Viscous cosmological fluids and Large-scale structure

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Outline

Motivation

- To study the cosmological implications of viscous fluid on background expansion evolutionary history as well as on Large-scale structure formation on the basis that the universe derives from Friedmann-Lemaître-Roberson-Walker (FLRW) metric on the assumption that the universe is spatially homogeneous and isotropic.

Background viscous fluids

For a spatially flat FLRW universe, we have fluid equations taken as:

$$\dot{\rho}_r + 3H(\rho_r + p_r) = 0$$
$$\dot{\rho}_d + 3H\rho_d = \delta H\rho_d$$
$$\dot{\rho}_\Lambda + 3H\big(\rho_\Lambda + p_\Lambda\big) = -\delta H\rho_d.$$

The expansion rate given as:

$$\dot{H} = -\frac{1}{2}\big(\rho_d + \rho_\Lambda + p_\Lambda\big),$$

with natural units adopted $(8\pi G = c = 1)$. Taking the equation of state inhomogeneous as given by :

$$p_\Lambda = w_\Lambda\rho_\Lambda - \zeta, \text{ where } \begin{cases} w_\Lambda = A_0\rho_\Lambda^{\alpha-1} - 1 \implies w_\Lambda = A_0 - 1, \quad \alpha = 1. \\ \zeta = \zeta_0\rho_{\Lambda 0}\left(\frac{\rho_\Lambda}{\rho_{\Lambda 0}}\right)^m \implies \zeta = \zeta_0\rho_\Lambda, \quad m = 1. \end{cases}$$

Background viscous fluids cont'

The ΛCDM can be recovered iff:

- $\delta = 0$; $A_0 = 0$; $\zeta_0 = 0$.

Evolution of energy densities in dimensionless form by use of fractional energy density parameter $\rho_i = 3H\Omega_i$ and $h = H/H_0$, are obtained as:

$$
\begin{aligned}
\Omega_r =& \frac{1}{h^2}\Omega_{r0}(1+z)^4 \\
\Omega_d =& \frac{1}{h^2}\Omega_{d0}(1+z)^{3-\delta} \\
\Omega_\Lambda =& \frac{1}{h^2}\Bigg\{(1+z)^{3(A_0-\zeta_0)} + \frac{\Omega_{d0}}{3(1+\zeta_0-A_0)-\delta} \\
& \times\left[\delta(1+z)^{3-\delta} - 3(1+\zeta_0-A_0)(1+z)^{3(A_0-\zeta_0)}\right]\Bigg\}
\end{aligned}
$$

Background viscous fluids cont'

Moreover, the expansion rate for such model is obtained as:

$$h^2 = \Omega_{r0}\left[(1+z)^4 - (1+z)^{3(A_0-\zeta_0)}\right] + \Omega_{d0}\left[\frac{3(1+\zeta_0-A_0)}{3(1+\zeta_0-A_0)-\delta}\right]\left[(1+z)^{3-\delta} - (1+z)^{3(A_0-\zeta)}\right] + (1+z)^{3(A_0-\zeta)}$$

The deceleration parameter yields:

$$q(z) = \frac{1}{2h^2}\left[\Omega_{r0}\left\{4(1+z)^4 - 3(A_0-\zeta_0)(1+z)^{3(A_0-\zeta_0)}\right\} + \Omega_{d0}\left(\frac{3(1+\zeta_0-A_0)}{3(1+\zeta_0-A_0)-\delta}\right)\left\{(3-\delta)(1+z)^{3-\delta} - 3(A_0-\zeta_0)(1+z)^{3(A_0-\zeta_0)}\right\} + 3(A_0-\zeta_0)(1+z)^{3(A_0-\zeta_0)}\right] - 1$$

Background viscous cosmology cont'

