Unifications in particle physics: from coupling crossing to asymptotic unification

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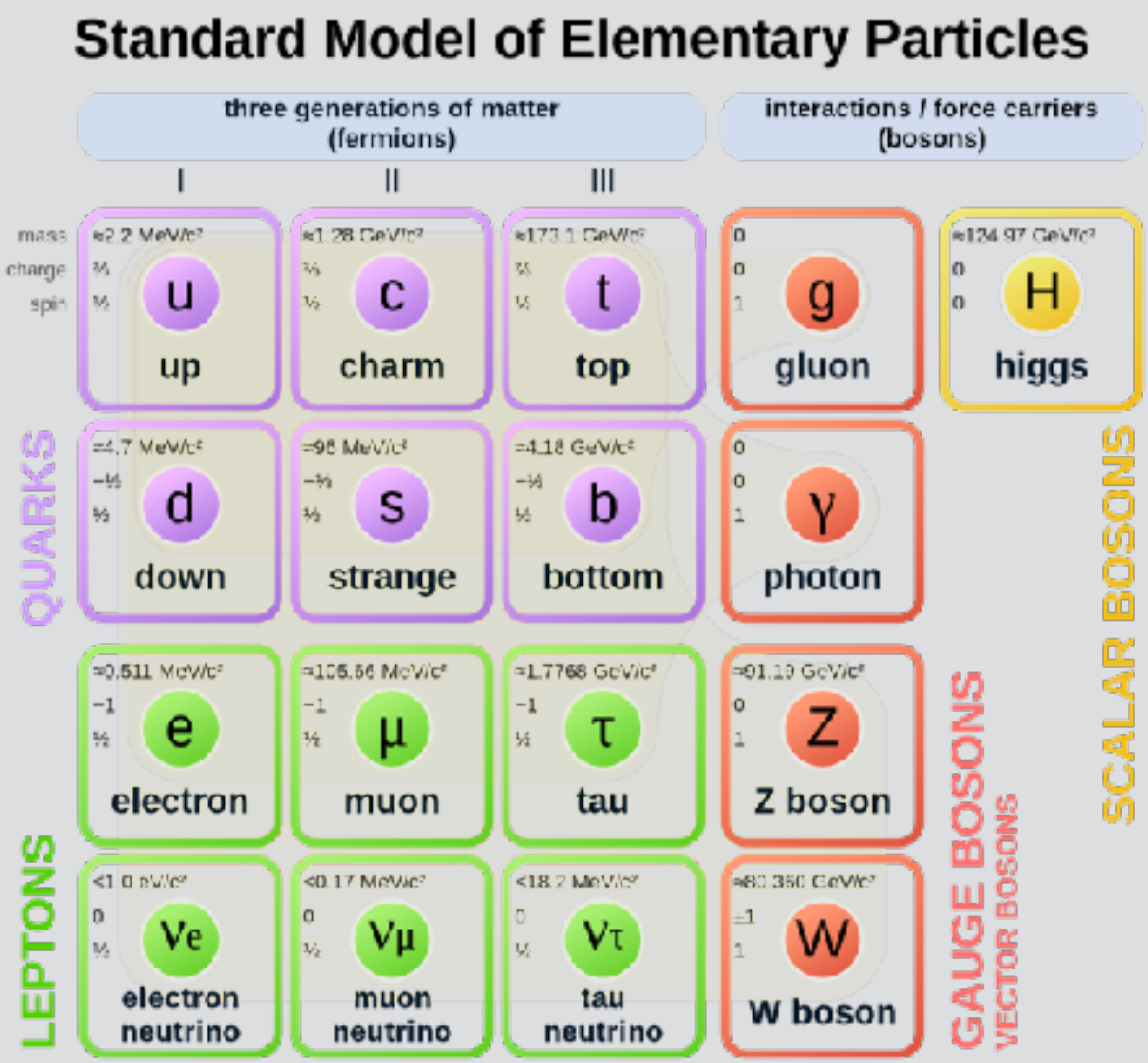
Particles and forces



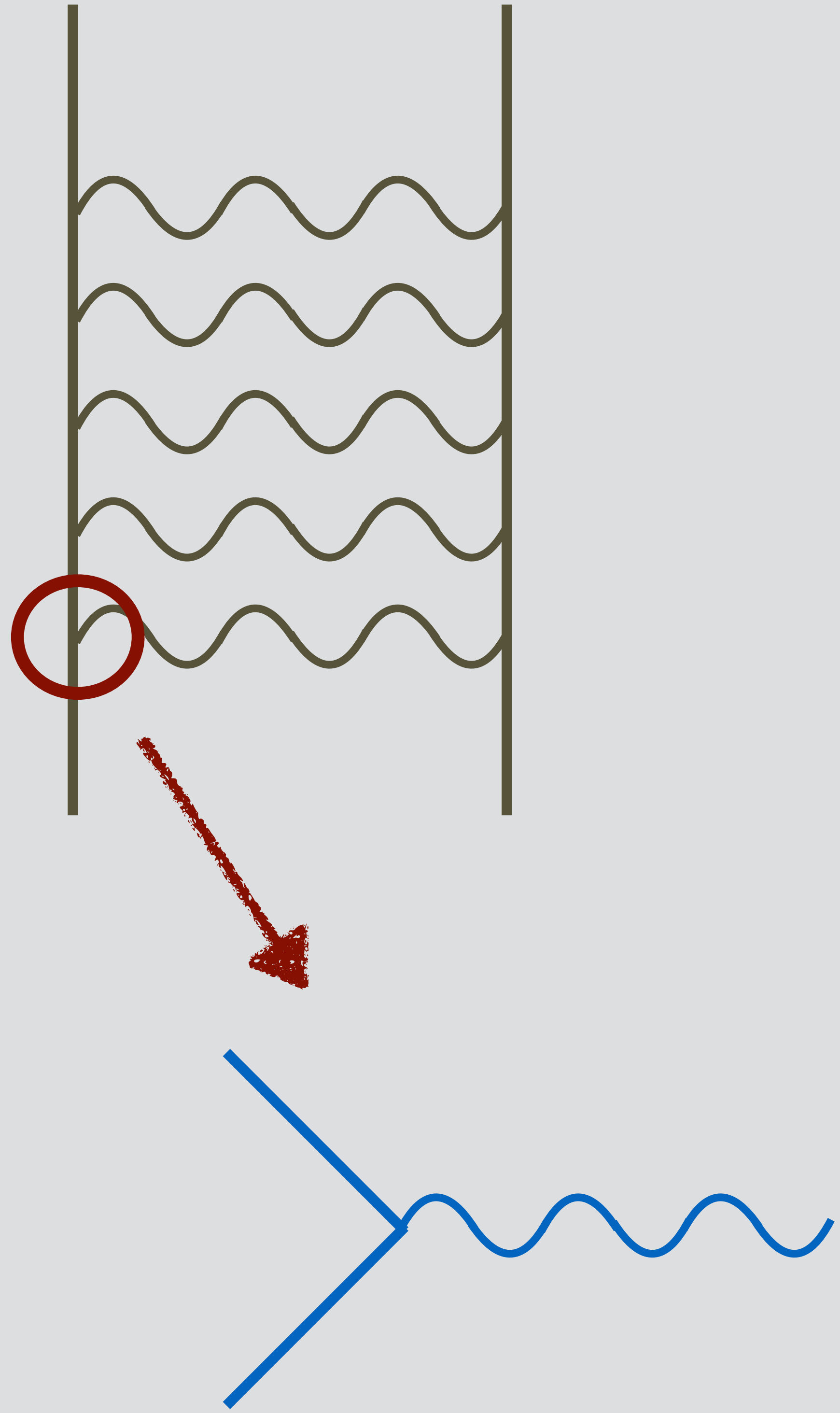
The present knowledge of fundamental forces of nature is called the Standard Model of particle physics and describes 3 types of interactions (electromagnetic, weak and strong forces) and 17 particles (12 matter, 4 mediators, 1 Higgs scalar)

