Weak Gravity Conjecture for dilaton de Sitter black holes in extra dimension

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SAIP2023 conference, July 4, 2023.

Introduction

- The Weak Gravity Conjecture
  In a flat space-time, for a single $U(1)$ gauge theory with gauge coupling g, there is a least one state that has a charge q bigger than its mass m, measured in Planck units

$$\boxed{\frac{m}{M_P} < \sqrt{2} g q}$$

(where M_P is the reduced Planck mass)
Arkani-Hamed, Motl, Nicolis, Vafa

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- For a first check we can look at the SM

  $$|gq| \sim 0.3, \quad m_e = 0.511kev, \quad M_P = 2{\times}10^{-18} \Rightarrow .42 > 2.5{\times}10^{-19}$$

Actually, all the fundamental particles of the SM verify this conjecture

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- One of the main motivations for the WGC is that it is a kinematic requirement that allows extremal BHs to decay.
- ✓ Since we are looking for charged objects, we consider the Reissner-Nordström metric:

$$ds^2 = (1 - \frac{2M}{r} + \frac{Q^2}{r^2})dt^2 + \frac{dr^2}{1 - \frac{2M}{r} + \frac{Q^2}{r^2}} + r^2 d\Omega^2$$

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⋆ $M > Q$: This solution has two horizons, and describe a BH

⋆ $M = Q$:The two horizons are combined,(extremal BH)

⋆ $M < Q$:There is no horizon, the solution is a naked singularity and it is a non-physical solution.

- Let's consider an extremal BH, $M = Q$

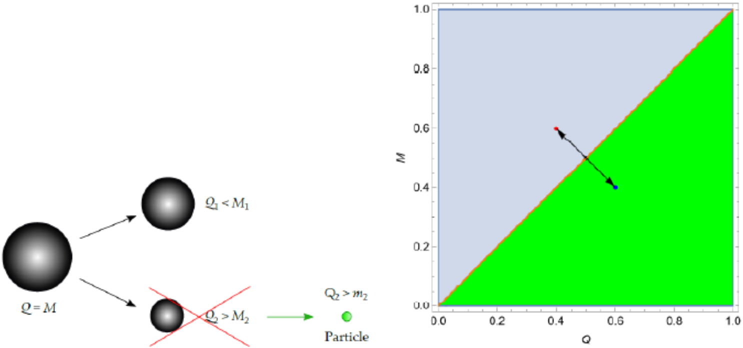

Figure: Extremal black holes can only decay into a particle that respects the WGC and a non-extremal black hole.

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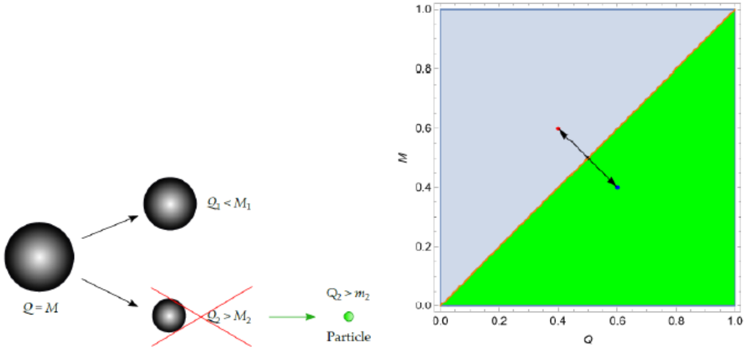

Figure: Extremal black holes can only decay into a particle that respects the WGC and a non-extremal black hole.

⋆ The WGC is thus seen to open up a possibility for extremal black holes to decay into non-extremal.

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- The simplest case is a cosmological constant $\Lambda \neq 0$
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This leads to the Einstein-Maxwell-Dilaton action in $4D$:

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This action on n-dimension

$$S = \int d^nx\sqrt{-g}[R - \frac{4}{n-2}\partial_\mu\phi\partial^\mu\phi - e^{\frac{4\alpha\phi}{n-2}}F^2 - V(\phi)]$$

Strategy:

- 1) First we need to construct black hole solutions.
- 2) Next, we need to find the relation between mass and charge for extremal black holes.
- 3) Finally, we identify the WGC inequality corresponding to the existence of a super-extremal state.

