Interacting viscous dark cosmology

B.G Mbewe[1], **A Abebe**[2]

[1]Centre for Space Research, North-West University, Potchefstroom 2520, South Africa
[2] Centre for Space Research, North-West University, Mahikeng 2735, South Africa

E-mail: bonang.mbewe@gmail.com

Abstract. Observational data show that the universe is dominated by the dark sector, which is comprising of dark matter ρ_{dust} and dark energy ρ_Λ. This is with budget allocation of 25% to dark matter while dark energy is about 70%. Since most of the existing work in the literature is limited to the study of background cosmological dynamics, the project aims at deriving the equations that govern the evolution of a universe filled with interacting viscous dark fluids and analyzing their behaviour as compared to ΛCDM universe. The approach will be to model and derive the background cosmological equations of interacting viscous fluids using the little rip, pseudo rip and bounce cosmology models and compare the results obtained with that of ΛCDM cosmology.

1. Introduction

Studies shows that the present universe is expanding in an accelerated fashion and this poses a great deal of challenges to fundamental physics as well as cosmology [1, 2, 3]. The challenge of cosmic acceleration of the universe is due to the belief that most of the energy that exist in the universe, exists by a form of new ingredient called dark energy which has a negative pressure. This notion has made way to various theoretical models of dark energy being proposed, with simplest model being the cosmological constant Λ with constant dark energy density and equation of state as $w_\Lambda = \frac{p}{\rho_\Lambda} = -1$; Shi et al [3].

The ΛCDM model is by far the simplistic and observational supported model, although it has two challenges being the fine-tunning problem and coincidence problem[1, 2, 4]; In fine-tunning the problem is that the the value of Λ is quite small as compared to the expectations of particle physicists, while in the coincidence problem we have, the magnitude of present energy density of dark matter Ω_m and present energy density of dark energy Ω_Λ being of the same order [5, 3].

The coincidence problem can be alleviated by consideration of interaction of fluids amongst one another since dark components are expected to not evolve separately. However, the interaction is supposed to be negligible at high red-shifts and to be important or significant at lower red-shifts in a sense that it enables us to choose the correct form of interaction term that leads to almost constant ratio between the energy density of dark matter and dark energy at low red-shifts [2, 5].

The ΛCDM is an idealized model which in practice is useful in many instances but in some instances incorrect. This then gives way for taking into account the viscous effects more especially when considering turbulence effects; or many other relativistic situations Brevik et al [6]. Taken from a hydrodynamic stance, the viscosity effect describes the deviation of the system from thermal equilibrium to the first order. Bulk viscosity in the cosmic fluid has a significant role more so in singularity phenomenon.

The objective of this article is to obtain the analytical solutions of the cosmological models that are induced by an inhomogeneous viscous fluid coupled with dark matter [7]. The article is structured as follows. In Section 2, we give equations that govern our evolutionary equation for viscous cosmological fluid coupled with dark matter. In Section 3, we discuss the little rip model and compare it to standard ΛCDM model. In Section 4, we discuss pseudo rip model in comparison to ΛCDM model. In Section 5 we discuss bounce cosmology and compares it to ΛCDM model. We then present our concluding remarks on the last section.

2. Background equations for viscous coupled dark fluids

Solving background equations for viscous interacting dark fluids in FLRW metric. The background equations for interacting viscous fluids are as follows; Brevik et al [7]:

$$\begin{cases} \dot{\rho}_\Lambda + 3H\big(p + \rho_\Lambda\big) = -Q, \\ \dot{\rho}_{dust} + 3H\rho_{dust} = Q, \\ \dot{H} = -\frac{\kappa^2}{2}\big(\rho_{dust} + \rho_\Lambda + p\big), \quad \text{where} \quad \kappa^2 = 8\pi G. \end{cases} \tag{2.1}$$

where the subscript "Λ" and "dust" refers to dark energy and dark matter respectively, and exchange of energy between dark energy and dark matter denoted by "Q" is given by

$$Q = \delta H \rho_{dust}. \tag{2.2}$$

with $\delta > 0$ constant.

We also have the expression for equation of state(E.O.S) given in inhomogeneous form as:

$$p = w\rho_\Lambda - 3H\xi. \tag{2.3}$$

where ξ is the bulk viscosity and w being the E.O.S parameter.

The E.O.S parameter is then given by,

$$w = A_0\rho_\Lambda^{\alpha-1} - 1. \tag{2.4}$$

with $A_0 \neq 0$ and $\alpha \geq 1$ being constants.