source: Wikipedia, Wikimedia Commons

Fundamental forces

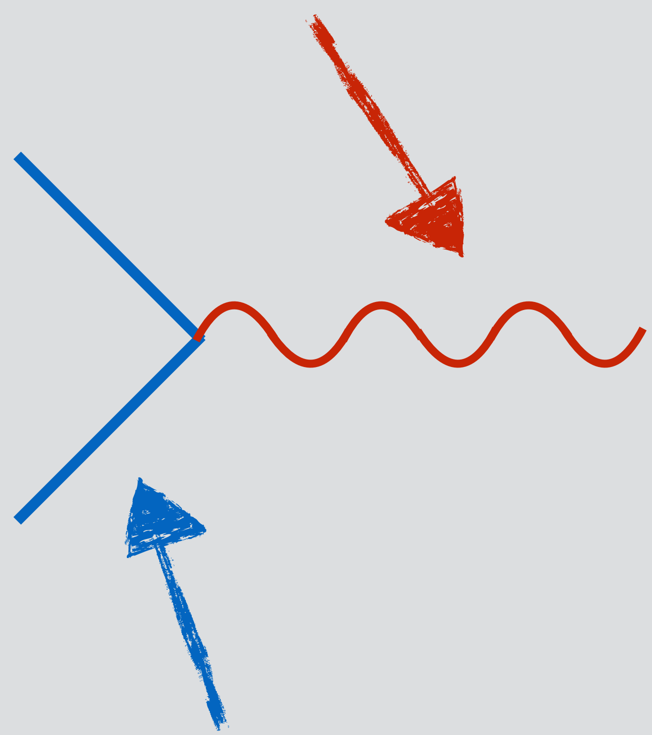
Interaction	Mediators	Relative strength to gravity	Long-distance behaviour (potential)	Range (m)
Weak	W+, W- and Z bosons	10^{33}	$\frac{1}{r}\, e^{-m_{\mathrm{W,Z}}\, r}$	10^{-18}
Strong	8 gluons	10^{38}	linear potential ~ r	10^{-15}
Gravitation	1 graviton (?)	1	$\frac{1}{r^2}$	∞
Electromagnetic	1 photon	10^{36}	$\frac{1}{r^2}$	∞

Forces in quantum theories



The potential is a semiclassical approximation: instantaneous action at distance is not consistent with special relativity and the effect of the potential is due to “mediator” exchange in quantum field theory. Mediators exchanges require interactions. Gravity not fully consistent with quantum field theory (not included in the SM), needs quantum gravity description.

Forces and groups



Interactions can be just chosen to match experimental results, but apart from electromagnetism, very difficult to guess.

In particle physics interactions come from symmetries: invariance with respect to local (=coordinate dependent) space-time transformations, called gauge symmetries, based on unitary Lie algebras

U(1) phase transformations $e^{i\alpha(x)}$ (electromagnetism)

SU(2) Pauli 2x2 matrices phase $e^{i\alpha_i(x)\,\sigma_i}$ (weak interactions)

SU(3) Gell-Mann 3x3 matrices $e^{i\alpha_a(x)\,\lambda_a}$ (strong interactions)

Symmetries of the Standard Model

SU(3) x SU(2) x U(1) are the combined symmetries of the Standard Model at high energy.

The Higgs mechanisms allows a spontaneous symmetry breaking to a residual symmetry:

SU(3) x $U(1)_{em}$

leaving the photon and the gluon massless

note that $U(1)_{em}$ is a subgroup of SU(2) x U(1)

The forces originating from these symmetries are not unified so they do not have a common origin.

Matter families

The SM contains 3 copies of the matter families/generations with the same quantum numbers under the SM gauge symmetry, so they can be freely rotated independently, there is a U(3)^5 global symmetry.

x 3

$$\begin{pmatrix} \nu \\ l \end{pmatrix}_L \quad l_R \quad \begin{pmatrix} q_u \\ q_d \end{pmatrix}_L \quad (q_u)_R \quad (q_d)_R$$

$$\underbrace{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}_{5}$$

The basic requirement of a "family" is anomaly cancellation = the quantum corrections respect the same gauge symmetries SU(3) x SU(2) x U(1)

Gauge coupling evolution

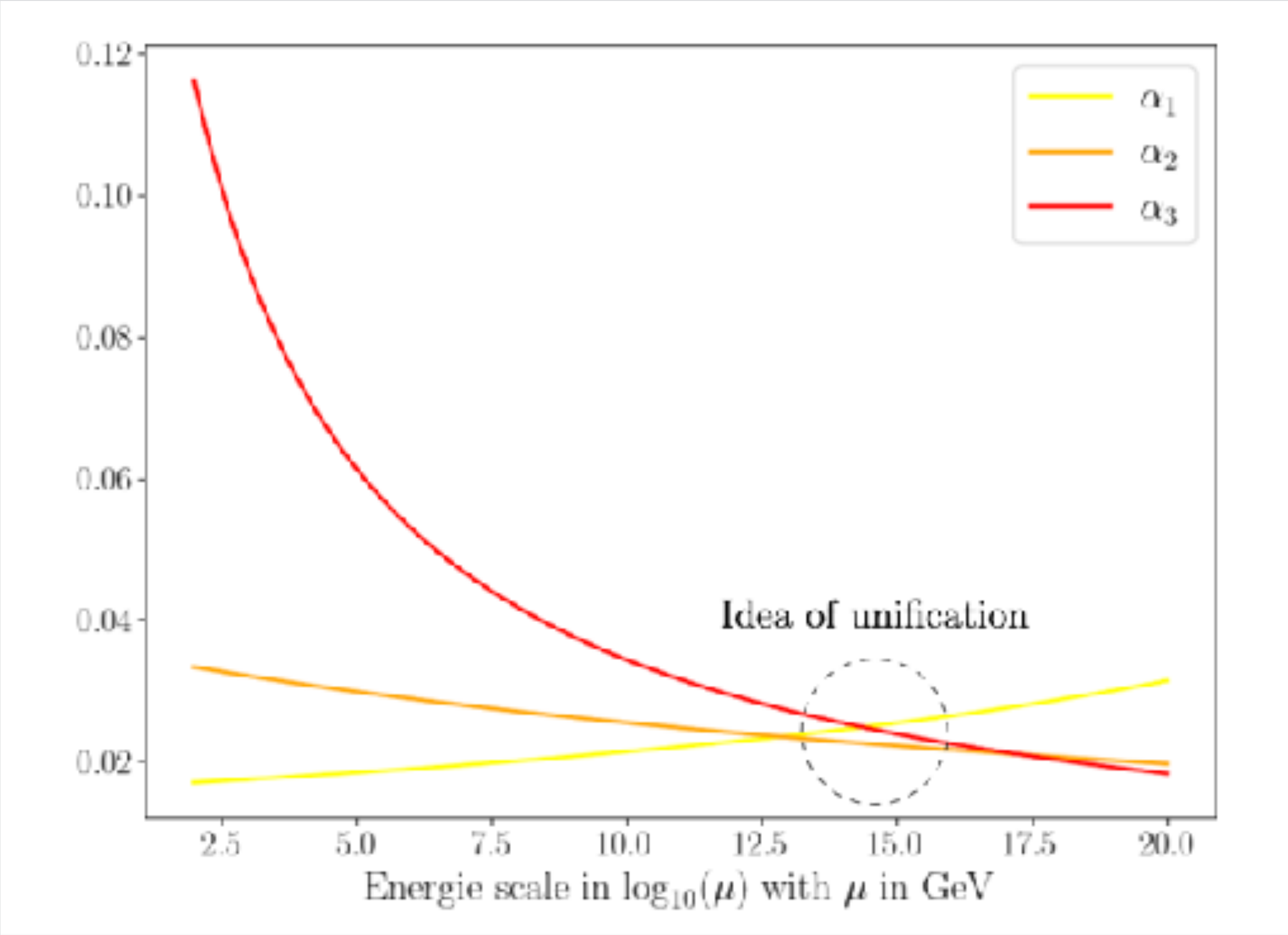
The quantum corrections also affect all the physical quantities we measure: masses and couplings are not constant values like in classical physics, their value depend on the energy at which the measurement is performed.

