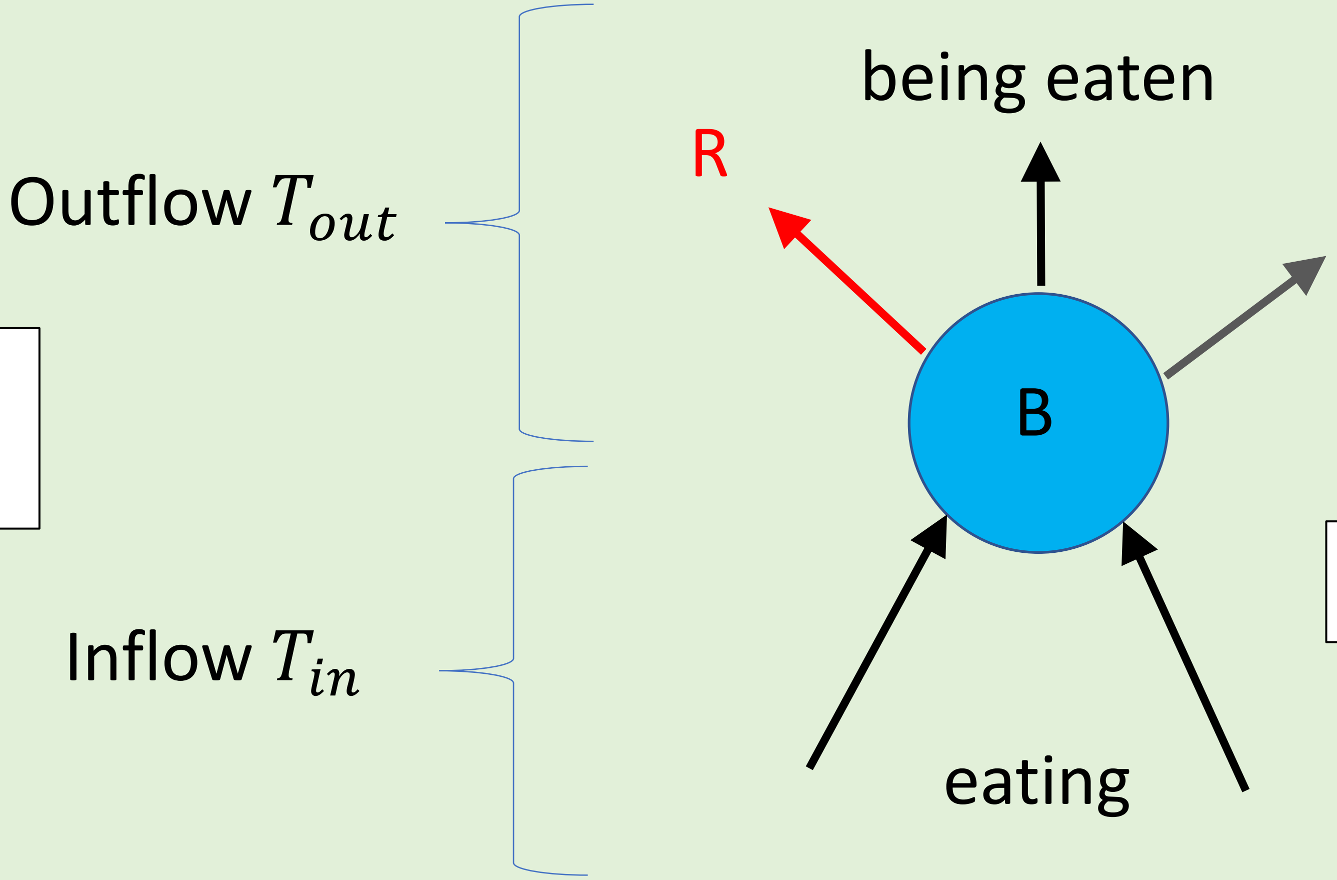
Flow of energy, mass, nutrients



Compartment (group of species)