$\Lambda = 0$ and $4D$

- $\alpha = 0$

$$g_{00} = (1 - \frac{r_+}{r})(1 - \frac{r_-}{r}) \Rightarrow \quad R.N.BH$$

⋆ No Naked singularity $\Rightarrow \quad Q^2 < M^2 \Rightarrow r_\pm = M + \sqrt{M^2 - Q^2}$,
$r = 0$ is a time- like singularity
$r_\pm$ are inner and event horizons

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- $\alpha \neq 0$

$$g_{00} = (1 - \frac{r_+}{r})(1 - \frac{r_-}{r})^{\frac{1-\alpha^2}{1+\alpha^2}}$$

⋆ No Naked singularity $\Rightarrow Q^2 e^{2\alpha\phi_0} < (1 + \alpha^2) M^2$.
r_- is a space- like singularity
r_+ is the only event horizon of the black hole.
Gibbons-Maeda Garfinkle-Horowitz-Strominger

dS $space$: $\Lambda > 0$, n dimensions

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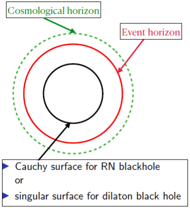
$$
\begin{aligned}
ds^2 \quad = \quad & -\left\{\left[1-(\frac{r_+}{r})^{n-3}\right]\left[1-(\frac{r_-}{r})^{n-3}\right]^{1-\gamma(n-3)} - H^2 r^2 \left[1-(\frac{r_-}{r})^{n-3}\right]^{\gamma}\right\} dt^2 + \\
& \left\{\left[1-(\frac{r_+}{r})^{n-3}\right]\left[1-(\frac{r_-}{r})^{n-3}\right]^{1-\gamma(n-3)} - H^2 r^2 \left[1-(\frac{r_-}{r})^{n-3}\right]^{\gamma}\right\}^{-1} \\
& \left[1-(\frac{r_-}{r})^{n-3}\right]^{-\gamma(n-4)} dr^2 + r^2 \left[1-(\frac{r_-}{r})^{n-3}\right]^{\gamma} d\Omega^2_{n-2}. \qquad (2.1)
\end{aligned}
$$

- ⋆ $H^2 = \frac{|\Lambda|}{3} \Rightarrow$ Hubble parameter. Gao-Zhang/ Elvang-Friedman-Liu
- ⋆ Note that r_- still indicates the coordinate of a singular surface,
- ⋆ r_+ is not an event horizon.
- ⋆ r_c is a cosmological horizon.

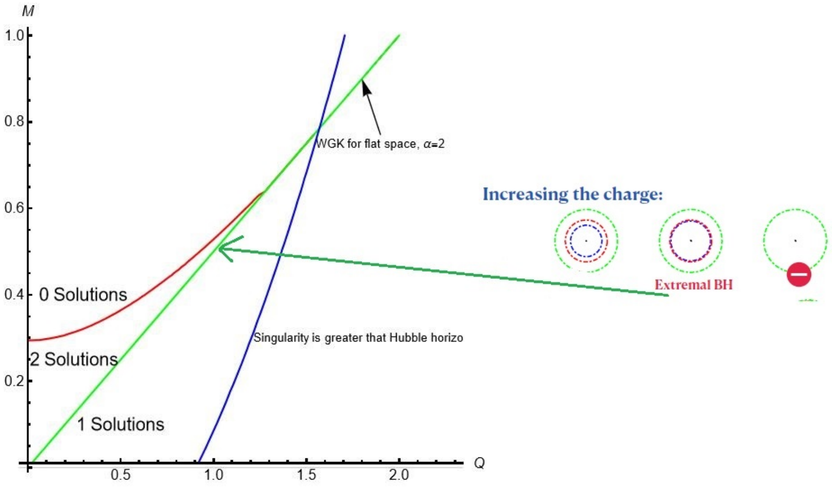


- The $(0,0)$ component of the metric :

$$f(r) = \left[1 - \left(\frac{r_+}{r}\right)^{n-3}\right] - H^2 r^2 \left[1 - \left(\frac{r_-}{r}\right)^{n-3}\right]^{\alpha^2 - \frac{(n-3)^2}{n-1}}$$