SM couplings:

$$\alpha_1 = \frac{e^2}{4\pi}$$ electromagnetic

$$\alpha_2 = \frac{g^2}{4\pi}$$ weak

$$\alpha_3 = \frac{g_s^2}{4\pi}$$ strong



SU(5)

SU(5) is the smallest example of unified gauge group for the standard model gauge interactions and an explicit model was fist constructed by Georgi and Glashow in 1973. This is based on the fact that 3 + 2 = 5:

$$\begin{pmatrix} SU(3) & * \\ * & SU(2) \end{pmatrix}$$

and a full generation of matter can be collected in just two objects (multiplets 5 and 10):

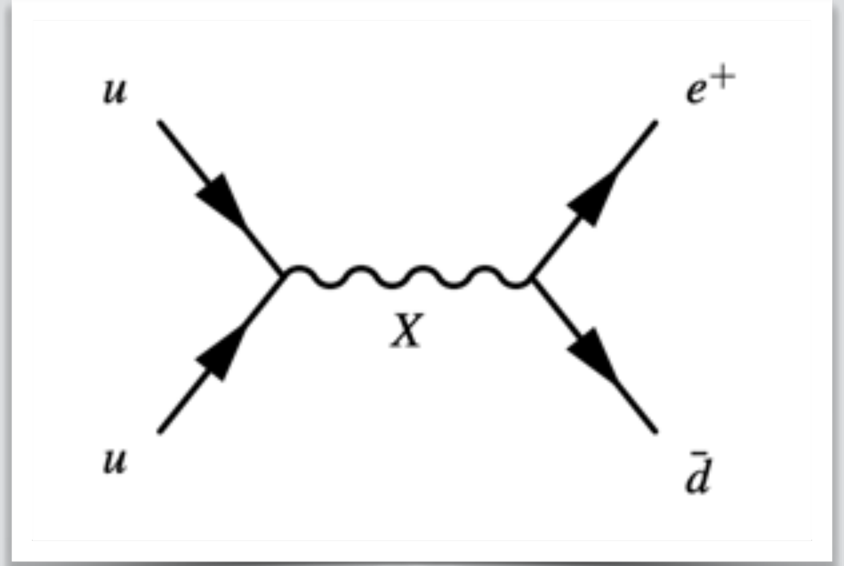
$$\begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ e^+ \\ -\bar{\nu}_e \end{pmatrix}, \quad \text{and} \quad \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & \bar{u}_3 & -\bar{u}_2 & u_1 & d_1 \\ -\bar{u}_3 & 0 & \bar{u}_1 & u_2 & d_2 \\ \bar{u}_2 & -\bar{u}_1 & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & e^+ \\ -d_1 & -d_2 & -d_3 & -e^+ & 0 \end{pmatrix}$$

SU(5)

SU(5) allows to assemble quarks and leptons in the same multiplets and to justify the quarks fractional charges. However the SU(5) unified force requires 24 mediators and some of them (called X and Y bosons) induce proton decay to an unacceptable rate.

$$A_M = \sum_{a=1}^{24} A_M^a T^a = \begin{pmatrix} \mathbf{G}_{ij} & \overline{\mathbf{X}}_i & \overline{\mathbf{Y}}_i \\ \mathbf{X}_i & W^3/2 & W^+/\sqrt{2} \\ \mathbf{Y}_i & W^-/\sqrt{2} & -W^3/2 \end{pmatrix}_M + \sqrt{\frac{3}{5}} B_M \begin{pmatrix} -\frac{1}{3}\mathbb{1}_{3\times 3} & 0 \\ 0 & \frac{1}{2}\mathbb{1}_{2\times 2} \end{pmatrix}$$

For example X exchange allows $p^+ \rightarrow e^+\pi^0$ $(uud \rightarrow e^+ d\bar{d})$



Grand Unification beyond SU(5)

- Many other options were tried in terms of larger GUT symmetry groups, as SO(10) and exceptional groups like E6. The models may also include supersymmetry.

- Supersymmetry helps building more realistic models

- Proton decay inevitable to some extent

- Gauge hierarchy problem, why are M_W and M_Z so much less than M_X and M_Y ?

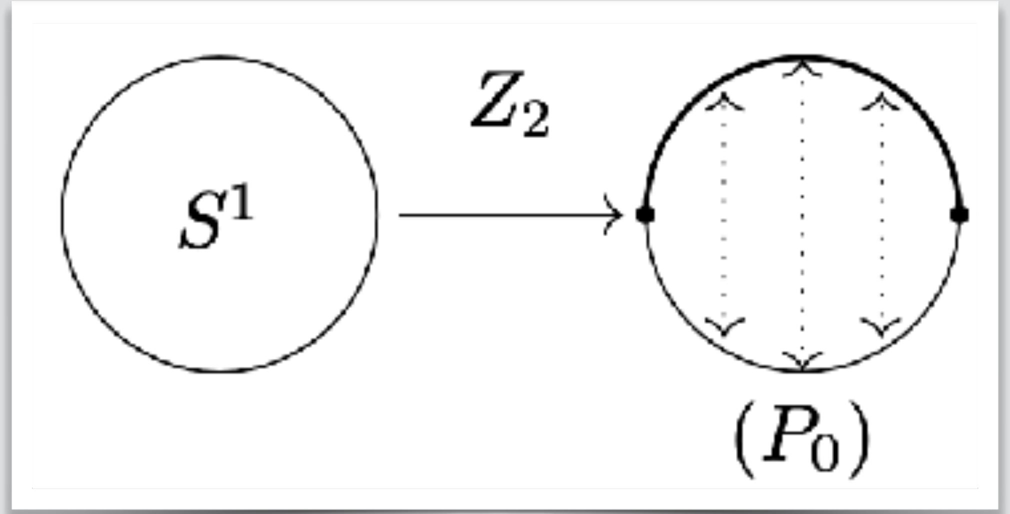
- Large scalar representations needed for the spontaneous symmetry breaking down to SU(3) x U(1)em

- GUT theories are still effective theories (Landau poles at very high energy), need to be completed (string theory, etc.)