Node in Network



Static network

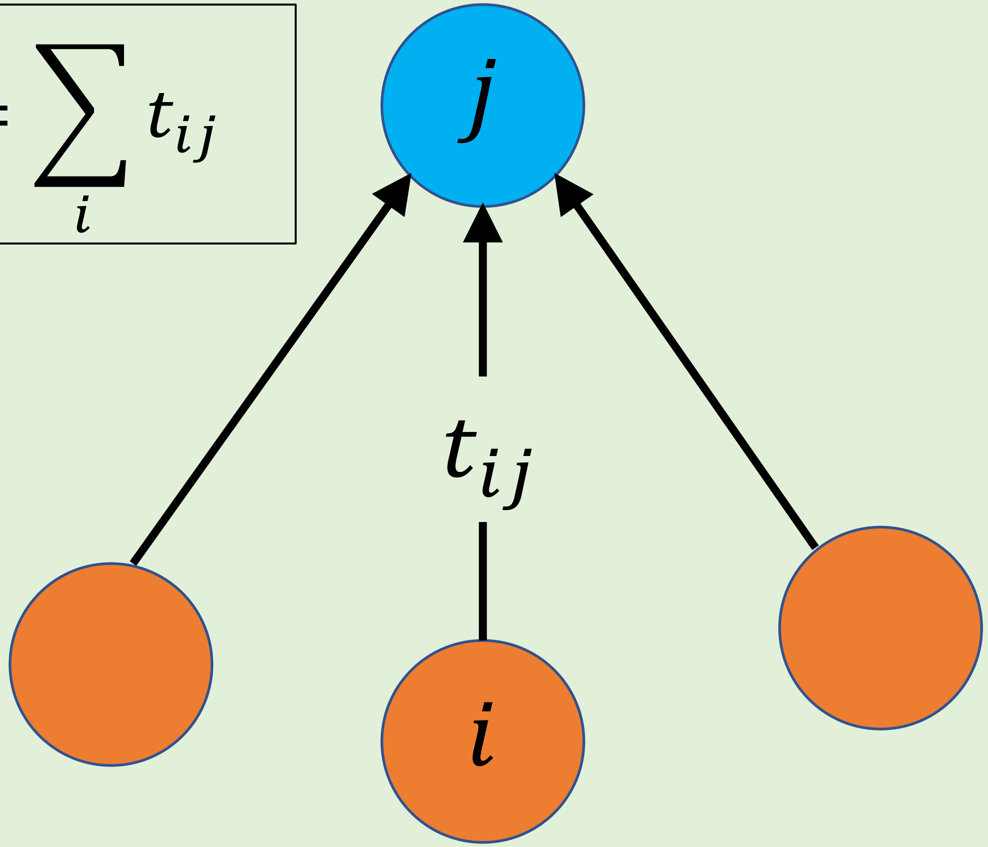
$T_{in} = T_{out}$

directed links

Trophic Transfer and Entropy

Trophic Transfer

$$T_{in} = \sum_i t_{ij}$$



Entropy (Information Theory)

- Measure of order

$$H = -\sum p_{ij} \log_2 p_{ij}$$

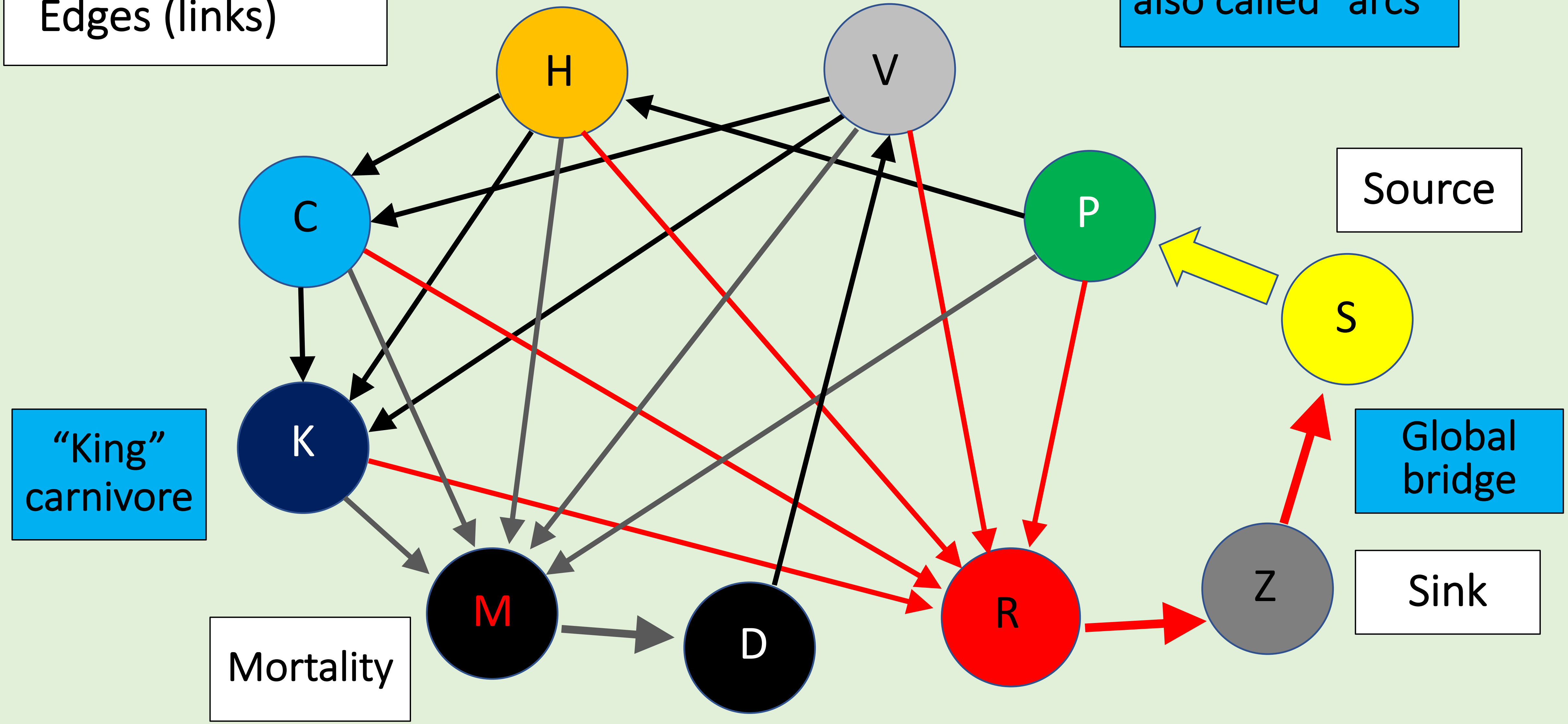
- Probability of energy through link (ij)

