**(2+1)D Graphene in a magnetic field in Noncommutative Geometry**

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**Abstract**

In this paper, we analyze the 2+1 dimensional Dirac massless equation under the impact of a uniform magnetic field in non-commutative geometry.
We solve the system analytically using a direct method in order to obtain the energy eigenvalues and the corresponding wave function.
In the end, we remark that the system has been influenced by the NC geometry.

**Keywords:** Graphene, Non-Commutative, Dirac massless equation.

**Introduction:**

The non-commutative space has found increasing interest in field and string theories [1]. Snyder was the first to explain the lack of change between the coordinates of space-time [2] and this principle offers stimulating scenarios in a number of unifying interaction theories called M-Theory [3-4] as well as in modern cosmology [5-7]. The use of the formula is not limited to the following areas, so that non-commutative theory can explain the fusion between ultraviolet and infrared radiation in very small areas [8], the Lorentz symmetry breaking [9], and the quantum physics of the field [10-11]. Extensive studies can also be found on the integration of the non-commutative relationships of ordinary quantum mechanics and classical geometry [12-14]. In addition, changing normal relationships, nondeterministic relationships, inspired by string theory, quantum gravity, and special relativity, have been the focus of very interesting research in recent years [15-18].

Graphite can be thought of as the accumulation of thick layers of carbon in an atom called graphene. The physics of graphene has drawn the attention of the theoretical scientific community as experimental observations revealed the existence of electrical charge carriers that behave like massless Dirac quasiparticles [19- 22]. The reason for this lies in the unusual molecular structure of graphene. Carbon atoms are arranged in a hexagonal lattice, similar to a honeycomb structure [23]. It has been observed that low-energy electronic excitations at the corners of graphene The Brillouin zone can be described by Dirac 2 + 1 fermions with a linear scattering ratio (massless) [19- 22]. This effect offers the prospect of testing multiple aspects of relativistic phenomena that generally require great energy, in experiments of physics of the condenses, such as chiral tunnels and Klein paradox [24-25].

**Graphene in NonCommutative space**

Before proceeding to the solution of the graphene equation, we define some relations of the non-commutative space.

 Non-commutative space is characterized by the fact that its coordinate operators are non-commutative, in contrast to the properties of the coordinates of ordinary space. The coordinates, in this case, satisfy the following relationships

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| (1) | $$\left[\hat{x}^{μ},\hat{y}^{v}\right]=iθ^{μv}$$ |

 Where $θ^{μv}$is an antisymmetrical tensor in the usual quantum mechanics and plays an analogous function to ℏ.

 We want to maintain the unity of the theory, we choose $θ^{0v}$=0 , which means that time remains as a parameter and that non-commutativity only affects physical space. The product of any two functions in the sense of this deformation is equivalent to the star Moyal product identified by

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| (2) | $$\left(f⋆g\right)\left(x\right)=\left.exp\left[\frac{i}{2}θ\_{ab}∂x\_{a}∂y\_{b}\right]f\left(x\right)g\left(y\right)\right|\_{x=y}$$ |

 Where $f$ and $g$ are two arbitrary and considered to be infinitely distinct functions.

 The effects of transition and momentum in this model can be expressed by the two transformations [26-28]:

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| (3) | $$x\_{i }⟶x\_{i}-\frac{1}{2ℏ}θ\_{ij}p\_{j }and p\_{i }⟶p\_{i}, i=\overbar{1,3}$$ |

With an antisymmetric tensor$θ$ parameter, selected as

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| (4) | $$θ\_{ij}=ϵ\_{ijk}θ\_{k} and θ\_{3}=θ$$ |

We can rewrite the transformation based on this fact (3), in the following condensed form.

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| (5) | $$θ\_{12}=-θ\_{21}=θ\_{3}=θ with \vec{ r}\rightarrow \vec{r}+\frac{\vec{θ}×\vec{p}}{2ℏ}$$ |

The electron in the quantum theory of graphene is a massless fermion that moves with a velocity $V\_{F} = (1.12 \pm 0.02) × 10^{6} m s^{-1}$ called Fermi velocity verify the relativistic massless Dirac equation. The discovery of graphene gives us the opportunity of testing various effects of QED, such as the “Klein paradox” because this effect is unobservable in particle physics. In this section, we are interested in solving the (1 + 2)-dimensional massless Dirac equation in the presence of an external constant magnetic field$A = \frac{B}{2} (-y, x, 0)$. In this case, the graphene equation read

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| $(\hat{α}.p)$Ψ(r)$=\frac{E}{V\_{F}}$Ψ(r) | (6) |

 Where $\hat{α}$ the usual Dirac matrice, given by

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| $$\hat{α}=\left(\begin{matrix}0&\hat{σ}\\\hat{σ}&0\end{matrix}\right)$$ | (7) |

 $\hat{σ}$ designates the Pauli matrices.