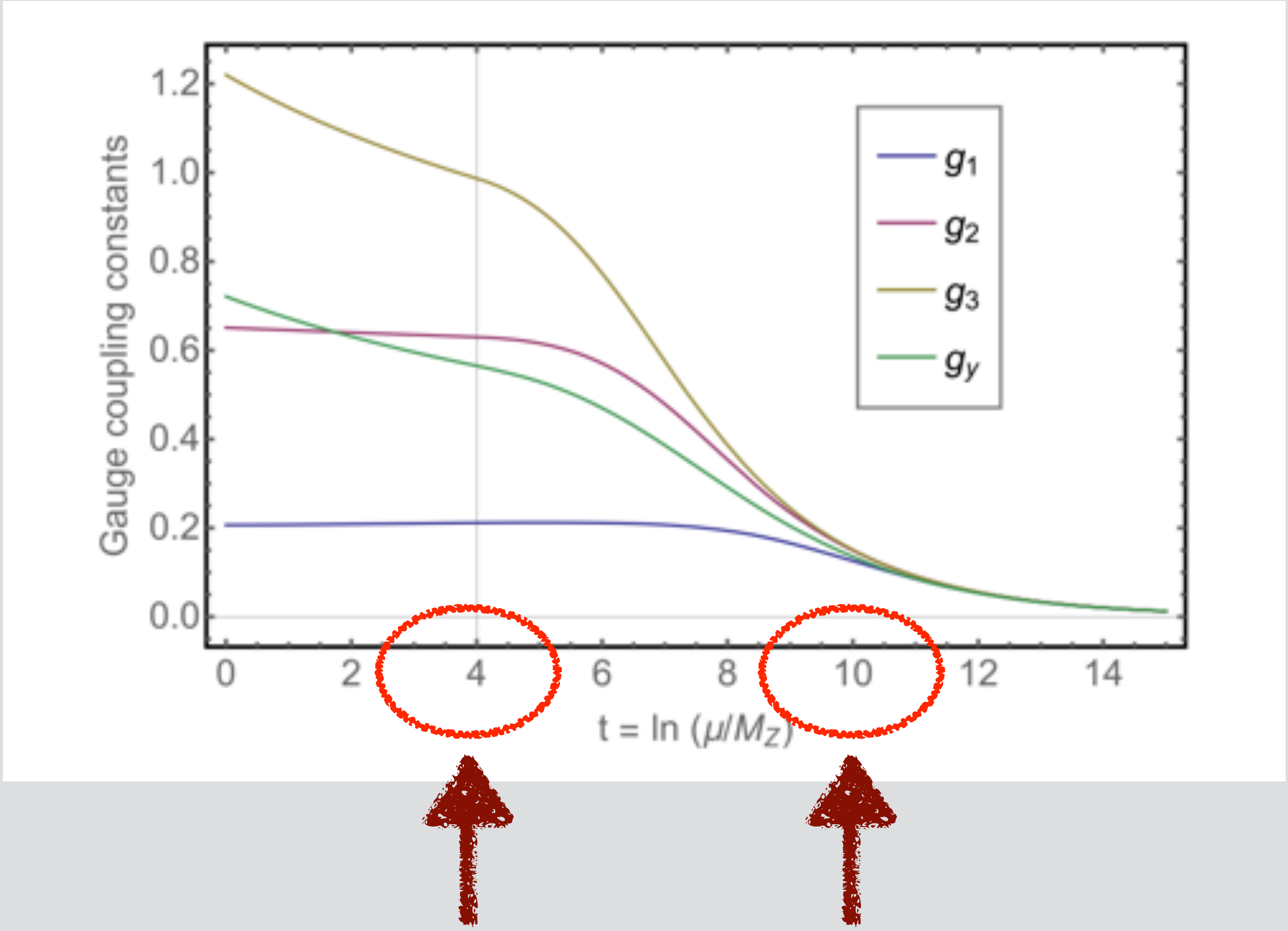
Asymptotic unification

Standard gauge couplings unification requires a specific scale and "crossing" of the evolution of couplings.
Asymptotic unification is NOT unification in the usual sense, rather at high energies, flow towards a fixed point.

An essential ingredient to obtain a fixed point in this set-up is not only a specific particle content of the theory but also the presence of extra compact dimensions. The simplest example is a circle at each point of our 4D space time. However fermions in 5D (M_4 x S^1) are not chiral. An extra Z_2 parity has to be imposed to obtain chiral SM fermions.



Asymptotic unification



1st KK mode kicks in at t ~4
"unification" sets around t~10 (~200 KK-modes)

Example in the EW sector with SU(3) ⊃ SU(2)xU(1)
for gauge and Yukawa couplings (5D model)

A gauge scalar from extra dimensions

$$A_M = (A_\mu, A_5)$$

Under orbifold projection vectors and scalars have opposite parity

$$A_M^+ = (A_\mu^+, A_5^-)$$

Contains a zero mode vector

$$A_M^- = (A_\mu^-, A_5^+)$$

Contains a zero mode scalar

5D gauge symmetry broken by parity projection (but 4D preserved)

Higgs scalar "protected" by the gauge symmetry (Hosotani mechanism), vev "geometrisation"

A SU(3) toy model

SU(2) U(1)

$$A_M^+ : \frac{1}{\sqrt{2}} \begin{bmatrix} W_3/2 & W^+/\sqrt{2} & 0 \\ W^-/\sqrt{2} & -W_3/2 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \frac{1}{2\sqrt{3}} \begin{bmatrix} B & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & -2B \end{bmatrix}$$

Higgs

$$A_M^- : \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & h^+ \\ 0 & 0 & h^0 \\ h^- & h^{0*} & 0 \end{bmatrix}$$

Fermion matter
bulk triplet

Weinberg angle, fermion mass

$$A_M^- : \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & h^+ \\ 0 & 0 & h^0 \\ h^- & h^{0*} & 0 \end{bmatrix}$$

Higgs doublet, but U(1) charge

$$\frac{1}{2\sqrt{3}}(1-(-2)) = \frac{\sqrt{3}}{2}$$

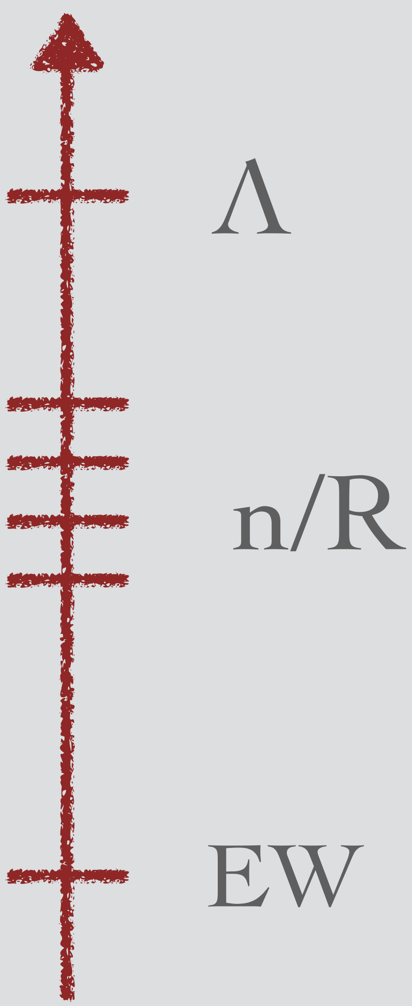
The group theory value for the Weinberg angle is not the correct one in SU(3)