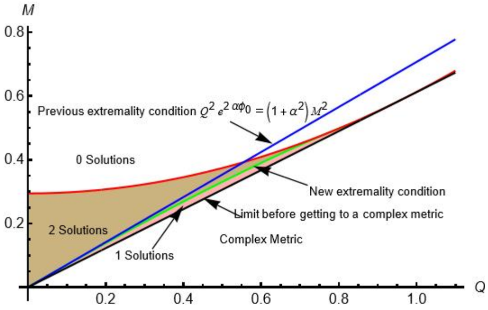
$$\alpha_c = \frac{(n-3)}{\sqrt{n-1}}$$

- $\Lambda > 0 \quad \alpha = 2\,(\alpha > \alpha_c = 1)$ and $n = 5$



- WGC in flat space and 5 dimensions: $r_+ = r_- \Rightarrow M^2 = \frac{Q^2}{4}$

- $\Lambda > 0 \quad \alpha = 1(\alpha = \alpha_c = 1)$ and $n = 5$



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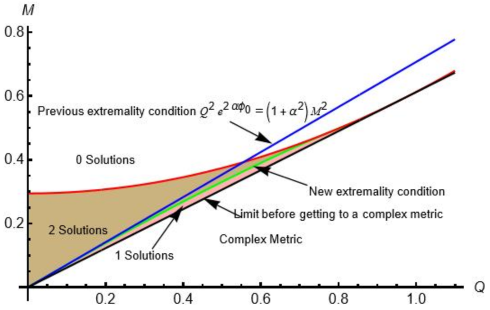
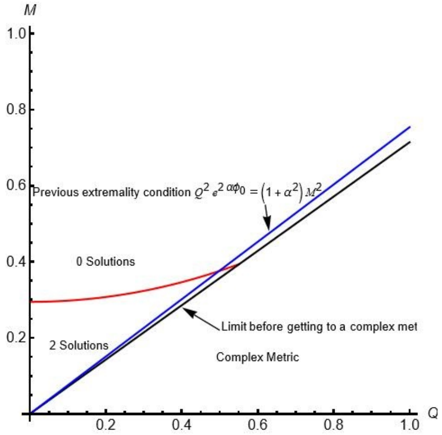
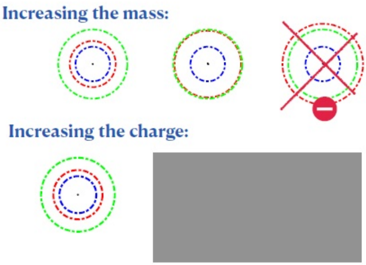


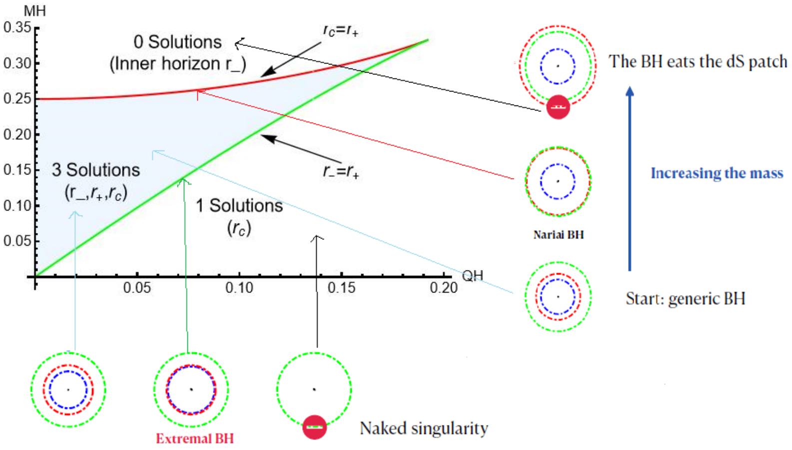
$$M^2 < \frac{Q^2}{2} + \frac{\sqrt{2} Q^2 H^2}{4\pi} - \frac{3 Q^4 H^4}{16\pi^2} + \mathcal{O}(H^6)$$