$$p_{ij} = \frac{t_{ij}}{\sum t_{ij}}$$

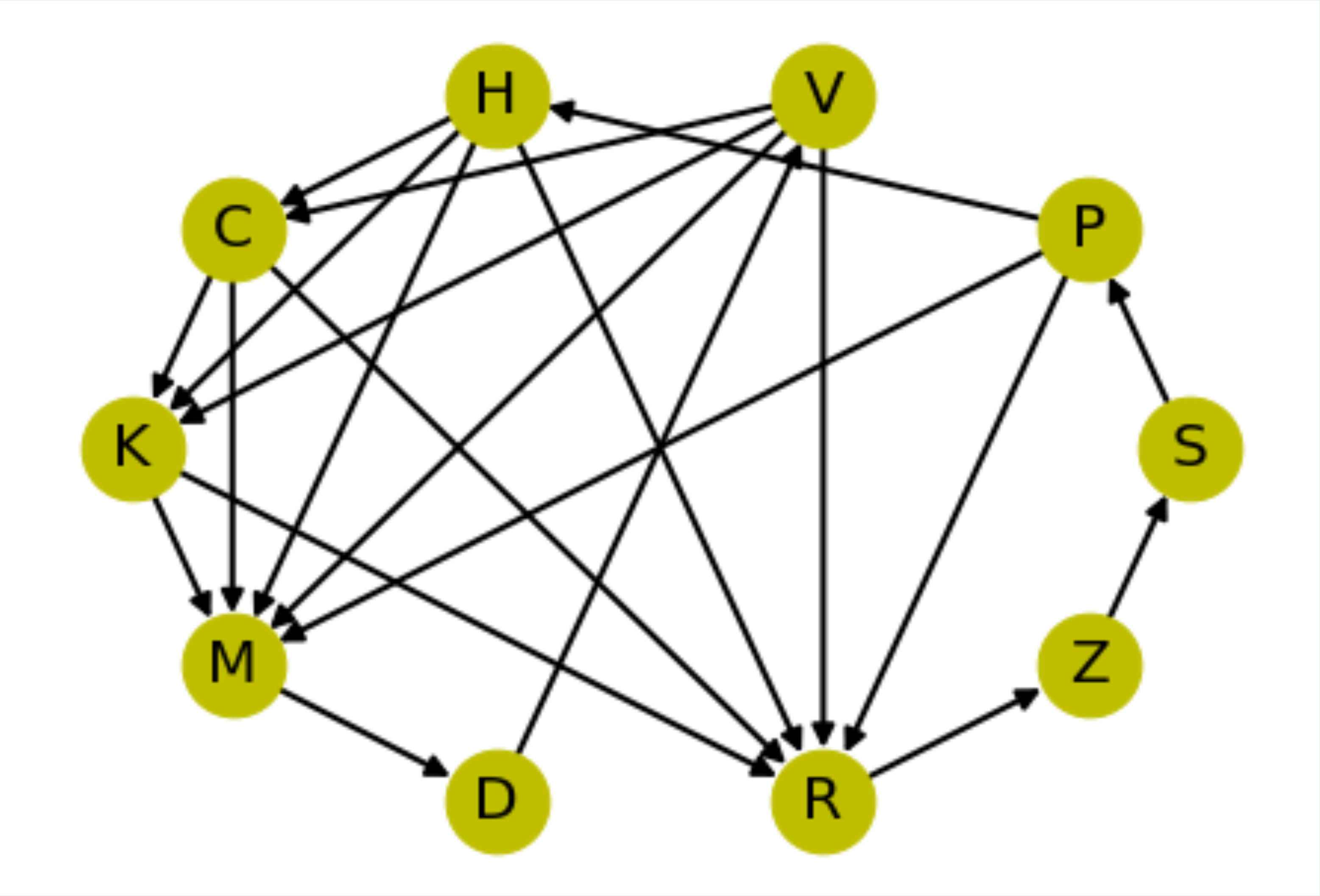
Directed Graph

Vertices (nodes)
Edges (links)

Directed edges also called "arcs"



Python: Networkx



Nodes

and

Edges

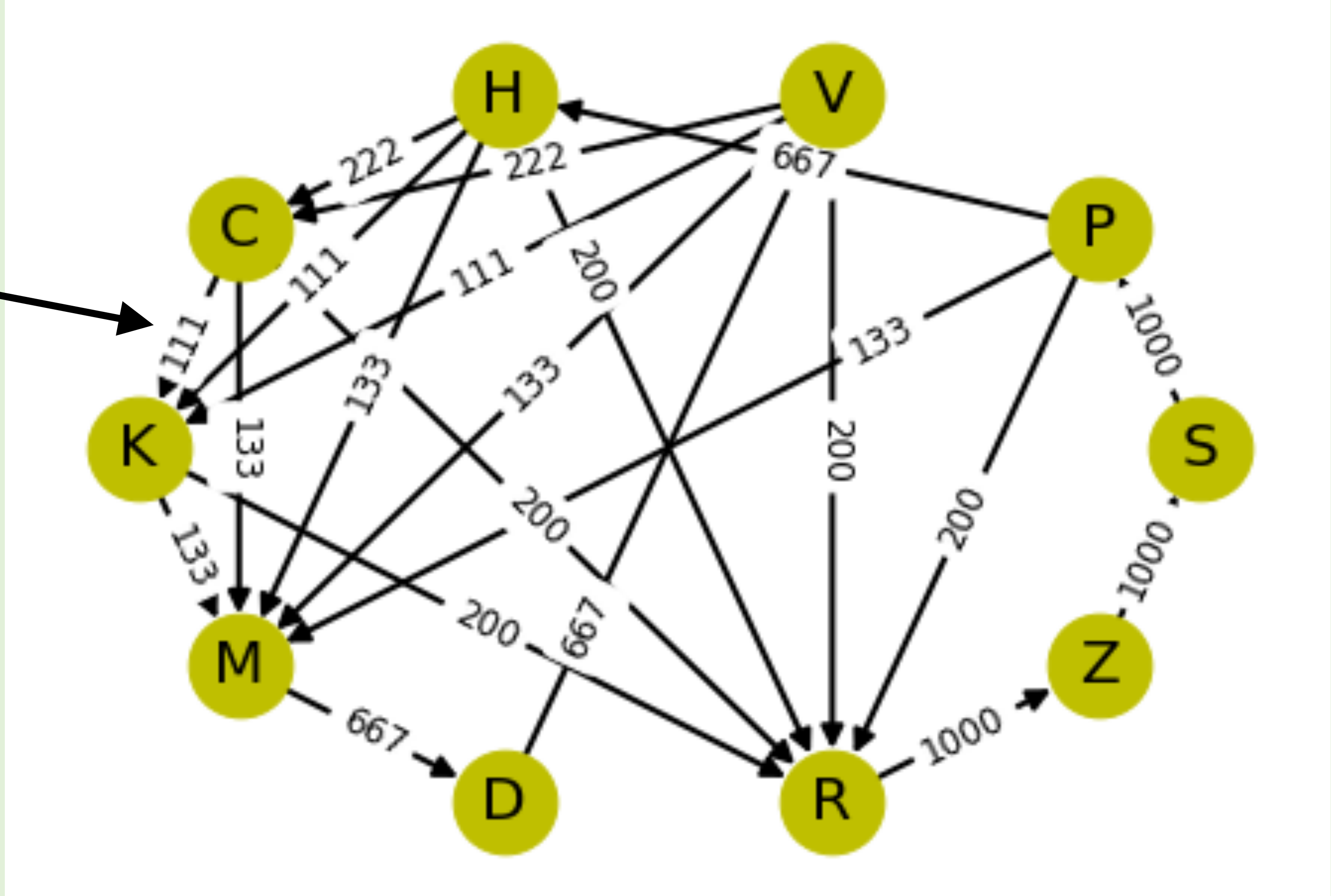
Network has:
$m = 10$ nodes
$n = 21$ edges