$$g' = \sqrt{3}\, g \rightarrow \sin^2\theta_W = \frac{3}{4}$$

Fermion mass related to W mass

$$m_f = m_W$$

Renormalisation group evolution

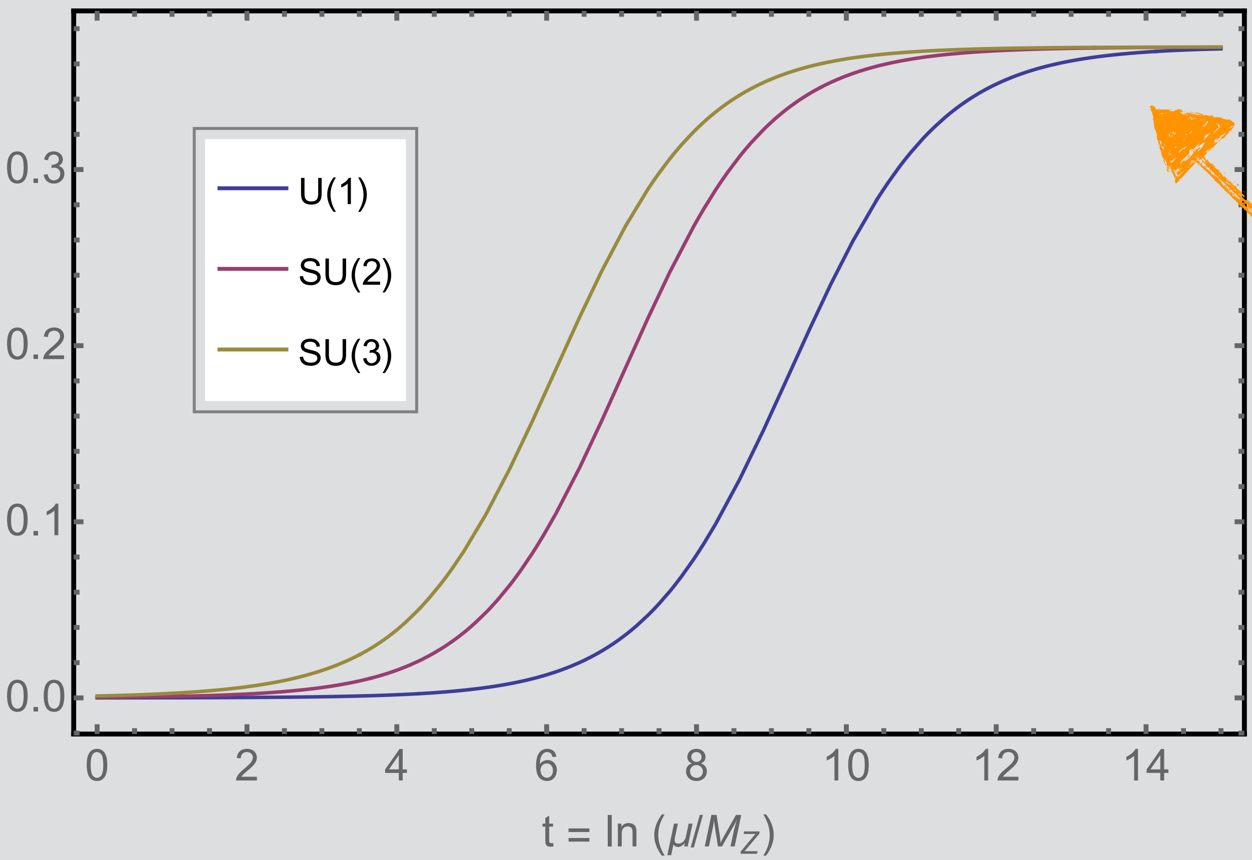


- Group theory prediction is at unification, need to run down at the EW scale
- Λ and EW not that far, but running in Xdim is fast, linear effect, not log
- Here Yukawa coupling is also a gauge coupling

	$SU(2)_L$ g	$U(1)_Y$ g'	Yuk. y	$SU(3)_c$ g_s
SU(3) GHU	g_{GHU}	$\sqrt{3}\, g_{GHU}$	$g_{GHU}/\sqrt{2}$	–
SM	0.66	0.35	1.0	1.2

Running of the gauge couplings

$$16\pi^2 \frac{dg_i}{dt} = b_i^{\rm SM}\ g_i^3 + (S(t)-1)\, b_i^{\rm GHU}\ g_i^3$$



S(t) encodes the sum of the KK contributions to the running

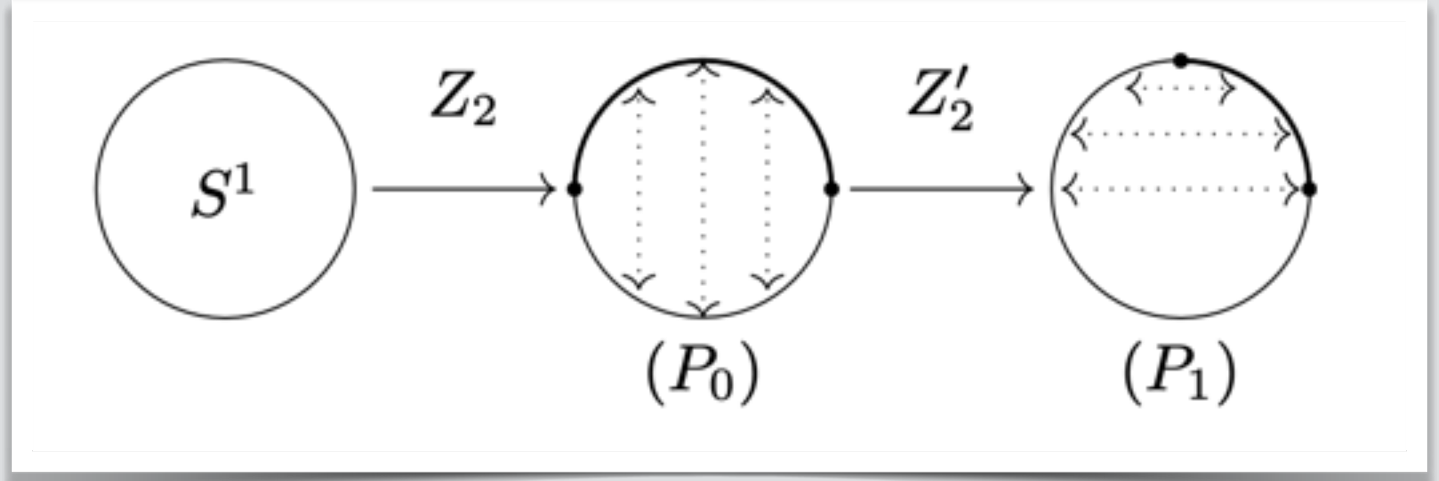
5d coupling is going to a constant ~ α_4 E (asymptotically safe)

$$b_i^{SM} = \left[\frac{41}{10}, -\frac{19}{6}, -7\right], \quad b_i^{SU(3)} = \left[-\frac{17}{6}, -\frac{17}{2}, -\frac{17}{2}\right]$$

Simplest aGUT : SU(5) embedding

- SU(5) gauge symmetry in the bulk, broken to SM, via orbifold boundary conditions

- A single extra dimension compactified on an orbifold $S_1/(Z_2 \times Z_2')$



- SM matter multiplets are NOT the usual SU(5) ones due to boundary conditions, need to duplicate the usual structure

Fermion non-unification

- Capital letter fields are the new "Indalo" fields
- Baryon and Lepton charge conserved
- No proton decay
- Dark matter candidate → Lightest Indalo

$$\psi_{1_{L/R}} = N, \qquad \psi_{5_{L/R}} = \begin{pmatrix} b \\ L^c \end{pmatrix}_{L/R}, \qquad \psi_{\bar{5}_{L/R}} = \begin{pmatrix} B^c \\ l \end{pmatrix}_{L/R},$$

$$\psi_{10_{L/R}} = \frac{1}{\sqrt{2}} \begin{pmatrix} T^c & q \\ & T^c \end{pmatrix}_{L/R}, \qquad \psi_{\overline{10}_{L/R}} = \frac{1}{\sqrt{2}} \begin{pmatrix} t & Q^c \\ & \tau \end{pmatrix}_{L/R},$$

