On the $R_h = ct$ Universe

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Introduction

The model is based on the flat the FLRW metric

$$ds^2 = -cdt^2 + a^2(t)\left[dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right]$$

The coordinates for this metric have been chosen so that t represents the proper time measured in the rest frame of a **co-moving** observer.

FLRW Equations

Expansion of the universe involves studying the behavior of the scale factor as described by the following equations of motion:

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 = \frac{8\pi G\rho}{3} \tag{1}$$

We can use this result to find a as a function of t. The spatial component of the EFEs gives the Friedmann acceleration equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) \tag{2}$$

Pressure here acts to increase the gravitational force, and so further decelerates the expansion.

- If $\rho c^2 + 3p > 0$ the universe is decelerating
- If $\rho c^2 + 3p < 0$ the universe is accelerating

Current observations indicate that the expansion of the universe is indeed accelerating.

Fluid Equation

We cannot use these two equations alone to describe the time evolution of the scale factor of the universe without an additional equation describing the time evolution of the density of material in the universe.

Energy conservation gives the fluid equation

$$\frac{d\rho}{dt} + 3H\left(\rho + \frac{p}{c^2}\right) = 0 \tag{3}$$

- The first term in the brackets corresponds to the density dilution due to volume increased
- The second term corresponds to the loss in energy because the pressure of the material has done work as the universe's volume increased

Equation of State

The term 'pressure' here, does not mean a pressure gradient – there are no such pressure forces in a homogeneous universe because density and pressure are the same everywhere. In cosmology the assumption is usually made that there is a unique pressure associated with each density.

The density and pressure of these components are related by an equation of state (EOS)

$$p_i = w_i \rho_i c^2$$

The equation-of-state parameter w_i is a constant. In more exotic cosmological models w_i may be a function of cosmic time t.

The $R_h = ct$ Universe

Since R_h is implicitly defined as a function of time, the gravitational horizon must itself be a proper distance $R_h(t) = a(t) r_h$ – where r_h remains constant as the universe expands.

$R_h = c/H(t)$ means that $\dot{a}$ must be a constant in time.

The acceleration is zero for a universe in which either $p = \rho = 0$ or $w = -1/3$.

This implies that

$$a \propto t \Rightarrow \frac{a}{a_0} = \frac{1}{1+z} = \frac{t}{t_0} = \frac{H_0}{H}$$
$$\Rightarrow H = H_0 (1+z)$$

Unlike the ΛCDM model which contains at least three parameters H_0, Ω_Λ and Ω_m, the $R_h = ct$ model depends only on the Hubble parameter H_0

Luminosity Distance

One of the most basic and most important measurements that can be performed is that of distance. There are several ways of measuring distances in the expanding universe.

For an object of known luminosity, it is possible to determine its luminosity distance. In the present discussion, the luminosity distance in the standard model is given as

$$d_L = R_h(t_0)(1+z)\int_0^z \frac{1}{\sqrt{\Omega_{m,0}\,(1+z)^3 + \Omega_{\Lambda,0}}}\,dz$$

The luminosity distance in the current proposal is given by the simple expression

$$d_L(z) = R_h(t_0)\,(1+z)\int_0^z \frac{1}{1+z}\,dz$$

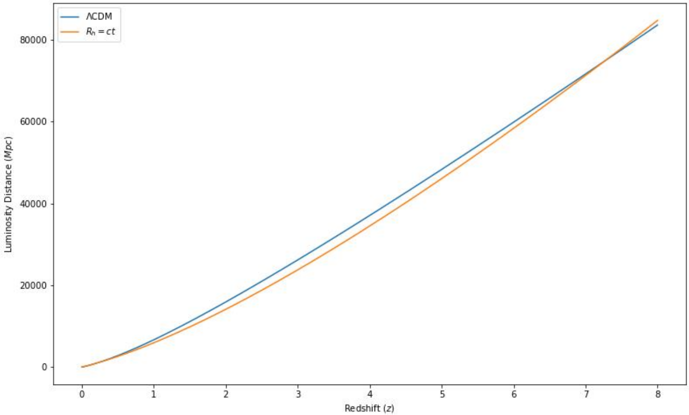

Figure 1: A graph of the luminosity distances.

Age of the Universe

Further constraints are placed on the $R_h = ct$ universe using the Union 2.1 SNe Ia data and the observed Hubble parameter data H(z).

Observations	H_0 (km/s/Mpc)
SNe Ia	70.01 ± 0.40
SNe Ia+H(z)	69.83 ± 0.40

Table 2: The best-fit values of the Hubble constant H_0 in the $Rh = ct$ universe

Age of the Universe

Recent observations show that the present cosmic age, in the standard model, is about $t_0 = 13.82$ Gyr.

It still suffers from the problem that some objects are older than the age of the universe.

