Investigating the effects of turbulence-induced tilt and lateral displacement on OAM modes

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Abstract. Free-Space Optical (FSO) communication links have utilized Orbital Angular Momentum (OAM) modes as channels in Mode Division Multiplexing (MDM) systems. OAM modes suffer from turbulence-induced OAM crosstalk which degrades the performance of FSO communication links. Turbulence predominantly cause lateral displacement and tilt on the beam wavefront. There are analytical models that characterize OAM crosstalk due to lateral displacement and tilt. We investigated the OAM spectrum due to lateral displacement and tilt angle, as the input beam is misaligned with respect to the measurement axis. An experimental setup to generate 11 OAM modes, $\ell \epsilon [-5, 5]$ was used to optically impose lateral displacement and tilt. We present the analytical expression and experimental results which show that our approach does correctly measure the OAM spectrum due to lateral and tilt angle misalignments of the input beam with some adjustments required.

1. Introduction

Orbital Angular Momentum (OAM) has been identified as another degree of freedom of light. A beam carrying OAM can be characterized with a complex amplitude term $\exp(i\ell\theta)$, where ℓ can take any integer value [1]. Such a beam is an OAM mode and Laguerre–Gaussian (LG) modes are one of the examples of such. OAM modes being orthogonal to each other can be used to multiplex communication channels and hence improving the channel capacity of free-space optical links [2]. Due to this orthogonality of OAM modes, they have been used for Mode Division Multiplexing (MDM).

In free-space optical links, a light beam is propagated through air to transfer information. Three primary atmospheric phenomena affect beam propagation namely absorption, scattering and turbulence. Absorption and scattering by gases and particles of the atmosphere that causes attenuation are wavelength dependent. The flow of turbulence induces lateral displacement and angle of arrival fluctuations (i.e tilt) as the fundamental abberations among many effects on the wavefront of beam [3, 4]. Lateral displacement and tilt manifest as a random beam movement at the receiver. Unfortunately, OAM mode propagating through turbulence spreads its energy to neighbouring modes. This phenomenon is known as OAM crosstalk causes errors at the receiver in FSO links.

In attempting to investigate the OAM crosstalk due lateral displacement and tilt, an analysis of OAM of a beam misaligned from a reference axis showed that a single OAM state becomes a superposition of a number of states [5]. A theoretical proof and numerical simulations showed the relationship of the dispersed OAM spectrum versus the tilt angle and lateral displacement but without experimental verification [6]. The change in the observed OAM spectrum was experimentally obtained as the input beam is misaligned with respect to the analyser [7].

In this work we look this topic from an alternative perspective. By assuming that lateral displacement and tilt angle are the main abberations of turbulence, we can simplify OAM in turbulence to OAM in lateral displacement and tilt angle. This work investigates the OAM spectrum due to lateral displacement and tilt angle. The experimentally obtained spectrum is compared with an OAM spectrum analytical expression. This investigation is necessary because allows a way to verify if the experimental setup conforms with the known theory about the relationship between lateral displacement and tilt angle with OAM spectrum. The measurements obtained will be used in the future to model the spectrum using machine learning instead of the analytical expression. The rest of this paper contains the background information, followed by the setup and approach used to obtain the measurement. Lastly, the preliminary result are shown of the spectrum when lateral displacement and tilt angles are imposed.

2. Background

2.1. OAM modes

OAM modes have intensities and phases profiles as depicted in Fig. 1. The spatial phase is twisted light spiral staircase. OAM beams are defined by the azimuthal index, ℓ, that can have any number of integer twists. When an OAM mode propagates through turbulence, its energy spreads to the neighbouring modes - OAM crosstalk. OAM modes are a subset of the Laguerre–Gauss (LG) modes that form an orthonormal basis in cylindrical coordinates.

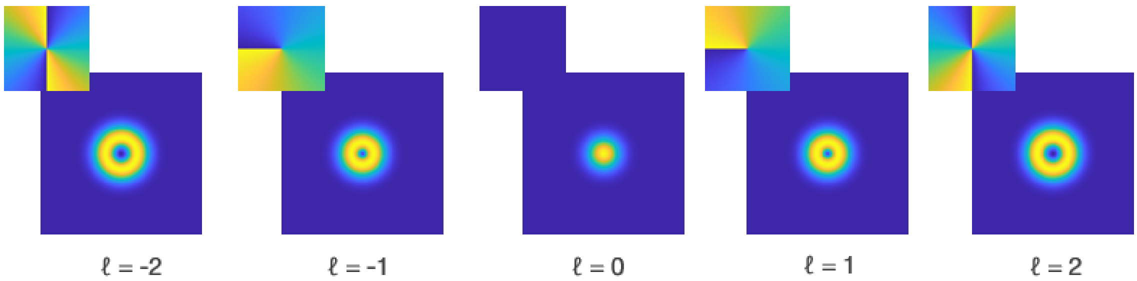


Figure 1: Intensity and phase (top left corner) of OAM beams with $\ell = 2$ to 2.