Particle content overview

Field	$(\mathbb{Z}_2, \mathbb{Z}_2')$	SM	Zero mode?	KK mass
l	$(+,+)$	$(\mathbf{1}, \mathbf{2}, -1/2)$	✓	$2/R$
L	$(+,-)$		–	$1/R$
τ	$(-,-)$	$(\mathbf{1}, \mathbf{1}, -1)$	✓	$2/R$
$\mathcal{T}$	$(-,+)$		–	$1/R$
N	$(-,-)$	$(\mathbf{1}, \mathbf{1}, 0)$	✓	$2/R$
q	$(+,+)$	$(\mathbf{3}, \mathbf{2}, 1/6)$	✓	$2/R$
Q	$(+,-)$		–	$1/R$
t	$(-,-)$	$(\mathbf{3}, \mathbf{1}, 2/3)$	✓	$2/R$
T	$(-,+)$		–	$1/R$
b	$(-,-)$	$(\mathbf{3}, \mathbf{1}, -1/3)$	✓	$2/R$
B	$(-,+)$		–	$1/R$
ϕ_h	$(+,+)$	$(\mathbf{1}, \mathbf{2}, 1/2)$	✓	$2/R$
H	$(-,+)$	$(\mathbf{3}, \mathbf{1}, -1/3)$	–	$1/R$
B_μ		$(\mathbf{1}, \mathbf{1}, 0)$		
W^a_μ	$(+,+)$	$(\mathbf{1}, \mathbf{3}, 0)$	✓	$2/R$
G^i_μ		$(\mathbf{8}, \mathbf{1}, 0)$		
A^μ_X	$(-,+)$	$(\mathbf{3}, \mathbf{2}, -5/6)$	–	$1/R$

Yukawa non-unification

$$\overline{\psi_1}\psi_{\bar{5}}\phi_5 = \overline{N}\phi_h l + \overline{N}H\mathcal{B}^c,$$

$$\sqrt{2}\bar{\psi}_5\psi_{10}\phi_5^* = \bar{b}\phi_h^* q - \bar{L}^c H^* q - \bar{L}^c\phi_h^*\mathcal{T}^c + \epsilon_3\bar{b}H^*T^c,$$

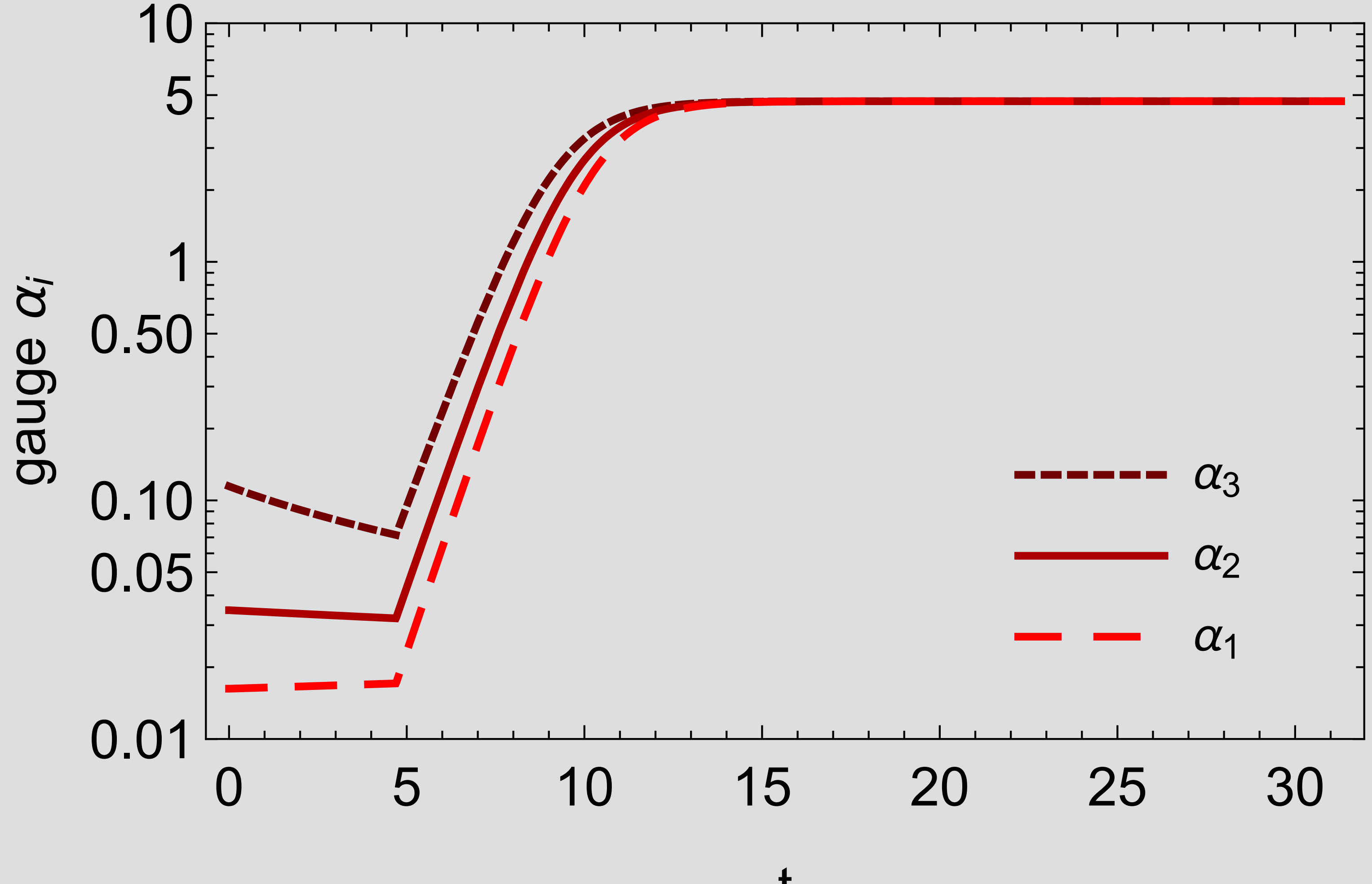
$$\sqrt{2}\overline{\psi_{\overline{10}}}\psi_{\bar{5}}\phi_5^* = -\bar{\tau}\phi_h^* l - \bar{Q}^c H^* l + \bar{Q}^c\phi_h^* B^c - \epsilon_3\bar{t}H^* B^c,$$

$$\frac{1}{2}\epsilon_5\overline{\psi_{\overline{10}}}\psi_{10}\phi_5 = \bar{t}\phi_h q + \bar{t}H\mathcal{T}^c + \bar{\tau}HT^c + \overline{Q^c}\phi_h T^c + \epsilon_3\bar{Q}^c Hq,$$

4 independent couplings, one for each SM Yukawa term

RGE SU(5) aGUT running

- All couplings constants flow to the same non-zero UV fixed point (- 2π/b5) asymptotically.
- Extra-dimensional one-loop factor is perturbative