Hubble parameter in terms of the scale factor is given as follows:

$$\begin{cases} H = \frac{\dot{a}}{a}, \\ a = \frac{a_0}{1+z} = \frac{1}{1+z}, \quad \text{where} \quad a_0 = 1. \end{cases} \tag{2.5}$$

Substitute equation (2.2) into (2.1) yields,

$$\begin{cases} \dot{\rho}_\Lambda + 3H\big(p + \rho_\Lambda\big) = -\delta H\rho_{dust}, \\ \dot{\rho}_{dust} + 3H\rho_{dust} = \delta H\rho_{dust}, \\ \dot{H} = -\frac{\kappa^2}{2}\big(\rho_{dust} + \rho_\Lambda + p\big). \end{cases} \tag{2.6}$$

Substitute equation (2.4) into (2.3) yields,

$$\begin{cases} p = A_0\rho_\Lambda^\alpha - \rho_\Lambda - 3H\xi, \\ p + \rho_\Lambda = A_0\rho_\Lambda^\alpha - 3H\xi. \end{cases} \tag{2.7}$$

Substitute equation (2.7) into (2.6) yields,

$$\begin{cases} \dot{\rho}_\Lambda + 3H\left(A_0\rho_\Lambda^\alpha - 3H\xi\right) = -\delta H\rho_{dust}, \\ \dot{\rho}_{dust} + 3H\rho_{dust} = \delta H\rho_{dust}, \\ \dot{H} = -\frac{\kappa^2}{2}\left(\rho_{dust} + \rho_\Lambda + p\right). \end{cases} \tag{2.8}$$

From FLRW metric Friedmann equation for universe composed of only dark fluids reads as follows for flat universe:

$$3H^2 = \kappa^2\left(\rho_{dust} + \rho_\Lambda\right) \implies 1 = \frac{\kappa^2}{3H^2}\left(\rho_{dust} + \rho_\Lambda\right). \tag{2.9}$$

The ΛCDM model can be recovered when and only when the following conditions are met:

- $\delta = 0 \implies Q = 0$
  This takes care of decoupling the fluids so the can evolve separately.
- $\xi = 0$
  The exotic fluid is allowed/permitted to be non-viscous.
- $A_0 = 0 \implies w = -1$
  Dark energy is now having a negative pressure together with the above condition 2, which is the driving force behind the accelerating expansion of the universe.

3. Little rip model

Little rip is characterized by an increasing Hubble parameter but in an asymptotic sense in remote time [7]. We have chosen the toy model for Hubble parameter to denote the little rip phenomenon as follows:

$$H(z) = H_0 - \lambda \ln(1+z), \tag{3.1}$$

with $H_0 > 0$ and $\lambda > 0$ constants.

The expression that governs the evolution of the bulk viscosity is given as follows:

$$\xi(H) = \frac{2\lambda}{3\kappa^2} + \frac{1}{3H}\left[\rho_{dust} + A_0\rho_\Lambda^\alpha\right], \tag{3.2}$$

where in expression (3.2) it can be noted that the bulk viscosity is dependent on the coupling of the dark fluids, also the first term being a constant since λ we have declared as a constant.

The analytical solutions for the energy densities expressions given by 2.8 yielded the following:

$$\begin{cases} \rho_{dust} = \rho_{dust,0}\left(1+z\right)^{3-\delta}, \\ \rho_\Lambda = -\rho_{dust,0}\left(1+z\right)^{3-\delta} + \frac{3H^2}{\kappa^2}. \end{cases} \tag{3.3}$$

To retrieve the standard ΛCDM model from 3.3, it can only be obtained by constraining a parameter λ as follows:

$$\lambda = \frac{H_0}{\ln(1+z)}\left[1 \pm \sqrt{1 - \frac{\kappa^2\rho_{dust,0}}{3H_0^2}\left(1 - \left(1+z\right)^{(3-\delta)}\right)}\right]. \tag{3.4}$$

Together, with allowing $\xi = \delta = A_0 = 0$ and $w = -1$, as this decouples the fluids and assures that the fluid is non-viscous and that the dark energy has the negative pressure. Although

3.4 is no longer a constant as declared in the initial setup, this rises another complex situation altogether as is now a variable instead of a constant, as such the ΛCDM can be recovered if and only if (3.4) is realized and taken as a constant.
The deceleration parameter equation reads as

$$q = -\frac{\lambda}{H_0 - \lambda \ln(1+z)} - 1. \tag{3.5}$$

4. Pseudo rip model
Pseudo rip model is a model characterized by the Hubble parameter that tends to a de Sitter space in remote future [7]. Taking the toy model to represent the analogue of pseudo rip as given below:

$$h = \frac{H}{H_0} = \sqrt{2\left[1 + \ln\left(\frac{H_1}{H_0}\right) - \frac{H_1}{H_0} + \frac{\lambda \ln(1+z)}{H_0}\right]}, \tag{4.1}$$

where $H_0 > H_1$ and λ they are positive constants.

The bulk viscosity equation that governs the evolution of the energy densities is given as

$$\xi(H) = \frac{2\lambda}{3\kappa^2}\left[\frac{H_0 - H}{H}\right] + \frac{1}{3H}\left(\rho_{dust} + A_0 \rho_\Lambda^\alpha\right). \tag{4.2}$$

It can be seen that the coupling of dark fluids term dependency is once again associated with the bulk viscosity.