With edge flow values

Flow vector $\boldsymbol{x}$
of n flow values

$$\begin{pmatrix} x_1 \\ . \\ . \\ x_k \\ . \\ . \\ x_n \end{pmatrix}$$

$x_k \equiv t_{ij}$

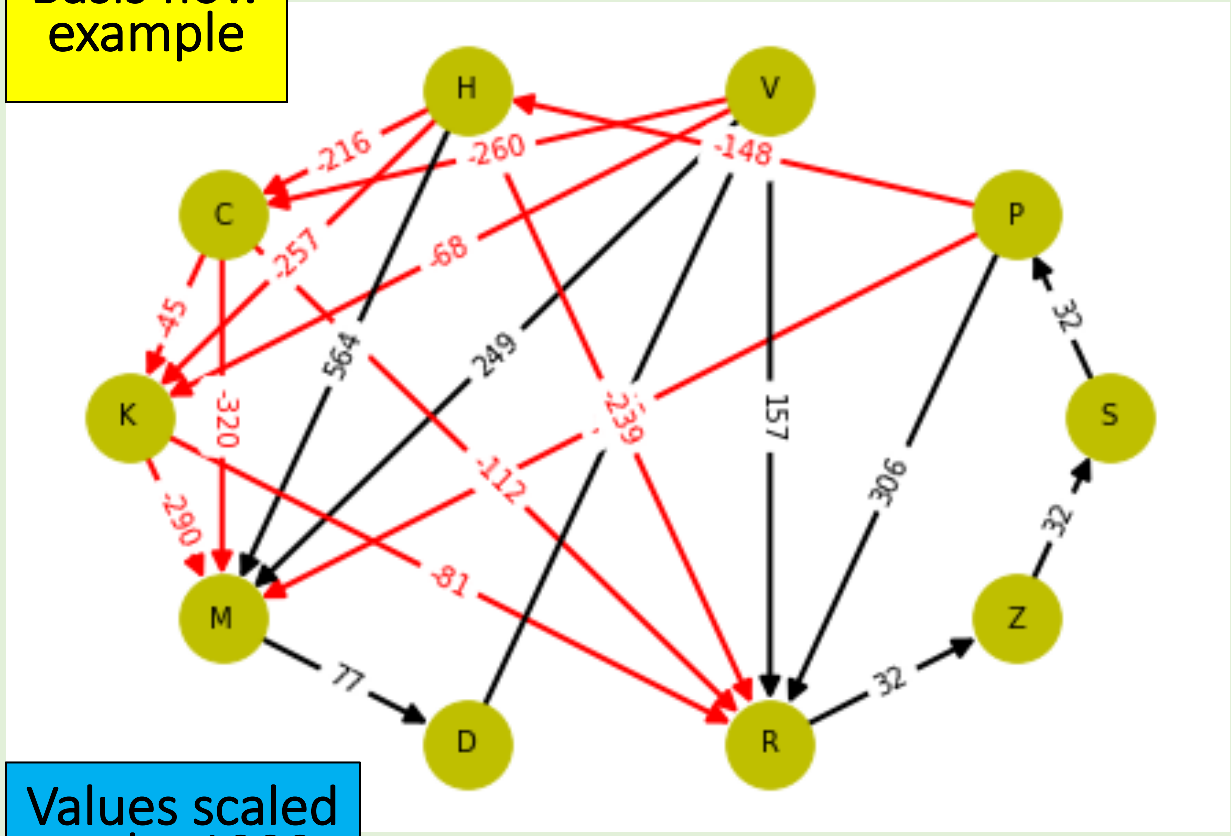


Linear Inverse Model (LIM)