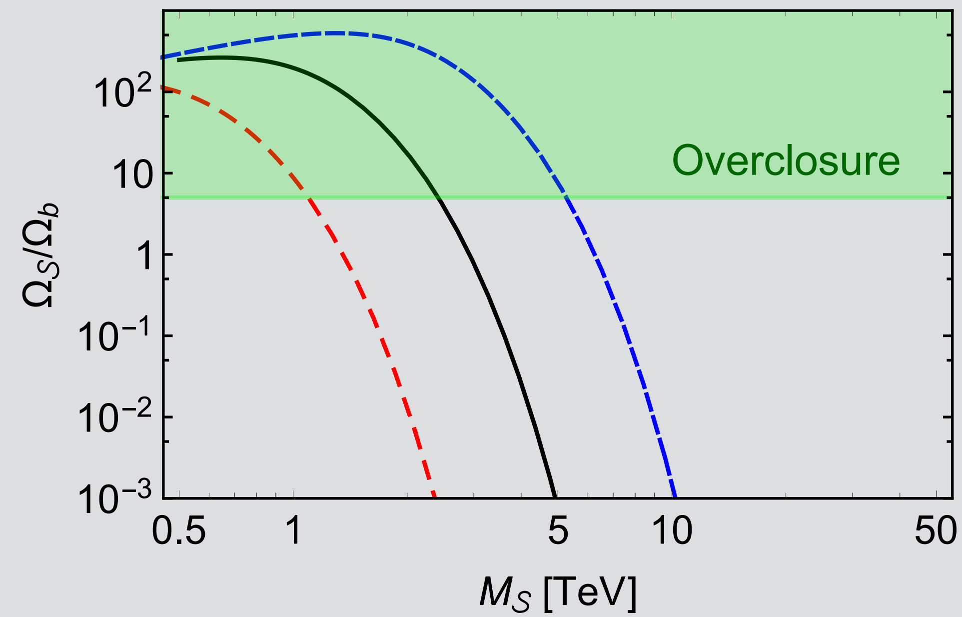


$$b_5 = -\frac{52}{3} + \frac{16}{3} n_g$$

n_g number of fermion generations in the bulk

$$\alpha_i = \mu R \alpha_{4D}$$

Baryogenesis, Indalogenesis and DM



assuming 1st order phase transition

- Indalo  particles carry both B and L charges, in values that are half of the SM unit charges
- It is not possible for them to decay into SM fields only and no proton decay
- The lightest state S is stable and a candidate for dark matter
- After the Indalo particles exit thermal equilibrium with the SM, they decay S + SM, and release some baryon number to the SM sector

E6 aGUT

Supersymmetry allows to generate fermions as gauge fields (gauginos)

In E6, the adjoint **78** contains the SM fermions (in vector-like pairs)

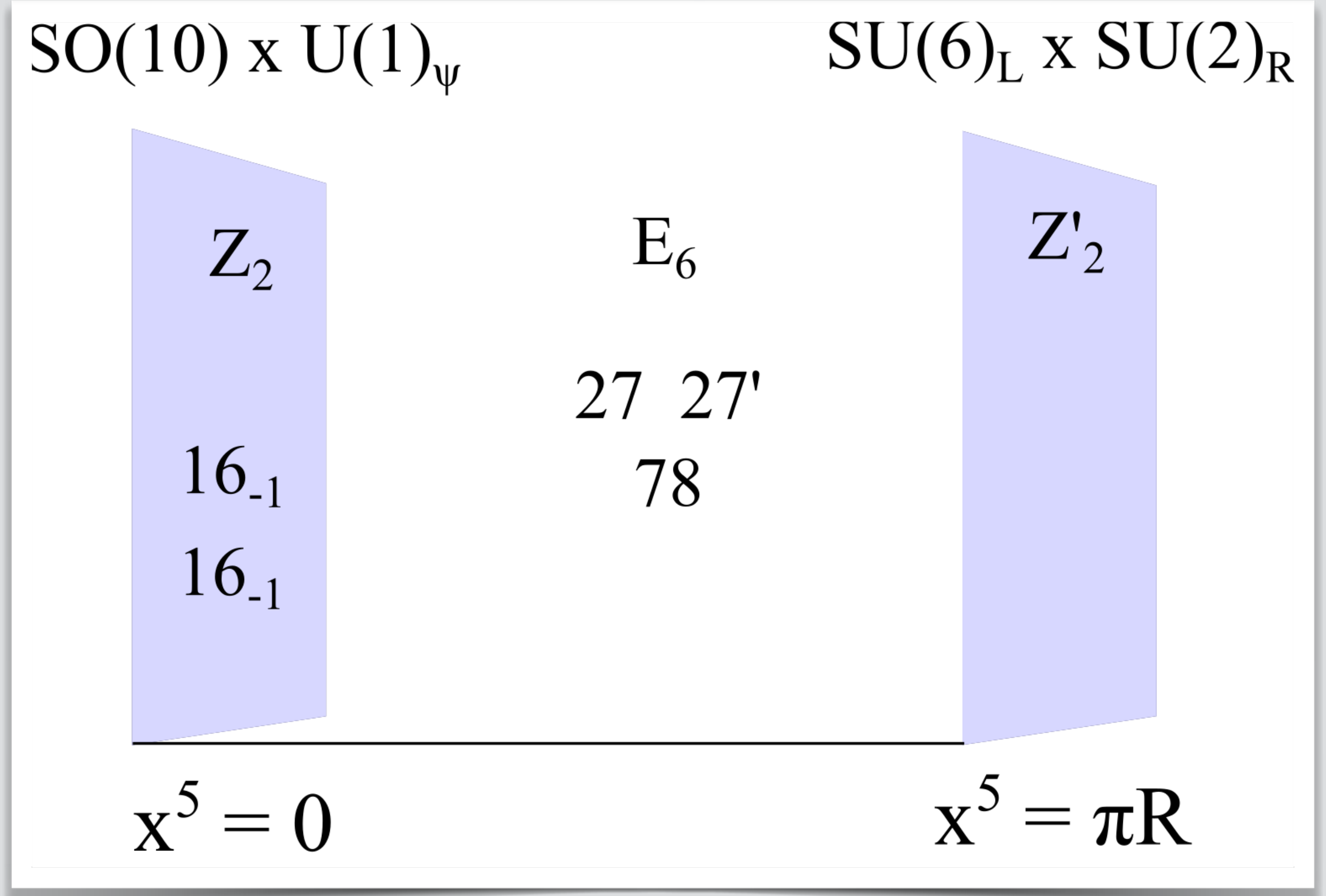
Right-handed SM fermions from the adjoint (gauginos)

Gives rise to a 4D **Pati-Salam** $SU(4) \times SU(2)_L \times SU(2)_R \times U(1)_\psi$

