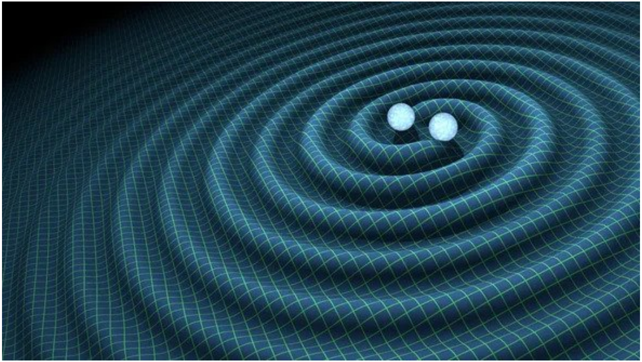
# Towards computing QNMs of Kerr black holes using PINNs


*presented by* **Anele Ncube**
*supervised by Alan S Cornell*


SAIP2023 | 04 July 2023

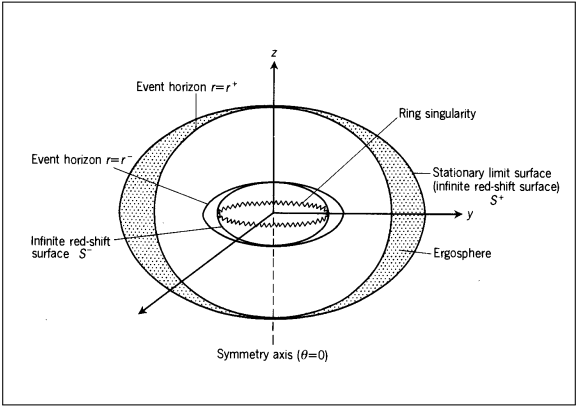
# Recap on QNMs



*Damped oscillations produced in response to some pert. e.g. BBH collision, formulated in BH perturbation theory, a precise description of GWs.*

Credit: Caltech

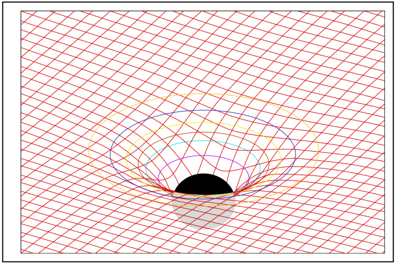
# Some properties of Kerr BHs




Kerr BH (d'Inverno, 1999)

- $J = 2Ma$ (BH angular momentum),
- $g_{ab} \to \eta_{ab}$ as $\rho \to \infty$ (asympt. flat).

# Grav. field/direct metric pert. $(s = -2)$ of a Kerr BH



Space-time curvature induced by BH

- BH perturbation theory (linear approx.):

$$g_{\mu\nu} = g^0_{\mu\nu} + \delta g_{\mu\nu},$$

where, $g^0_{\mu\nu}$ = Kerr bkg, $\delta g_{\mu\nu}$ = metric pert., $\delta g_{\mu\nu} \ll g^0_{\mu\nu}$.

# Teukolsky equation

- Radial ODE:

$$\Delta(r)R''(r) + (s+1)(2r-1)R'(r) + V(r)R(r) = 0,$$

- Angular ODE:

$$(1-u^2)S''(u) - 2uS' + \left[a^2\omega^2u^2 - 2a\omega su + s + A - \frac{(m+su)^2}{1-u^2}\right]S(u) = 0,$$

where $R(r) = \psi_4 = -\ddot{h}_+ + i\ddot{h}_\times$ (Weyl scalar), $S(u) =$ spheroidal harmonics.

# Astrophysically relevant BCs and eigenvalues

- BCs/asymptotic behaviour:

$$R(x) = \begin{cases} e^{-i(\omega - \frac{ma}{2Mr_+})x}/\Delta^s, & x \to -\infty \\ e^{+i\omega x}/r^{2s+1}, & x \to +\infty \end{cases}.$$

analytically intractable eigenvalue problem

- Eigenvalues:

$$\omega = \omega_{n\ell m} = \omega_{Re} - i\omega_{Im},\ A = A_{n\ell m},$$

where $\omega_{Re}$ = oscill. freq., $\omega_{Im} \propto$ damp. rate. $A$ = sep. const.

# Using PINNs to solve the Teukolsky eqn.

*what has been done?*

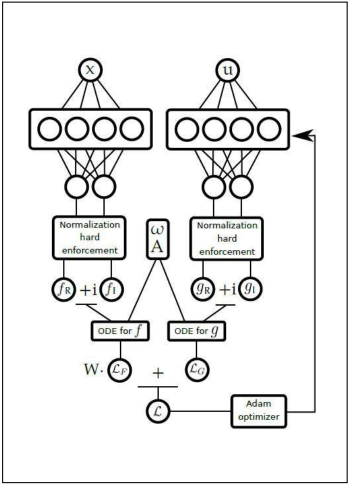
- **PINN algorithm**:

$$\theta^*, \hat{\psi}^*, \hat{\lambda}^* = \text{argmin}_{\theta,\psi,\lambda} \mathcal{L}(\theta, \hat{\psi}, \hat{\omega}),$$

where $\hat{\psi}^*, \hat{\lambda}^*$ approx. satisfy PDE.

- **Schwarzschild:** $(a = 0, \ell < 0, n = 0)$; $< 0.01\%$ err. Cornell *et al.*, (2022) PRD **106** 124047.