We may assume that the four-component spinor Ψ is of the form Ψ(r) = (Ψa(r), Ψb(r)).

On substitution of Ψ (r) as given into (6), we get the following equations for the two-component spinors Ψa(r) and Ψb(r):

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| $$\hat{σ}p^{-}Ψ\_{b}\left(r\right)=\frac{E}{V\_{F}}Ψ\_{a}\left(r\right)$$$$\hat{σ}p^{-}Ψ\_{a}\left(r\right)=\frac{E}{V\_{F}}Ψ\_{b}\left(r\right)$$ | (8) |

With $p^{-}=p-eB×\left(r+\frac{θ×p}{2ℏ}\right).$

These two equations can be used to eliminate $Ψ\_{b}\left(r\right)$in favor of$Ψ\_{a}\left(r\right)$, so that we can have

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| $$(\hat{σ}.p^{-})(\hat{σ}.p⁻)Ψ\_{a}\left(r\right)=\frac{E^{2}}{V\_{F}^{2}}Ψ\_{a}\left(r\right)$$ | (9) |

According to the following relations $(\hat{σ}.A)(\hat{σ}.B)=A.B+i\hat{σ}.(A×B)$

The equation (9) can be written as

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| $$((p^{-}.p⁻)+iσ.(p^{-}×p⁻))Ψ\_{a}\left(r\right)=\frac{E^{2}}{V\_{F}^{2}}Ψ\_{a}\left(r\right)$$ | (10) |

After a straightforward calculation of eq(10),we obtain

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| $$\left[\left(1+\frac{eBθ}{2ℏ}+\frac{e²B²θ²}{16ℏ²}\right)p²+\left(\frac{e²B²}{4}\right)r²-\left(eB+\frac{e²B²θ²}{4ℏ²}\right)L\_{z}-\left(eBℏ+\frac{e²B²θ}{4}\right)σ\_{z}-\frac{E²}{V\_{F}^{2}}\right]Ψ\_{a}\left(r\right)=0$$ | (11) |

To solve the eq.(11), we introduce the polar coordinates in position space $(r,ϕ)$, and we use the following ansatz $Ψ\_{a}\left(r\right)=exp⁡(im\_{l}φ)R\_{n,l}(r)χ\_{τ}$, where $n$ is the radial quantum number, $m\_{l},$ and $τ=\pm 1$ are, respectively, the eigenvalues of angular momentum and spin operators, and $χ\_{+1}^{T}=(1,0)$, $χ\_{-1}^{T}=(0,1)$ are the spin functions; to obtain

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| $$\left[\left(\frac{d}{dr}\right)^{2}+\frac{1}{r}\frac{d}{dr}-\frac{m\_{l}^{2}}{r²}-\frac{ηr²}{ℏ²}+ε\right]R\_{n,l}(r)=0$$ | (12) |

While the parameters $η, ε$are constants, we just name them after reorganizing the equation.

We use the appropriate transformations by employing a direct method in order to obtain the following energy spectrum such as:

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| $$E\_{n,m\_{l}}=\frac{ℏV\_{F}}{l\_{B}}\left[(2n+m\_{l}+1)\sqrt{\left(l\_{B}⁴+\frac{θl\_{B}²}{2}+\frac{θ²}{16}\right)}-τ\left(1+\frac{θ²}{4l\_{B}}\right)-m\_{l}\left(l\_{B}+\frac{θ²}{4ℏl\_{B}}\right)\right]^{1/2}$$ | (13) |

In addition, we notice here that $l\_{B}= \sqrt{ℏ/eB}$ is called magnetic length.

Moving to the wave function, which gives such

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| $$ψ\_{a}(r,ϕ)=Cr^{-\frac{1}{2}}exp\left(-\frac{r^{2}}{a^{2}}\right)(-n,\frac{3}{4}+\frac{m\_{l}}{2},\frac{r^{2}}{a^{2}})\frac{r^{2}}{a^{2}}^{\frac{1}{4}+\frac{m\_{l}}{2}}F(-n,\frac{3}{4}+\frac{m\_{l}}{2},\frac{r^{2}}{a^{2}})exp(ilϕ)$$ | (14) |

If we test these obtained results by putting$θ=0$, we get the ordinary case in the commutative space.

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