The field of an LG beam, with an azimuthal index, ℓ, and a radial index p, in a cylindrical coordinate system, is given by

$$U(r, \phi, z) = E_0(\sqrt{2}\frac{r}{\omega(z)})^{\ell} L_p^{\ell}(\frac{2r^2}{\omega(z)^2})\frac{\omega_0}{\omega(z)} \exp[-i\psi_{p\ell}(z)] \times \exp[i\frac{k}{2q(z)}r^2] \exp(i\ell\phi) \tag{1}$$

where $L_p^{\ell}(\cdot)$ is the associated Laguerre polynomial, E_0 is a constant electric field amplitude, $\omega(z)$ is the beam radius, ω_0 is the beam radius at the beam waist, $z_0 = \frac{\pi\omega_0^2}{\lambda}$ is the Rayleigh range, $q(z) = z - iz_0$, is the complex beam parameter and $\psi_{p\ell}(z) = (2p + |\ell| + 1)\tan^{-1}(\frac{z}{z_0})$ is the Gouy phase shift.

2.2. OAM in turbulence

As a Gaussian beam propagates through turbulence, three main effects that will take place:- beam spreading causing the beam to get bigger as it propagates, beam wander causing the centroid position of the beam to change at the receiver and scintillation resulting in the beam changing its shape. In addition to these effects, an OAM mode will spread its energy neighbouring modes - OAM crosstalk when propagating in turbulence. Atmospheric turbulence induces mainly tilt and lateral displacement abberations due to pressure fluctuations and temperature gradients. Fig 2 shows lateral displacement and tilt angle on a beam. Therefore, the system of OAM modes in turbulence will be simplified to OAM modes in tilt and lateral displacement since turbulence is stochastic in nature.

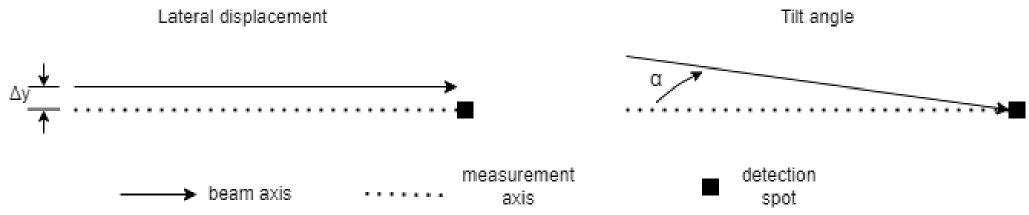


Figure 2: Illustration of lateral displacement and tilt angle. The beam axis is the axis the beam propagates through turbulence. The measurement axis is the axis at which the intensity of the beam is measured at the detector spot. Lateral displacement, Δy is the offset of the beam axis vertically (or horizontally) parallel to the measurement axis. Tilt, α is when the beam axis is at angle upon beam arrival at the detector spot. Therefore, lateral displacement and tilt causes the beams axis to be misaligned with the measurement axis.

2.3. Modal decomposition

Modal decomposition is a standard technique used to obtain the modal power spectra to fully characterize complex field distributions using a few mode weight coefficients. This technique can be used to reveal the information of crosstalk present in a particular LG field in an optical system. In modal decomposition an unknown optical field (the mode to be detected) is expressed as a linear combination of orthogonal basis functions.

$$U(x) = \sum_{\ell} c_{\ell} \Phi_{\ell}, \tag{2}$$

where U(x) is the unknown field at the receiver, ℓ are the mode indices, c_{ℓ} is the complex expansion coefficient and Φ_{ℓ} is the chosen basis function.

The main aim is to calculate the complex expansion coefficients, c_{ℓ}. With the aid of a digital hologram encoded on the spatial light modulator (SLM) and a Fourier lens the complex expansion coefficient measurements can be achieved. The coefficient can be extracted from the optical field intensity at the optical axis of the Fourier plane. This optical field intensity will be directly proportional to the c_{ℓ}^2.

3. Methodology

A standard modal decomposition setup with two SLMs was used to perform the experiment, depicted schematically in Fig 3. A 632.8 nm collimated Gaussian laser beam is transformed into the required mode using the first SLM. The required mode is imaged onto the second SLM which performs the modal decomposition to a camera at the Fourier plane of the decomposition lens.