With the use of equations (4.1,4.2) together with equation (2.8), the equations governing the evolution of the energy densities is obtained as follows:

$$\begin{cases} \rho_{dust} = \rho_{dust,0}\left(1+z\right)^{3-\delta}, \\ \rho_\Lambda = -\rho_{dust,0}\left(1+z\right)^{3-\delta} + \frac{3H_0^2}{\kappa^2}\left(h + \frac{1}{2}\right) - \frac{3H_0^2}{\kappa^2}\left(h_0 - \frac{1}{2}\right)e^{-2(h_0+h)}, \end{cases} \tag{4.3}$$

where h_0 in expression (4.1) evaluated at $z = 0$ and not necessarily a Hubble constant.

The ΛCDM model can be retrieved by constraining the λ as follows:

$$\lambda = \frac{H_0}{2\ln\left(1+z\right)}\left[\left(\frac{\rho_{\Lambda,0} + \rho_{dust,0}\left(1+z\right)^{3-\delta}}{2\left(\rho_{\Lambda,0} + \rho_{dust,0}\right)\left(1-h_0\right)} + \frac{\left(h_0 - 1\right)}{2}\right)^2 - h_0^2\right]. \tag{4.4}$$

The expression (4.4) can be noted that is not a constant even though we assumed it a constant in the initial setup, but see it becoming a variable once a ΛCDM model is invoked.

The deceleration parameter is given by:

$$q = -\frac{h_2}{h^2} - \frac{h_2}{h} - 1, \tag{4.5}$$

where $h_2 = \frac{\lambda}{H_0}$.

5. Bounce cosmology model

Bounce cosmology is a model where singularities do not occur but instead the model behaves in a cyclic manner [8]. In that regards the model goes from accelerated collapse era to an accelerated expansion era without any display of singularities [7]. Taking the scale factor in an exponential form as a toy model, we have the Hubble parameter given as follows:

$$H = 2n\beta \ln(1+z)^{-\frac{2n-1}{2n\beta}}, \tag{5.1}$$

with $n > 0$, $\beta > 0$ as constants.

By use of equation (2.8) to obtain the bulk viscosity expression we obtain expression governing the evolution of the bulk viscosity as follows:

$$\xi = \frac{1}{3H}\left(\rho_{dust} + A_0\rho_{\Lambda}^{\alpha}\right) + \frac{2(2n-1)}{3\kappa^2}\ln(1+z)^{-2n\beta}. \tag{5.2}$$

Thus the analytical expression that gives the evolution of the energy densities is given as:

$$\begin{cases} \rho_{dust} = \rho_{dust,0}\left(1+z\right)^{3-\delta}, \\ \rho_{\Lambda} = -\rho_{dust,0}\left(1+z\right)^{3-\delta} + \frac{3H_0^2}{\kappa^2}\exp\left(\frac{2n-1}{2n\beta^2}\ln\left(1+z\right)^2\right). \end{cases} \tag{5.3}$$

To retrieve the ΛCDM model, we constrain the parameter β as:

$$\beta = \ln\left(1+z\right)\sqrt{\frac{2n-1}{2n\ln\left(\frac{\rho_{\Lambda,0}+\rho_{dust,0}\left(1+z\right)^{3-\delta}}{\rho_{dust,0}+\rho_{\Lambda,0}}\right)}}. \tag{5.4}$$

Again, is evident that the parameter β depends on red-shift and is not a constant. However, if we are to retrieve the ΛCDM case then equation (5.4) must be realized together with $A_0 = \xi = \delta = 0$ and $w = -1$.

The deceleration parameter is obtained as:

$$q = -\frac{2n-1}{H}\ln(1+z)^{-2n\beta} - 1. \tag{5.5}$$

6. Conclusions

In this work we studied late-time cosmology, where the viscous dark fluids were coupled together and allowed to exchange energy amongst one another in a flat Friedmann-Lemaitre-Robertson-Walker universe. It was evident for the models investigated that there were correlations in the bulk viscosity as they all have dependencies of coupling between dark matter(assumed to be composed of dust) and dark energy. Furthermore, by a closer look at the deceleration parameter we note that the universe is in an accelerating era as the value of $q < 0$ at all red-shifts for little rip and pseudo rip models with exception for bounce cosmology model. However, the drawback in that is that the work does not suffices to be used to go back in time and test the theory as it is modelled to study late-time cosmology and fails at testing the current epoch and the early universe.

For the little rip model, we see that the deceleration parameter for any choice of λ is always negative and as such this depicts late-time cosmology where the universe is in an accelerated expansion era at all times and thus explains the universe to be dark energy dominated. The same applies to the pseudo rip model; hence the two models cannot be used as a tool to be used in predicting the early universe as well as the present epoch. However, for the bounce cosmology the deceleration parameter can be positive at some point and that requires the choice of n to be $n \in (0, \frac{1}{2})$ for decelerated epochs. All other values of n give an accelerated expansion solution.Thus the bounce cosmology model can be used as a substitute to investigate the history of the universe. This however, is a preliminary result and needs further investigations.

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