- Constraints (equalities): $\sum a_{ik} x_k = 0$ (per node i)
- Inequalities: $x_k > 0$ for all edges (negative flows are unphysical)
- Solve with matrix methods ($A\boldsymbol{x} = \boldsymbol{0}$ and inequalities)
- Gives basis:
  - Set of basis flows (represented as vectors)
  - Each basis flow is a solution
  - Flow solution = weighted sum of basis flows
- $n - m$ free variables (+1 for global scale): 12 in this example

Singular Value Decomposition

Basis flow example

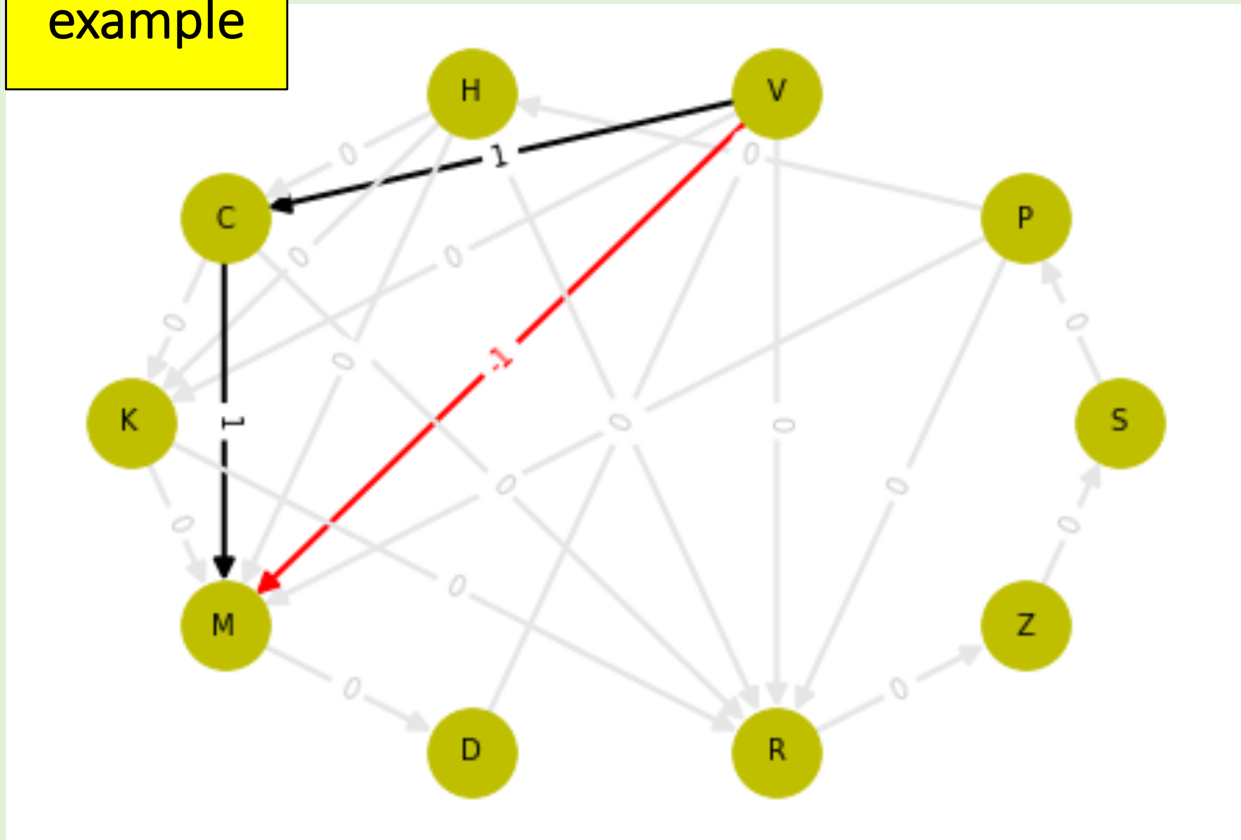


Values scaled up by 1000

- Orthonormal basis, very nice, but …
- Many negative flows
  - Plants eat herbivores
  - Herbivores eat carnivores
  - Carnivores rise from the dead
  - Heat recovered
- Must exclude negative flows
- (Same with QR decomposition)