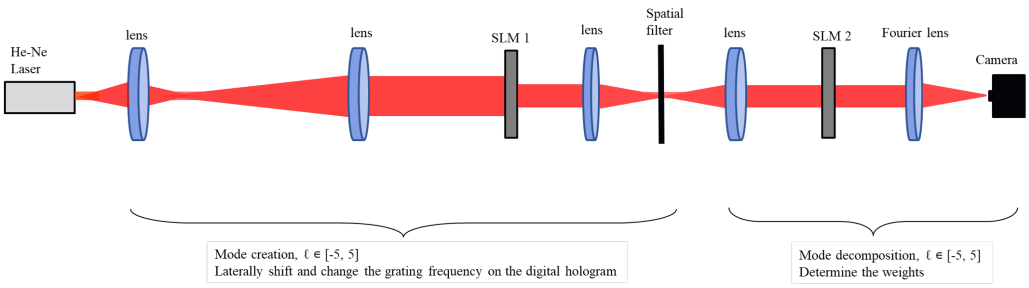


Figure 3: Schematic experimental setup: Lens before SLM 1 enlarges the beam. The first SLM is illuminated with horizontally polarized light and encoded with holograms that perform complex amplitude modulation and impose tilt angle and lateral displacement. An incoming LG mode from SLM 1 selected by the spatial filter is modally decomposed by the second SLM addressed by detecting hologram. At the Fourier plane the on-axis intensity is detected by the camera.

The first SLM was used to laterally displace and tilt the created mode. Fig 4 illustrates how the lateral displacement and tilt were optically imposed by using the first SLM. By offsetting the position of the created mode on the cartesian plane on the hologram, lateral displacement was imposed. Tilt angle was imposed on the created mode by varying the grating frequency of the blazed grating on hologram. The setup used, but not limited to 11 $LG^{\ell}_{p=0}$ modes, $\ell \,\epsilon\, [-5, 5]$.

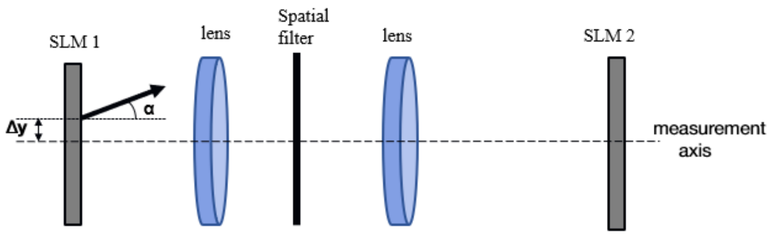
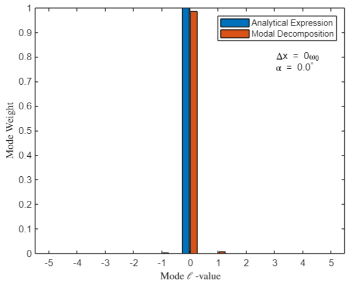


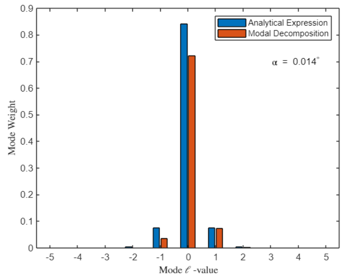
Figure 4: Experiment schematic of how the SLM will be used to impose both lateral displacement and tilt angle on the hologram on the first SLM. During mode creation, the center of the mode created is moved by changing the amount lateral displacement, $\Delta\ y$. The blazed grating on the first SLM will be varied to change the amount of tilt imposed. Both lateral displacement and tilt angle are with respect to the measurement axis. The measurement axis is determined before lateral displacement or tilt angle is imposed.

4. Preliminary results

Several different combinations of lateral displacement and tilt angle with respect to the measurement axis for OAM modes were experimentally imposed. The measured spectrum are similar to the spectrum calculated using the crosstalk analytical expression was as detailed in [6]. The spectrum for a mode index value of $\ell = 0$ was considered considered to make the comparison possible. The corresponding results are shown in Fig. 5 and 6. As expected individually imposing lateral displacement or tilt angle results in a spectrum symmetrical about the input mode [6]. As the amount of lateral displacement and tilt angle increases the spectrum broadens. Results shows that the mode, $\ell = 0$ is more sensitive to tilt angles than displacement. For future work, machine learning will be used to model the spectrum using these measurements instead of the analytical expression. The use of machine learning could model the spectrum in the presence of additional higher order abberations because of turbulence. Such a model might be useful for combating turbulence in MDM.