The age of a flat universe is given by

$$t(z) = \frac{1}{H_0} \int_0^{\infty} \frac{1}{(1+z)^2} = \frac{1}{H_0}$$

Using the best-fit values $H_0 = 70.01 \pm 0.40$ and $H_0 = 69.83 \pm 0.40$ $km/s/Mpc$ gives $t_0 = 13.97 \pm 0.08$ Gyr and $t_0 = 14.01 \pm 0.08$ Gyr, respectively.

Linear Perturbation Theory

It makes no sense to talk about distances to structures without discussing how they formed.

Define a density perturbation $\delta(t, \vec{x})$ such that

$$\delta \equiv \frac{\delta\rho}{\rho_0} = \frac{\rho(t, \vec{x}) - \rho_0(t)}{\rho_0(t)}$$

where ρ_0 is the unperturbed density of the homogeneous background and $\delta\rho$ is the much smaller perturbation about that background.

The goal here to study how the perturbations grow so we need some dynamical equations.

Linear Perturbation Theory

The equation governing the dynamics of the perturbations is

$$\frac{d^2\delta}{dt^2} + (2 - 3w)\, H \frac{d\delta}{dt} = \frac{3}{2} H^2 \,(1 + 3w)\,(1 - w)\,\delta + \frac{c_s^2 \nabla^2 \delta}{a^2} \tag{4}$$

where $w \equiv c_s^2 = d_\rho p$ is the sound speed squared in $c = 1$ units.

The second term on the left suppresses the growth of perturbations; the combined term on the right account for the gravity and pressure terms, respectively.

Linear Perturbation Theory

In the linear regime, we can form the wave-like decomposition

$$\delta(t,\vec{x}) = \frac{V}{(2\pi)^3}\int_k \delta_k(t)e^{\iota k\cdot x}\ d^3k \tag{5}$$

The Fourier component δ_k depends only on cosmic time t, and $\vec{k}$ and $\vec{x}$ are the comoving wavevector and distance, respectively.

Linear Perturbation Theory

For an $R_h = ct$ universe, the dynamical equation for δ_k is

$$\frac{d^2\delta_k}{dt^2} + \frac{3}{t}\frac{d\delta_k}{dt} - \frac{k^2c^2}{3H_0^2t^2}\delta_k = 0 \tag{6}$$

whose solution is

$$\delta_k(t) = C_1 t^{-1+\sqrt{1+k^2c^2/3H_0^2}} + C_2 t^{-1-\sqrt{1+k^2c^2/3H_0^2}} \tag{7}$$

At large fluctuations ($k \to 0$), the perturbations decay as

$$\delta_k \propto a^{-2} \tag{8}$$

Power Spectrum

It is possible to study the formation of structure by studying the power spectrum of this model – which, fortunately for us, is simply the volume average of the square of Fourier components of the density perturbations: i.e.

$$P(k) \equiv \langle |\delta_k|^2 \rangle = \frac{1}{V} \int_R \zeta(r) e^{\iota k \cdot r} \, d^3 r \tag{9}$$

where $\zeta(r)$ is the correlation function. The integral is rather involed but a plot of the numerical solution is attached below

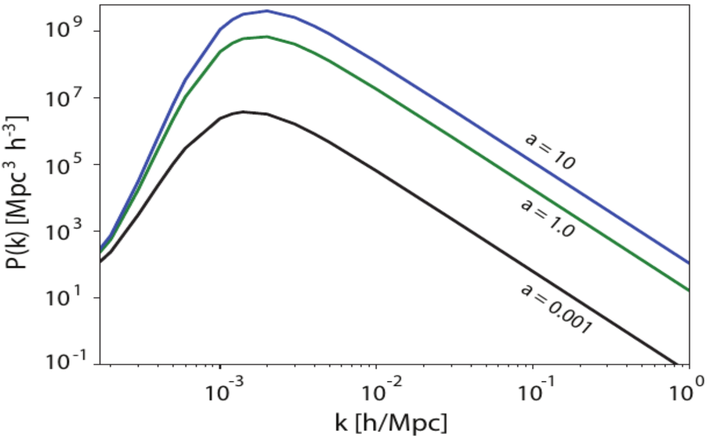

Figure 2: The power spectrum of the $R_h = ct$ Universe. The shape remains unchanged but the maximum slowly tends to higher values of k for higher values of a.

Conclusion

- The ΛCD model comprises dust, radiation, and 'dark energy' – each of which evolves in a different way – so the solution to the Friedmann equation will be more complicated and it is not easy to obtain a simple analytic solution
- The standard model tells us the universe is undergoing accelerated expansion
- The $R_h = ct$ model says the universe is not accelerating but it does not rely on the ad hoc assumption of early inflation and late-time dark energy
- The current proposal fails the acceleration test but fits luminosity distance data reasonably well and predicts a reasonable age of the universe
- There is also the issue of $3w = -1$. It leads to an equation of state that is not physically realisable