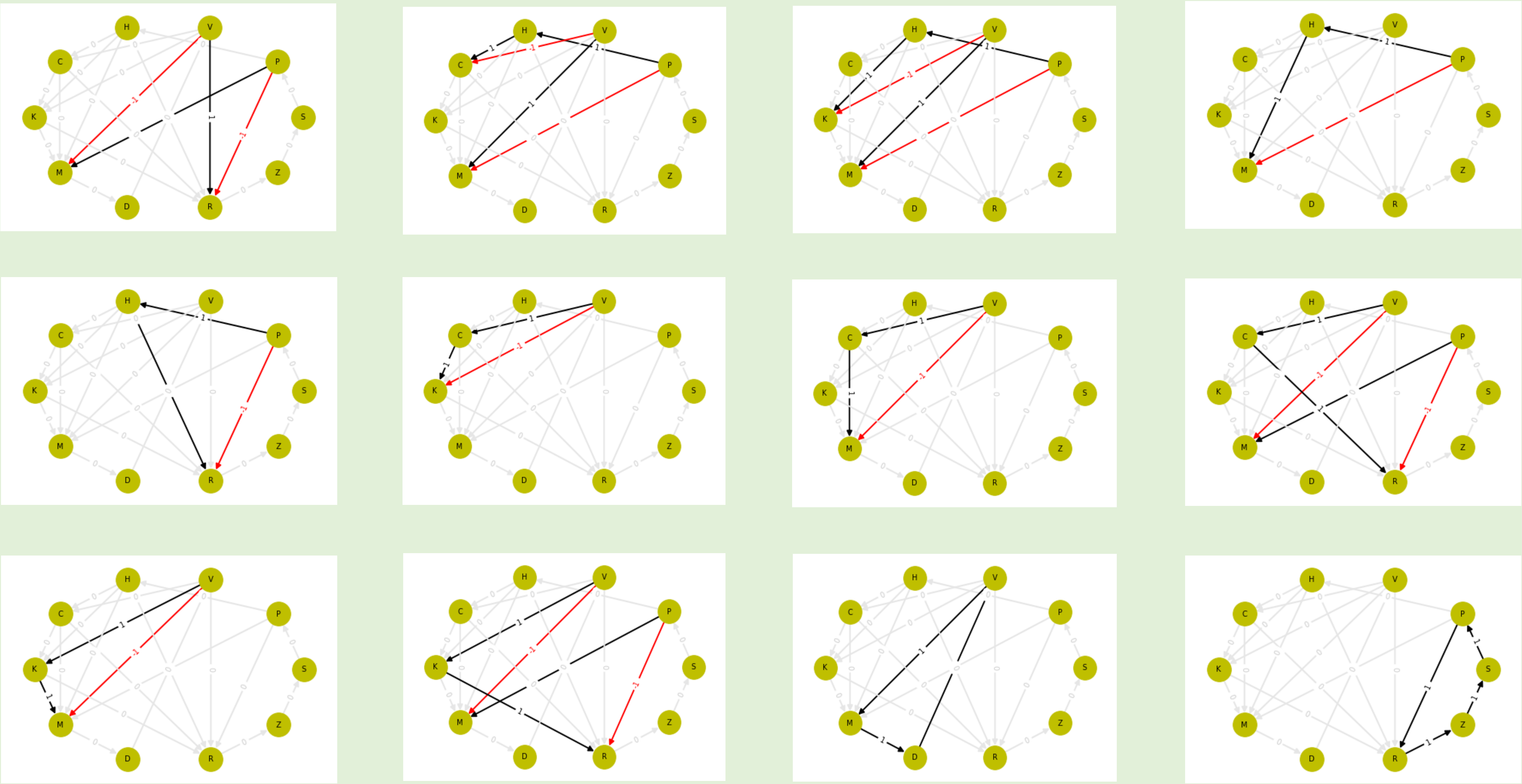
Reduced Row-Echelon Form

Basis flow example



- Simpler basis (simple cycles), but …
- Still has negative flows
- As for other LIM, basis not interpretable (no meaning)

Set of 12 basis flows

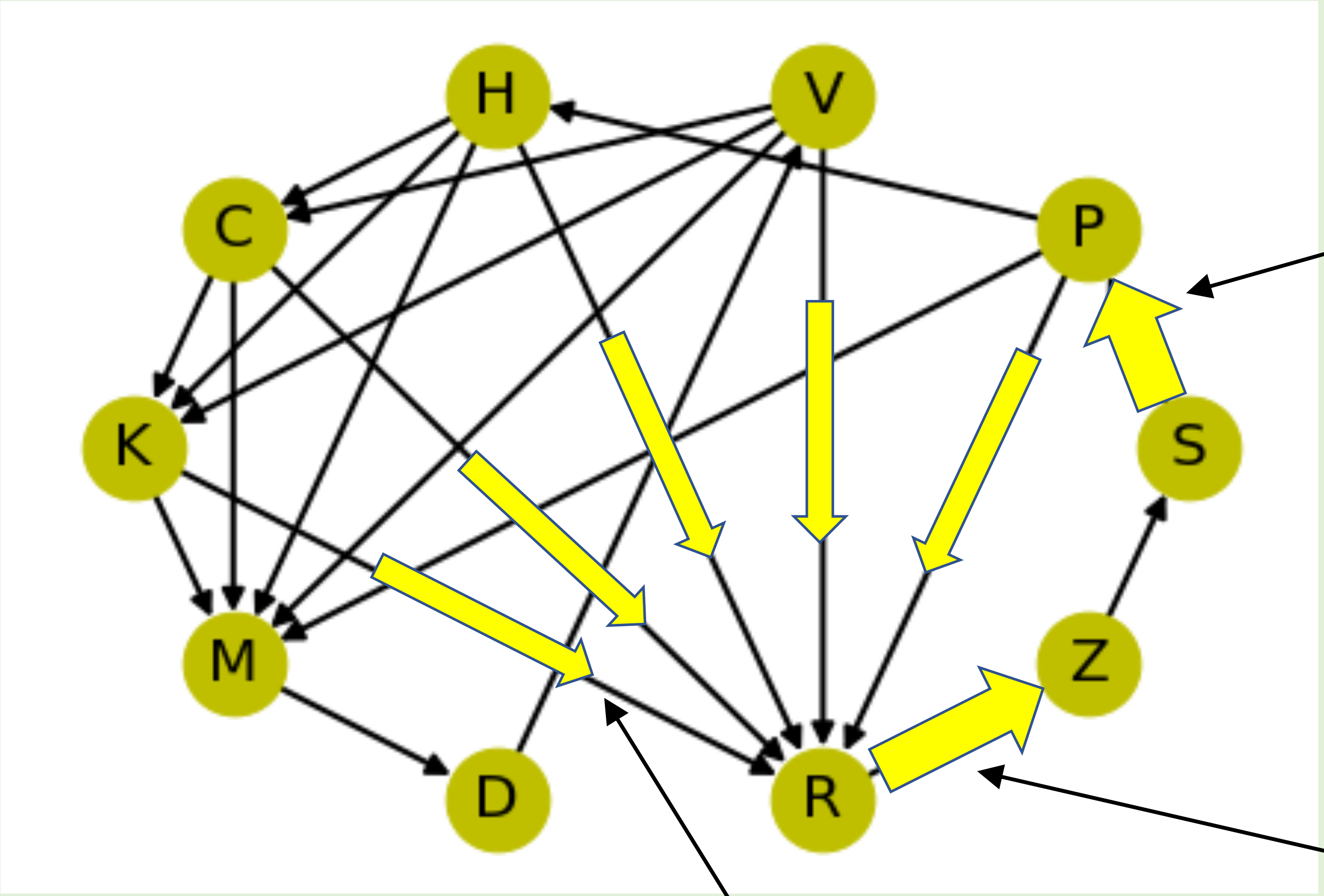


“Physics” Approach

- Avoid negative flows (unphysical, biologically implausible)
- Mechanisms (respiration, mortality)
- Conservation of energy