Left-handed fermions and Higgs(es) from the 27

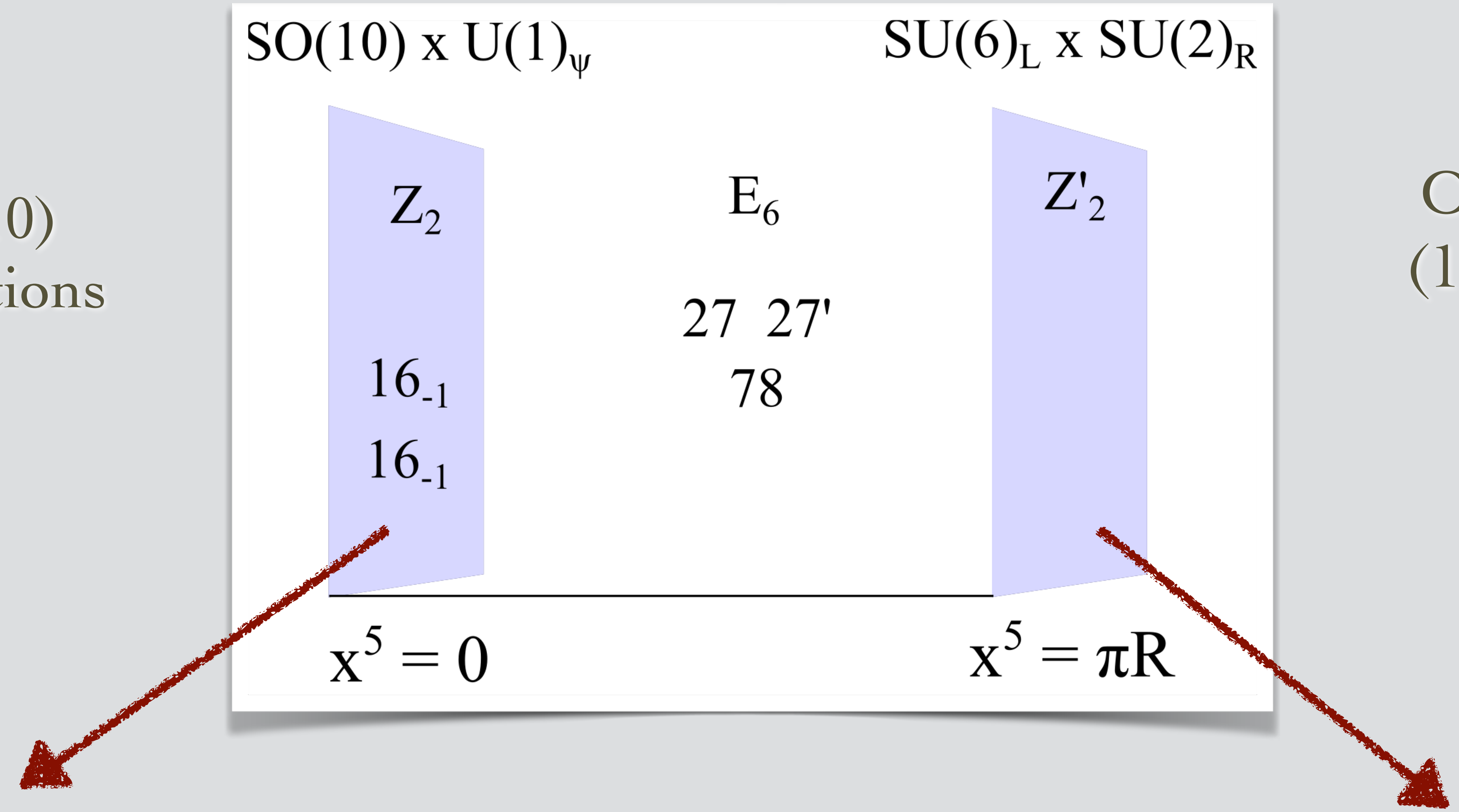
27' to give mass to unwanted states

see 2302.11671 for details



E6 aGUT

SO(10) generations

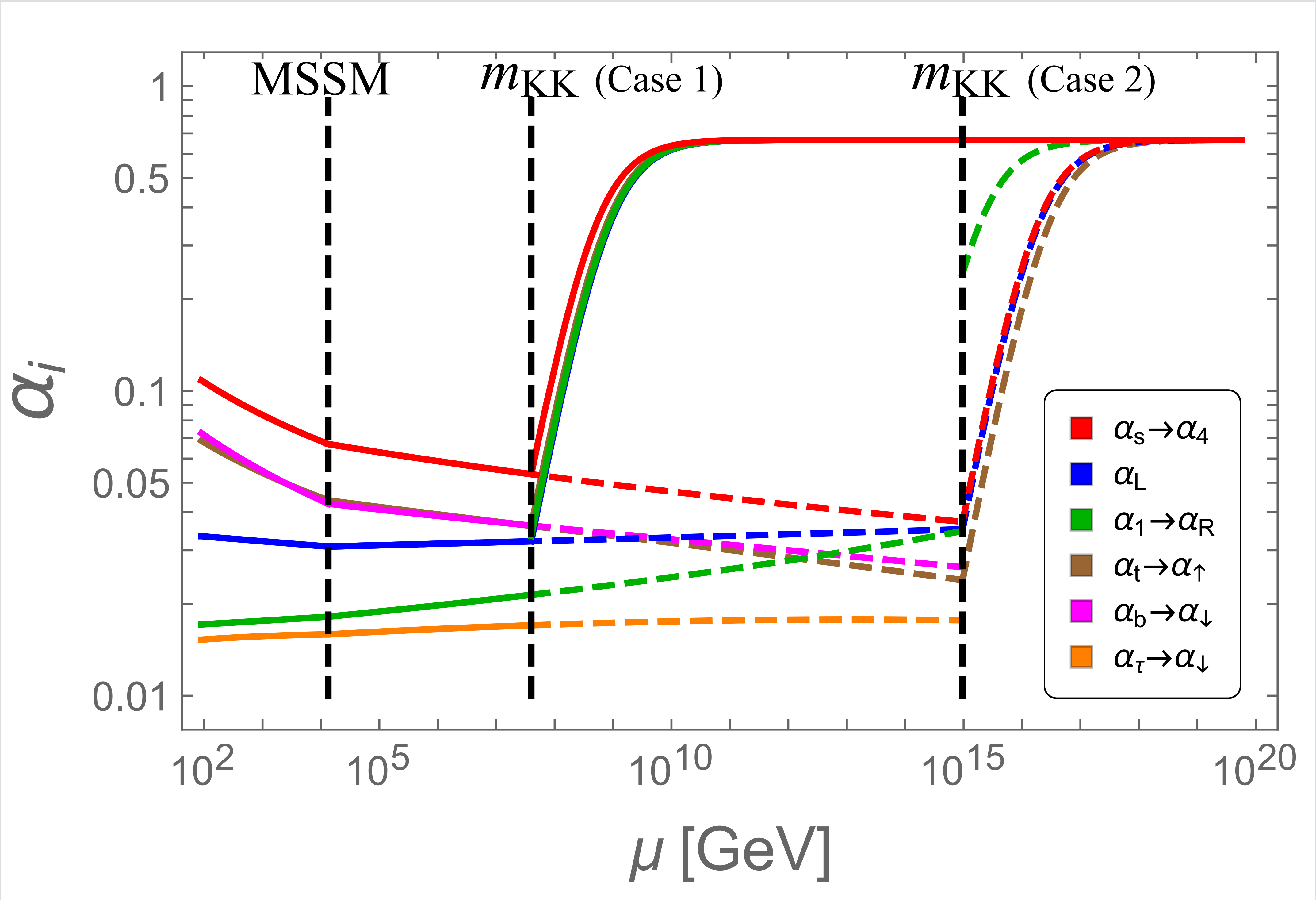


One gen in (15,1)+(6,2)

Model 1 :
Predicts 2 generations
"Usual" SO(10) model building
Scale high by proton decay

Model 2 :
Light generations preserve baryon number
Number of generations not predicted
Scale can be lowered (1000's TeV) from PS

Fixed point



MSSM
m_{KK} (Case 1)
m_{KK} (Case 2)
1
0.5
0.1
0.05
0.01
α_i
10^2
10^5
10^{10}
10^{15}
10^{20}
μ [GeV]
$\alpha_s \to \alpha_4$
α_L
$\alpha_1 \to \alpha_R$
$\alpha_t \to \alpha_\uparrow$
$\alpha_b \to \alpha_\downarrow$
$\alpha_\tau \to \alpha_\downarrow$

Conclusions

- New paradigm on (asymptotic) unification (aGUT)
- A extra-dimensional motivated examples
- No proton decay
- Yukawa evolution constrained
- A dark Matter candidate (the lightest  -field)
- Baryogenesis can be reproduced (typical mass 2-6 TeV)
- Flavour observables under study