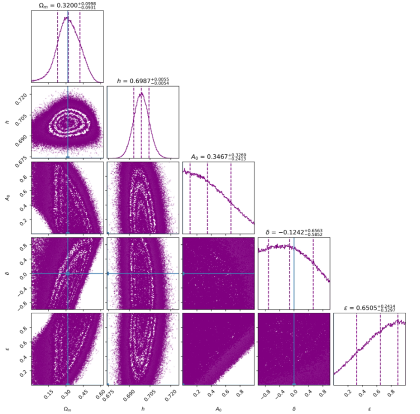

Figure: MCMC results constraining the background parameters

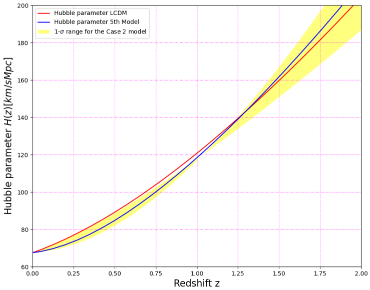

Figure: Evolution of Hubble parameter for the viscous model in comparison to ΛCDM model.

Background viscous cosmology cont'

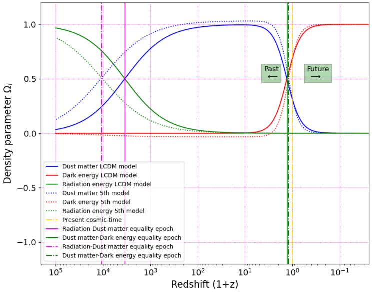

Figure: Evolution of fractional energy densities in redshift space

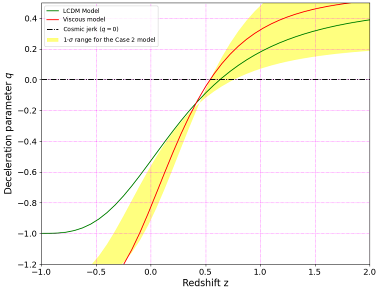

Figure: Evolution of deceleration parameter.

Cosmological Perturbations

1+3 Covariant formalism approach was used in the study for the cosmological perturbations. The spatial gradient variables are given as:

$$D_a^d = \frac{a}{\rho_d}\nabla_a \rho_d \ ; \ D_a^\Lambda = \frac{a}{\rho_\Lambda}\nabla_a \rho_\Lambda \ ; \ Z_a = a\nabla_a \Theta$$

The evolution equations for the spatial gradient variables written in redshift space with the scalar and harmonic decomposition taken reduces to:

Cosmological perturbations Cont'

$$
\begin{aligned}
\Delta'_m =& \left[1-\frac{\delta}{3}\right]\frac{\mathcal{Z}}{h(1+z)}+\left[\frac{\delta}{3}-1\right]\left[\frac{w_d\Omega_d}{(1+w_t)\Omega_t}\right]\frac{3}{1+z}\Delta_d\\
&+\left[\frac{\delta}{3}-1\right]\left[\frac{w_\Lambda\Omega_\Lambda}{(1+w_t)\Omega_t}\right]\frac{3}{1+z}\Delta_\Lambda\\
\Delta'_\Lambda =& \left[\frac{\delta\Omega_d}{3\Omega_\Lambda}+A_0-\frac{\Sigma}{3\Omega_\Lambda}\frac{1}{h^2}\right]\frac{\mathcal{Z}}{h(1+z)}\\
&+\left[\frac{\delta\Omega_d}{3\Omega_\Lambda}-\left\{\frac{\delta\Omega}{3\Omega_\Lambda}+A_0-\frac{\Sigma}{3\Omega_\Lambda}\frac{1}{h^2}\right\}\left\{\frac{w_d\Omega_d}{(1+w_t)\Omega_t}\right\}\right]\frac{3}{1+z}\Delta_d\\
&-\left[\left\{\frac{\delta\Omega_d}{3\Omega_\Lambda}-\frac{\Sigma}{3\Omega_\Lambda}\frac{1}{h^2}-\zeta_0\right\}+\left\{\frac{\delta\Omega_d}{3\Omega_\Lambda}-\frac{\Sigma}{3\Omega_\Lambda}\frac{1}{h^2}+A_0\right\}\tfrac{w_\Lambda\Omega_\Lambda}{(1+w_t)\Omega_t}\right]\\
&\times\frac{3}{1+z}\Delta_\Lambda
\end{aligned}
$$