- **Kerr:** $(a = 0 - 0.4999, \ell > 2, n < 2)$; err.$< 1\%$ Luna *et al.*, (2023) PRD **107** 064025.




*Schematic of PINNs (Luna et al., 2023)*

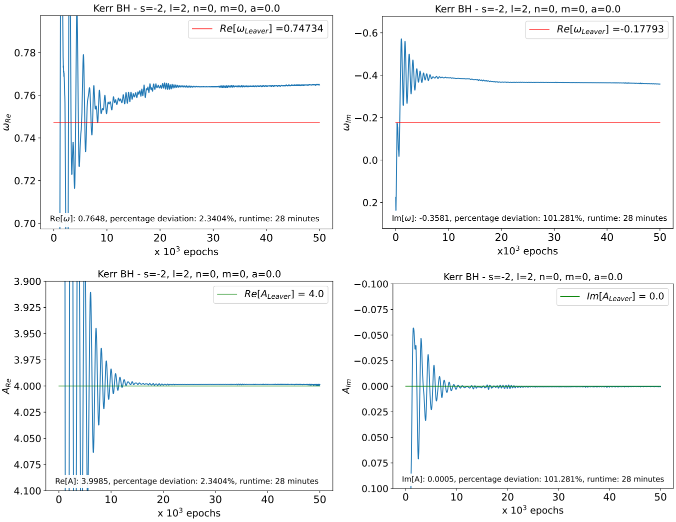
# Preliminary results

*PINN approx. of grav. pert. of a Kerr BH*
$(s = -2, \ell = 2, n = 0)$.

| $a$ | $\omega_{CFM}$ (Leaver, 1985) | $\omega_{PINN}$ (Luna et al., 2023) | $\omega_{PINN}$ (prelim.) |
|---|---|---|---|
| 0 | $0.74734 - 0.17793i$ | $0.74981 - 0.17812i$ (0.33%)(0.11%) | $0.7648 - 0.3581i$ (2.34%)(101.28%) |
| 0.3 | $0.77611 - 0.17199i$ | $0.77758 - 0.17332i$ (0.19%)(0.78%) | $0.8052 - 0.3666i$ (3.75%)(113.15%) |
| 0.4999 | $0.85023 - 0.14365i$ | $0.84589 - 0.14354i$ (0.51%)(0.07%) | $0.9935 - 0.5041i$ (16.85%)(250.93%) |

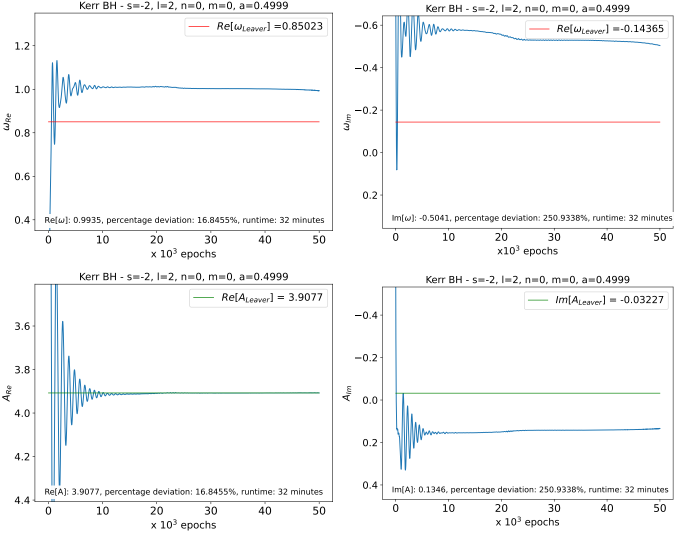
- **Baseline set-up**: 2 NNs, 2 hidden layers 50 nodes each, 50K epochs, sin activation fnc, Adam optimiser, 100 pts.

# Results for the Schwarzschild limit ($a = 0$)



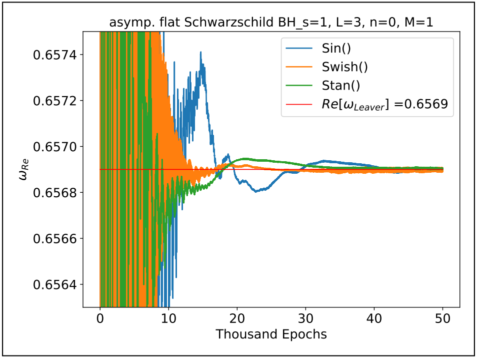

*The evolution of* $\{\omega_{n=0}, A_{n=0}\}$ *for a Kerr BH.*

# Results for the near extremal limit ($a = 0.4999$)




*The evolution of* $\{\omega_{n=0}, A_{n=0}\}$ *for a Kerr BH.*

# Towards computing overtones using PINNs




*Tunable activation functions for faster convergence*

- **Supervised learning:** use a "labelled" dataset from col. methods.
- e.g. from Ripley (2022) *Class. Quantum Grav.* **39** 145009.
- **Hyperparameter tuning**, e.g. use of tunable activation functions.

# Summary

- Recap on QNMs.
- Teukolsky eqn - gravitational field pert. of a Kerr BH.
- Using PINNs to solve the Teukolsky eqn.
- Prelim. results: Leaver (1985) vs. Luna et al (2022) vs. reprod.
- Potential enhancements of the PINN algorithm.

*Thank you. Any Questions?*