- $\Lambda > 0 \quad \alpha = 0.5$ and $n = 5$





- $\Lambda > 0 \quad \alpha = 0\ (\alpha < \alpha_c = 1)$ and $n = 5$



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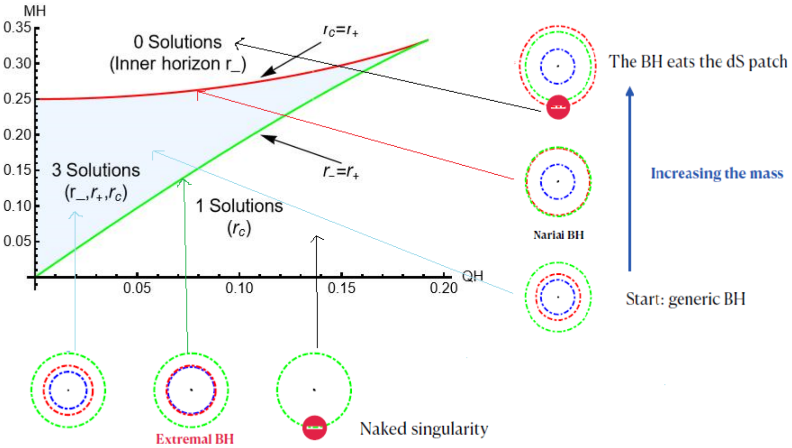

- In flat 5 dimensions $\Rightarrow r_- = r_+ \Rightarrow M^2 < \frac{3}{4}Q^2$.
- Now in De sitter space, by expansion of equation of green line

$$M^2 < \frac{3}{4}Q^2 + \frac{\sqrt{3}}{2\pi}Q^3H^2 - \frac{3}{4\pi^2}Q^4H^4 + \mathcal{O}(H^6)$$

Conclusion

- we investigated extremal black hole solutions in Einstein-Maxwell-Dilaton theory in extra dimensions.

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$$\alpha > \alpha_c \quad \Rightarrow \quad Q^2 e^{2\alpha\phi_0} = (1+\alpha^2)M^2,$$

$$\alpha = \alpha_c \quad \Rightarrow \quad M^2 = \frac{Q^2}{2} + \frac{\sqrt{2}Q^2H^2}{4\pi} - \frac{3Q^4H^4}{16\pi^2} + \mathcal{O}(H^6),$$

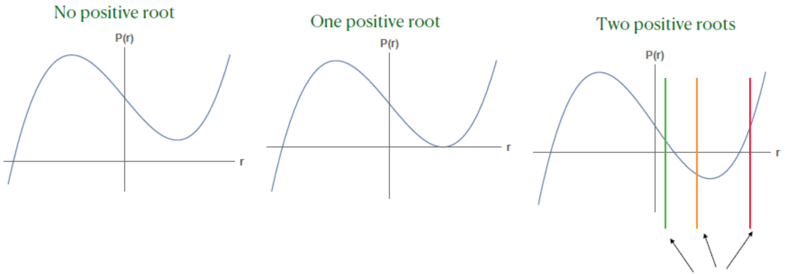
$$0 < \alpha < \alpha_c \quad \Rightarrow \quad for \quad M^2 = \frac{3Q^2(2-\alpha^2)}{8},$$

the metric is comlex and the extremality is never reached

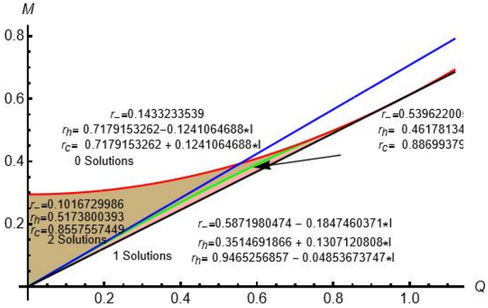
$$\alpha = 0 \quad \Rightarrow \quad M^2 < \frac{3}{4}Q^2 + \frac{\sqrt{3}}{2\pi}Q^3H^2 - \frac{3}{4\pi^2}Q^4H^4 + \mathcal{O}(H^6),$$

Thank you for your attention!