- Try iterative algorithm (only positive flows)

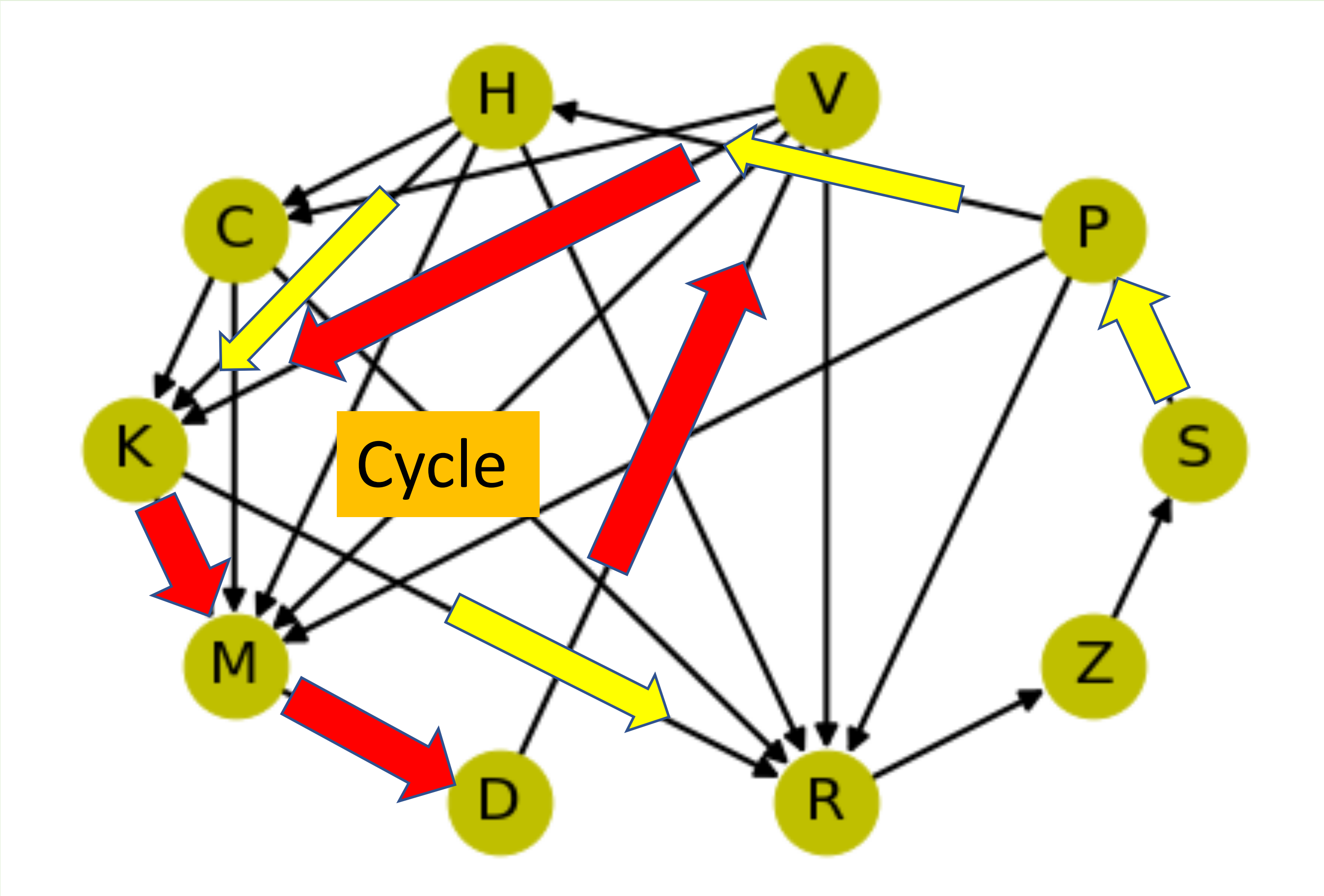
Iterative Solution



Continue until S-flow = Z-flow

Send energy backwards

Split among in-edges



Iteration needed for cycles

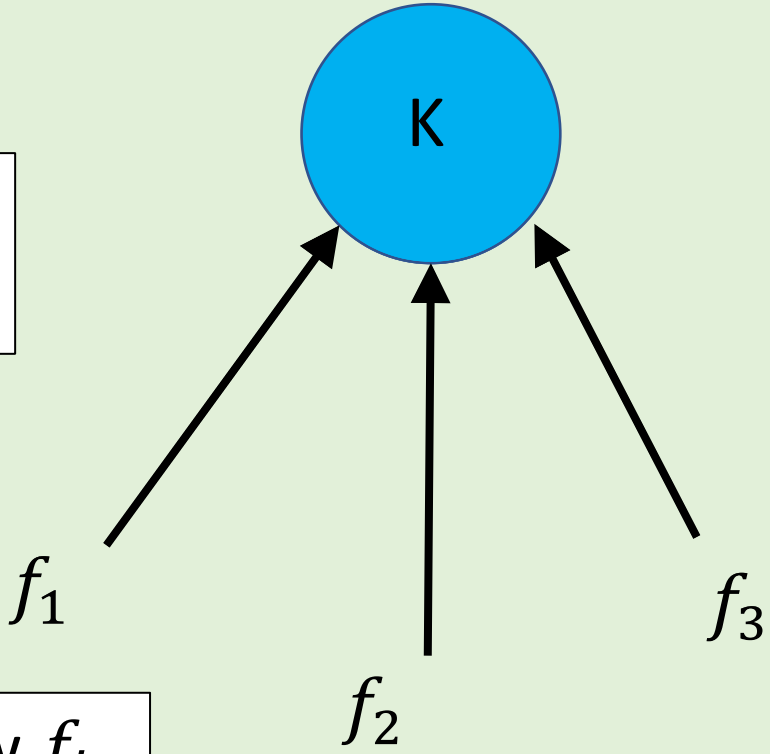
“Diet fraction”

Constraint

$$\sum f_i = 1$$

One per node

Flow along edge
$x_i = f_i T_{in}$

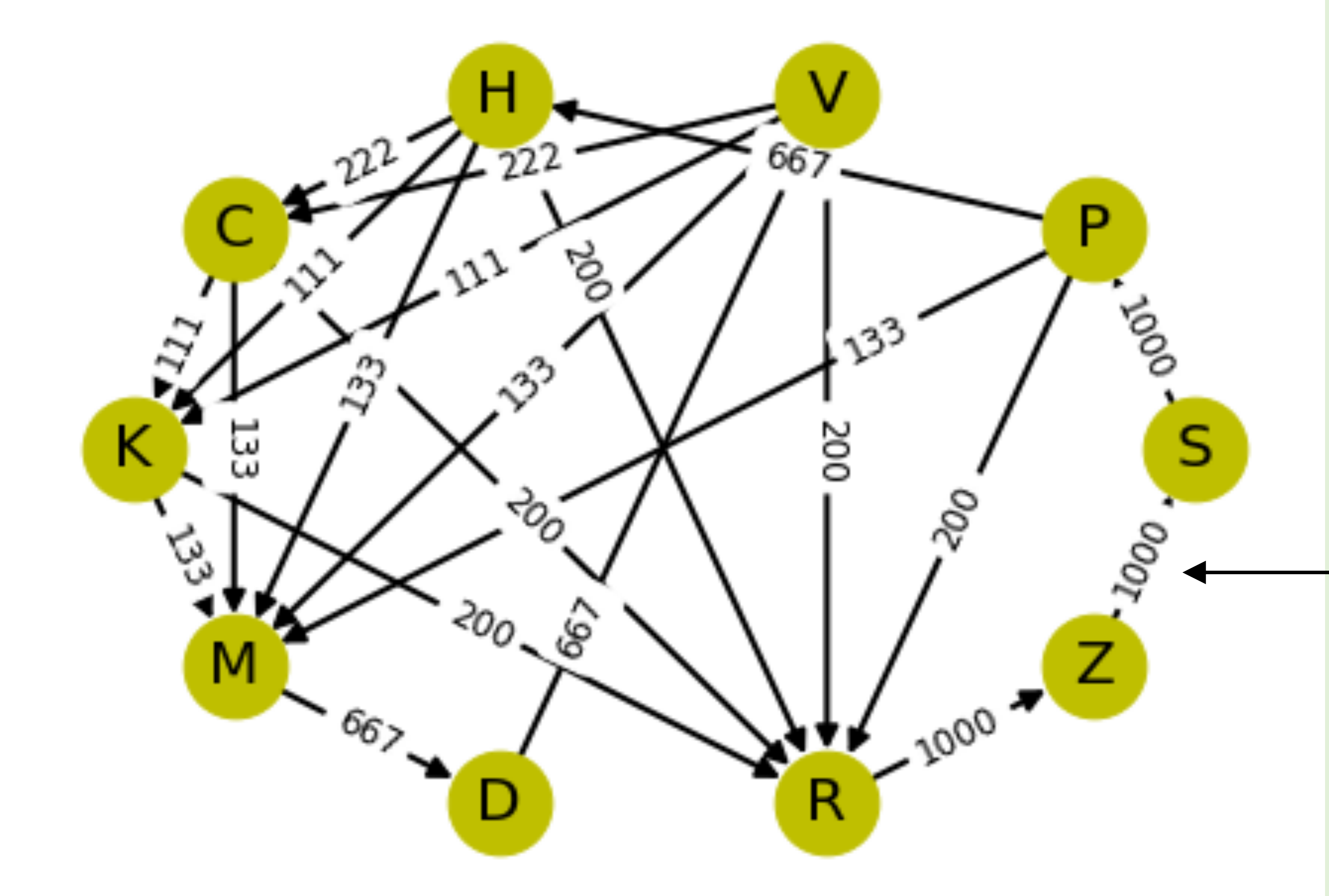


$\{f_k\}$ ∀ edges has
$n - m$ free variables
Just like LIM

Fraction of inflow f_i

$$H = -\sum f_k \log_2 f_k$$

Solution for equal diet fractions



But mechanisms need attention …

Next step

$t_{ZS} = 1000$

CONCLUSION

- Iterative method: No negative flows
- Diet fraction is interpretable
- Can specify biomass, respiration, mortality
  - Also interpretable
- Ecosystem structure: many interesting questions
- Ecological networks can be applied to other systems