Towards two-mode mode averaging with Orbital Angular Momentum modes

Alice V. Drozdov and Mitchell A. Cox*

School of Electrical and Information Engineering, University of the Witwatersrand, Johannesburg, South Africa, 2000

E-mail: mitchell.cox@wits.ac.za

Abstract. Higher order structured modes can be used for Mode Division Multiplexing (MDM) in Free Space Optical communication; however, they are highly susceptible to atmospheric turbulence. Atmospheric turbulence causes crosstalk between the modes which decreases the power within the transmitted mode. Mode averaging could assist in mitigating the effects of turbulence by transmitting and receiving two modes, between which crosstalk occurs. Mode averaging used at the receiver and transmitter simultaneously has been shown to increase the performance of a system when used with multiple Orbital Angular Momentum (OAM) modes. Using multiple OAM modes limits the number of modes available for MDM, therefore, it would be more favourable to make use of only two modes. Additionally, modes for mode averaging are often chosen for their simplicity. Choosing more appropriate modes could further improve performance. We investigate the use of OAM modes with un-normalised and normalised beam sizes in two-mode mode averaging by first determining the mode combinations that provide the most received power for each transmitted mode and then using these combinations to compare the total received power of a system using mode averaging to one without averaging. We show that mode averaging can increase the power received by a maximum of 61% at a Strehl Ratio of 0.6 when combining modes optimally.

1. Introduction

Free space optical (FSO) communication has become a popular research area due to the promise of fast, cheap and secure internet [1]. FSO transfers information using a laser beam which propagates through the air. This laser beam is usually in the form of a Gaussian or Gaussian-like beam.

There is an interesting alternative to using a Gaussian beam in FSO. Higher order mode sets can be perceived as "patterns of light" with different spatial intensity and phase structures. The Laguerre-Gaussian (LG) and Orbital Angular Momentum (OAM) mode sets are most commonly studied [2; 3]. Higher order modes can be used for Mode Division Multiplexing (MDM), which involves the superposition of orthogonal modes carrying different signals [4]. They are, however, highly susceptible to atmospheric turbulence [5]. As a mode passes through turbulence, turbulence distorts the wavefront and amplitude of the beam, causing crosstalk between different modes [6]. The crosstalk decreases the power and intensity of the transmitted mode. In communication systems, this causes fading errors and thus reduces the data-rate [7].

Multiple methods to prevent these fading errors have been researched. These include adaptive optics [8], spatial diversity [9] and multiple-input-multiple-output digital signal processing [7].

Alternatively, a method known as *Mode Diversity* can be used. By transmitting and/or receiving a number of adjacent modes between which crosstalk occurs, the errors caused by MDL can be mitigated [10]. Since "true" mode diversity as used in communication systems, harnesses the statistical independence of different channels over time [11], we believe that this name is confusing for this particular use case. *Mode Averaging* is more descriptive as there is a similarity to aperture averaging.

Mode averaging has been used at the receiver [12; 13] and transmitter [10] separately as well as at both simultaneously [14; 15]. In this work, we focus on mode averaging used at both the transmitter and receiver. Mode averaging using several OAM modes has shown to increase the performance of a system [14; 15]. However, using many OAM modes can reduce the number of modes available for MDM [16]. More modes could be provided by only using two mode combinations for mode averaging. Mode averaging systems could further be improved by choosing more appropriate modes for the mode combinations. This is in contrast to a simple multi-mode or "bucket"-type receiver. In many of the previous papers, the modes for mode combinations have mainly been chosen for their simplicity. However, in [17], modes are chosen using an exhaustive search and the effective fading figure. Modes could also be chosen by investigating the mode combinations which provide the maximal total received power.

Therefore, we determine the OAM mode combinations that provide the most received power at different turbulence strengths in two-mode mode averaging systems. These combinations are then used to compare the total received power of a system using mode averaging to one without averaging to determine the efficiency of such a system.

2. Preliminaries

2.1. Orbital Angular Momentum Modes

OAM modes are a subset of LG modes. LG modes are a set of higher-order modes that are solutions to the Helmholtz equation calculated using cylindrical coordinates [2]. The expression for LG modes at $z = 0$ is given by

$$\mathrm{LG}_p^\ell(\rho,\phi,0) = C_{l,p}^{\mathrm{LG}} \left(\frac{\rho\sqrt{2}}{w_0}\right)^{|\ell|} L_p^{|\ell|}\left[\frac{2\rho^2}{w^2(z)}\right] \exp\left(-\frac{\rho^2}{w^2(z)}\right) \exp[-i\ell\phi]. \tag{1}$$

where $C_{l,p}^{\mathrm{LG}}$ is the normalisation constant, $L_p^{|\ell|}[\cdot]$ is the generalised Laguerre polynomial and w_0 is the beam waist. LG modes are characterised by two indices namely the azimuthal (ℓ) and radial (p) indices. When $p = 0$, the modes are known as OAM modes. In OAM modes, the l parameter determines the beam propagation factor, M^2, which affects the beam's propagation characteristics [18], and is given by

$$M^2 = |\ell| + 1. \tag{2}$$

The diameter of the beam increases according to the second moment radius calculated using $r^2 = w_0\sqrt{1 + M^2}$ [19]. This diameter can be normalised in the near field using the following equation

$$w_L = \frac{w_0}{\sqrt{1 + M^2}} \tag{3}$$

where w_L is the radius of a beam with the beam size normalised to the size of the Gaussian beam within the mode set.