(a)



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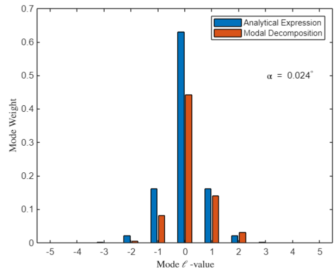

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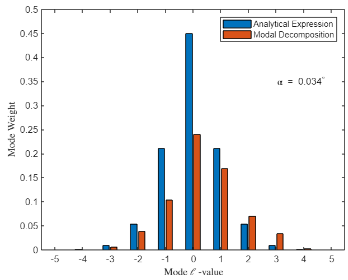

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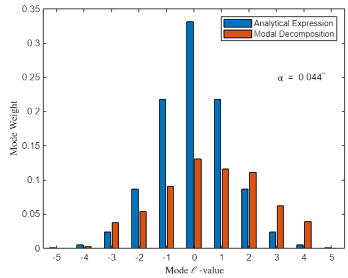
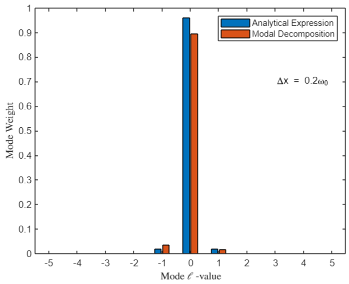


(e)

Figure 5: The spectrum over 11 states from $\ell = 5$ to 5 for tilt angles. A mode index value, $\ell = 0$ was the input mode. (a) shows the spectrum when the measurement axis is aligned with beam axis for a mode index value of $\ell = 0$ i.e no lateral displacement or tilt angle. (b)-(e) shows the experimentally obtained and crosstalk analytical expression spectrums when tilt angle imposed is increased the same tilt angle and input mode value.

However, there are mismatches in the spectrums probably due to the precision of the setup as indicated by the crosstalk present in the aligned setup as shown in Fig.5(a). The additional aberrations arising from the SLMs and the other optical elements used within the experimental setup as well as the orientation of the tilt angle produced by the first SLM.



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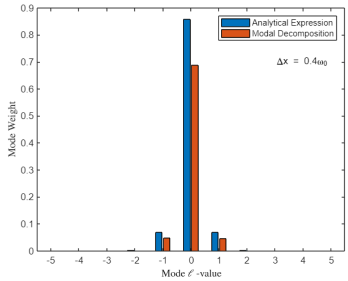

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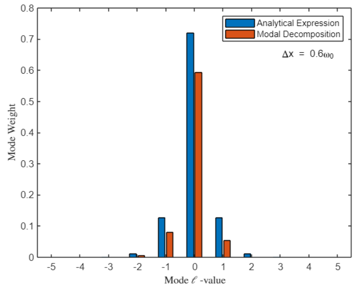

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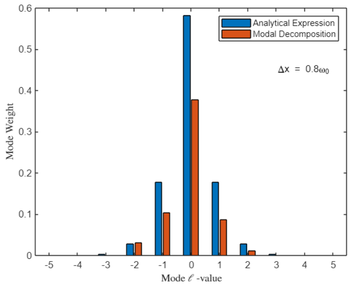

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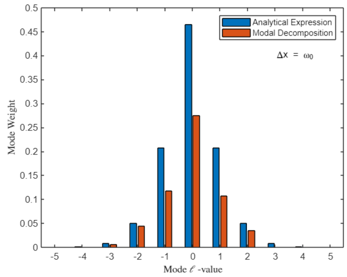

(e)

Figure 6: A case when lateral displacement imposed was increased. A mode index value, $\ell = 0$ of beam waist, $\omega_0 = 0.35$mm was the input mode. (a)-(b) shows the spectrum obtained from the experimental setup and crosstalk analytical expression for the same amount of lateral displacement.

Conclusion

The OAM spectrum of various beams, with lateral displacement and tilt with respect to the measurement axis were analysed. Results were presented on mode value of $\ell = 0$ but similar results were obtained for other ℓ values. These results corresponded to the spectra obtained from the crosstalk analytical expression, thereby confirming that our approach yields a useful measurement of the OAM spectrum. An example of this is for analysing the OAM crosstalk due to turbulence which may reveal interesting features within the OAM spectrum. Future work will be attempting to model the spectrum using machine learning and these measurements instead of the analytical expression. In the presence of additional higher order abberations machine learning model might be useful for mitigating the effects of turbulence in MDM.

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