Cosmological perturbations cont'

$$
\begin{aligned}
\mathcal{Z}' =& \frac{2}{1+z}\mathcal{Z} \\
&+ \left[\frac{\Omega_d}{2} - \left\{1 + \frac{1}{2}\left(\Omega_d - 2\Omega_d + 3(A_0 - \zeta_0)\Omega_\Lambda\right)\right\}\left\{\frac{w_d\Omega_d}{(1+w_t)\Omega_t}\right\}\right. \\
&\left. - \left\{\frac{w_d\Omega_d}{(1+w_t)\Omega_t}\right\}\left\{\frac{\kappa^2(1+z)^2}{3h^2}\right\}\right]\frac{3h}{1+z}\Delta_d \\
&+ \left[\frac{\Omega_d}{2} + \frac{3}{2}(A_0 - 1 - \zeta_0)\Omega_\Lambda - \right. \\
&\quad \left\{1 + \frac{1}{2}\left(\Omega_d - 2\Omega_\Lambda + 3(A_0 - \zeta_0)\Omega_\Lambda\right)\right\}\left\{\frac{w_\Lambda\Omega_\Lambda}{(1+w_t)\Omega_t}\right\} \\
&\left. + \left\{\frac{w_\Lambda\Omega_\Lambda}{(1+w_t)\Omega_t}\right\}\left\{\frac{\kappa^2(1+z)^2}{3h^2}\right\}\right]\frac{3h}{1+z}\Delta_\Lambda
\end{aligned}
$$

Cosmological perturbations cont'

The evolution equations in ΛCDM reduces to:

$$\begin{aligned}
\Delta_d' =& \left[1 - \frac{\delta}{3}\right] \frac{\mathcal{Z}}{h(1+z)} + \left[\frac{\delta}{3} - 1\right] \left[\frac{w_d \Omega_d}{(1+w_t)\Omega_t}\right] \frac{3}{1+z} \Delta_d \\
\mathcal{Z}' =& \frac{2}{1+z}\mathcal{Z} + \left[\frac{\Omega_d}{2} - \left\{1 + \frac{1}{2}\Big(\Omega_d - 2\Omega_d + 3(A_0 - \zeta_0)\Omega_\Lambda\Big)\right\}\left\{\frac{w_d \Omega_d}{(1+w_t)\Omega_t}\right\}\right. \\
& \left. - \left\{\frac{w_d \Omega_d}{(1+w_t)\Omega_t}\right\}\left\{\frac{\kappa^2 (1+z)^2}{3h^2}\right\}\right] \frac{3h}{1+z} \Delta_d,
\end{aligned}$$

where we have used normalized parameters given as
$\mathcal{Z} = \frac{Z}{H_0}$; $\Sigma = \frac{\zeta}{H_0^2}$.

Cosmological perturbations cont'

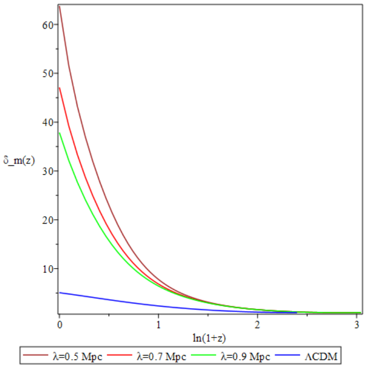

Figure: Evolution of dust matter perturbations with positve coupling strength.

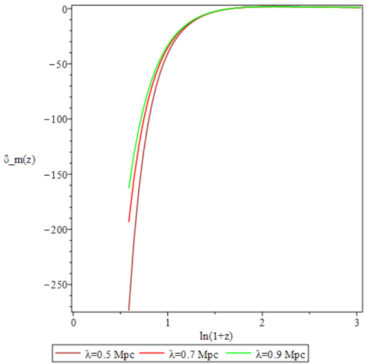

Figure: Evolution of dust matter perturbations with positve coupling strength.

Concluding remarks

- The background dynamics of the viscous model showed an agreement with ΛCDM model for a support of expanding and accelerating universe.
- In the evolution of energy densities, the viscous model predicts a longer era for dust dominated epoch in comparison to the ΛCDM case.
- As expected, the perturbations of viscous model yield for more structure formation.
- Other models are in progress, models such as little rip, pseudo rip, bounce cosmology and imperfect fluid system (where the total fluid of the universe does not preserve conservation of energy) universe.

Thank you!