2.2. Atmospheric Turbulence

Atmospheric turbulence is caused as a result of the mixing of air with different temperatures and pressures, which causes random space and time-varying changes in the refractive index. Turbulence can be simulated by generating random phase screens using statistical models of which the simplest is the Kolmogorov model [20]. The power spectral density of the refractive index of air is described by

$$\Phi_n^K(\kappa) = 0.033 C_n^2 \kappa^{\frac{-11}{3}}. \tag{4}$$

In the equation, κ is the angular spatial frequency and C_n is the refractive-index structure parameter. Turbulence affects beams by distorting their wavefronts (and thus intensities, after propagation). In higher order modes, this causes the spreading of energy or coupling of power between different modes which is known as crosstalk. Crosstalk decreases the amount of power within the transmitted mode. This attenuation is known as Mode Dependent Loss (MDL) and is calculated as

$$\mathrm{MDL}_n = 1 - \frac{S_n}{S_0}. \tag{5}$$

where n represents the transmitted mode index, S_n is the intensity of mode n in the presence of turbulence and S_0 is the intensity of the mode in the absence of turbulence. The MDL is related to the Strehl Ratio (SR) which is a simple measure for turbulence strength defined by

$$SR = \frac{\langle I(\mathbf{0}) \rangle}{I_0(\mathbf{0})}, \tag{6}$$

where $\langle I(\mathbf{0}) \rangle$ is the average on-axis intensity of the beam with turbulence and $I_0(\mathbf{0})$ is the on-axis intensity without turbulence. The angle brackets, $\langle \cdot \rangle$, represent an ensemble average. The lower the SR, the greater the turbulence strength.

2.3. Modal Decomposition

Any field, U, can be expressed as a sum of modes that form part of a complete basis, given by

$$U(\boldsymbol{x}) = \sum_n c_n \Phi_n(\boldsymbol{x}), \tag{7}$$

where $\boldsymbol{x}$ represents the transverse spatial coordinates, and $\Phi_n(\boldsymbol{x})$ is the mode n within the complete basis [21]. c_n is the complex expansion coefficient for mode n of a particular basis where the amplitude of the expansion coefficient describes the degree to which the field correlates to the basis mode. The crosstalk and MDL measurements are given by this. The phase of c_n is called the inter-modal phase.

We can determine these expansion coefficients using the inner-product between the initial field and the modes of the complete basis. This so-called modal decomposition of the initial field is given by an inner product

$$c_n = \langle \Psi_n | U \rangle = \int d^2\boldsymbol{x} \Psi_n^*(\boldsymbol{x}) U(\boldsymbol{x}). \tag{8}$$

where $\Phi_n^*(\boldsymbol{x})$ is the complex conjugate of $\Phi_n(\boldsymbol{x})$.

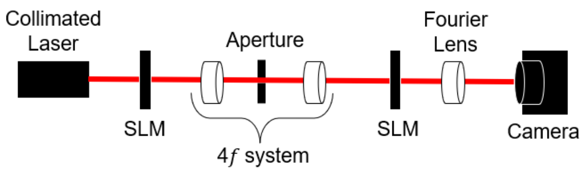

Figure 1. Experimental setup that is analogous to the simulation method used (SLM: Spatial Light Modulator)

3. Methodology

One of the key performance metrics in a communication system is the signal to noise ratio. If we assume the noise floor is some constant, the greater the received signal power the better the signal to noise ratio. Ignoring other potential complexities which are out of scope, in this work we strive to maximise the received power.

Due to the sheer quantity of data, the results presented here have been acquired using an accurate simulation. We plan to re-measure and verify interesting cases using a physical experimental setup in future. A setup that is analogous to the simulation method can be seen in Fig. 1. In the setup, the collimated laser beam is modulated with a mode combination using the first Spatial Light Modulator (SLM). The modes are combined such that the power within each mode is equal. For modes with normalised beam sizes, their beam sizes are normalised by using Eq. 3 to the size of the Gaussian beam before they are combined and used to modulate the laser beam. Simulated turbulence with turbulence strengths ranging from SR=0.1 to 0.9 in steps of 0.1 is also added to the laser beam using the first SLM. Turbulence is simulated using a hundred phase screens for each SR. A $4f$ system with an aperture is used to acquire the first diffraction order from the SLM. Modal Decomposition is performed using the second SLM and Fourier Lens [21]. The total received power is calculated using the modal decomposition results. The mode combinations which provide the largest received power for a particular SR are determined by combining all the modes two modes at a time and determining the received power for each combination for that SR. The best combinations are then used to compare a system with mode averaging to one without mode averaging (i.e. a single mode system).

The method used to determine the best mode combinations is computationally expensive and, thus, only allows for a limited number of OAM modes to be studied. The studied OAM modes have ℓ values ranging from -5 to 5 and M^2 values ranging from $M^2 = 1$ to 6.