- In order to find the BH horizons, we look for the roots of $f(r)$,
  - ⋆ We are interested only in solutions of $f(r)$in the region $r > r_{-}$ outside the singularity



Two, one, or zero positive roots depending on the position of $r = r_{-}$

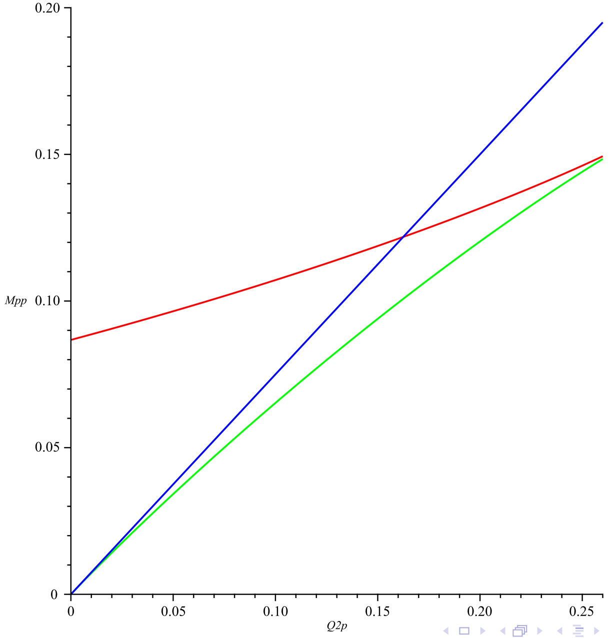

M
0.8
0.6
0.4
0.2
Q
0.2
0.4
0.6
0.8
1.0
r_=0.1433233539
rh= 0.7179153262−0.1241064688*I
rc= 0.7179153262 + 0.1241064688*I
0 Solutions
r_=0.1016729986
rh=0.5173800393
rc=0.8557557449
2 Solutions
1 Solutions
r_=0.5871980474 − 0.1847460371*I
rh=0.3514691866 + 0.1307120808*I
rh= 0.9465256857 − 0.04853673747*I
r_=0.53962200
rh= 0.46178134
rc= 0.88699379

- The relation between the physical quantities mass M and Q^2 and the integration constants r_+, r_-:

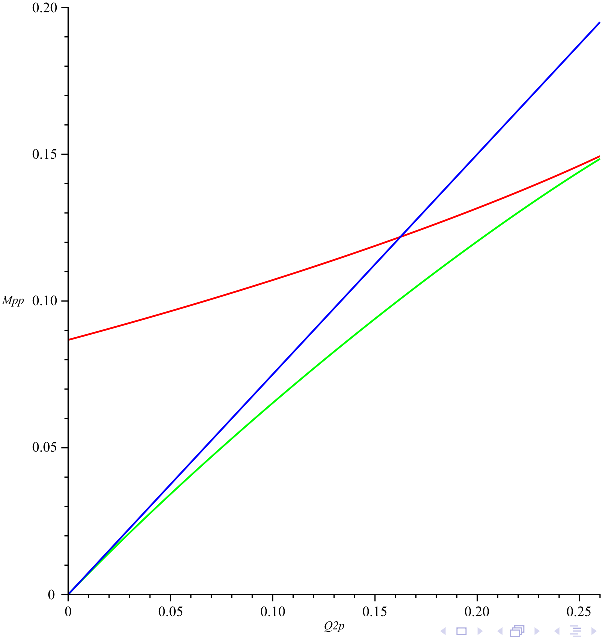
$$r_- = \frac{Q^2\left(\alpha^2+n-3\right)\left(\frac{\sqrt{8(n-2)Q^2(\alpha^2-n+3)+M^2(n-3)^2}+M(n-3)}{(n-3)(n-2)}\right)^{3-n}}{(n-3)^2(n-2)}$$

$$r_+ = \frac{\left(\sqrt{M^2(n-3)^2-8(n-2)Q^2\left(-\alpha^2+n-3\right)}+M(n-3)\right)}{(n-3)(n-2)}$$

$$M^2 \geqslant \frac{8(n-2)Q^2\left(-\alpha^2+n-3\right)}{(n-3)^2}$$



0.20
0.15
Mpp
0.10
0.05
0
0
0.05
0.10
0.15
0.20
0.25
Q2p



0.20
0.15
Mpp
0.10
0.05
0
0
0.05
0.10
0.15
0.20
0.25
Q2p