4. Results and Discussion

In Fig. 2, the total received powers when a single un-normalised or normalised mode is transmitted and combined with other OAM modes at different turbulence strengths are shown. We can see that mode averaging can make a significant improvement on the received power. In Fig. 2(a), it can be seen by taking note of the asterisks (*) that un-normalised beams with a lower M^2 are best combined with adjacent beams (adjacent ℓ values) at lower SRs. This is, however, no longer the case for beams with a higher M^2 which are best combined with Gaussian beams as can be seen in Fig. 2(b).

At lower M^2, the crosstalk between modes is able to mitigate the MDL of the transmitted beam, however, the strength with which turbulence affects a beam is dependent on it's size [6]. Since the beam sizes are not normalised, turbulence will affect the beams with higher M^2 and, thus, larger diameters more. As a result more crosstalk will occur when the M^2 is high. Additionally, the crosstalk of modes with a higher M^2 will spread more [22]. Due to all of this, when the M^2 is high, the crosstalk of adjacent modes no longer has the ability to mitigate the effects of MDL. Gaussian beams already have a low MDL, therefore, when the M^2 of a transmitted beam is high it becomes more favourable to combine the beam with a Gaussian beam rather than the adjacent beam. At higher SRs, beams with a lower M^2 are also combined best with Gaussian beams (Fig. 2(a)). This is as a result of the increase in crosstalk that occurs as the turbulence strength increases [6].

On the other hand, beams with normalised beam sizes are best combined with adjacent beams. This is the case for all SRs and beams with any M^2 as can be seen in Fig. 2(c) and (d). Beams with normalised beam sizes are affected by turbulence similarly. This means that the MDL of all the beams is equal at a particular SR, therefore, when mode averaging is used, crosstalk between the beams is the only factor that will affect the total received power. Since the most crosstalk occurs between adjacent beams, the OAM modes are best combined with

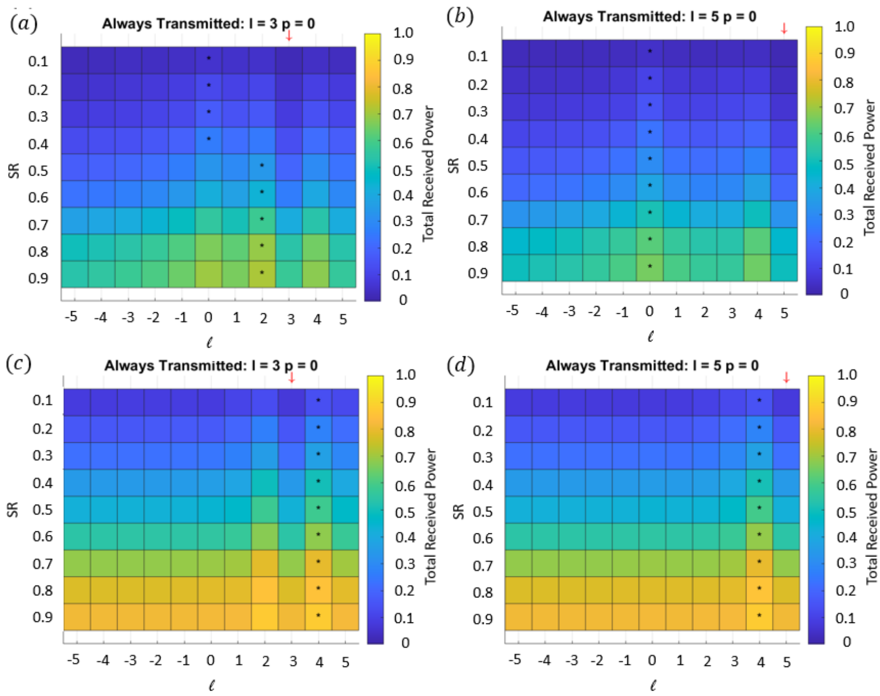

Figure 2. Total powers received when a single OAM beam is transmitted and combined with other beams. The mode combined with the transmitted mode which provides the largest received power is shown with an asterisk. In (*a*) and (*c*) the transmitted mode is $\ell = 3$ and has a lower M^2 whereas in (*b*) and (*d*) the transmitted mode is $\ell = 5$ and has a higher M^2. (*a*) and (*b*) show results for un-normalised beams whereas (*c*) and (*d*) show results for normalised beams.

adjacent beams.

Using the best mode combinations and comparing the total received power for different modes with and without mode averaging, it was found that mode averaging does improve the total received power. For OAM modes with un-normalised beam sizes at an SR of 0.6 it was found that there was a maximum increase of 61% in the power. Furthermore, for OAM modes with normalised beam sizes this increase was 23%. Un-normalised OAM modes benefit more from mode averaging than normalised OAM modes, however, this is mainly the case for modes with higher M^2. Nevertheless, the power received for un-normalised modes is always less than the power received for normalised modes in the case of both mode averaging and no mode averaging.

However, it must be noted that in this work, we did not take into account the divergence of the beam and its effect on the results. Although the results show that using a normalised beam size provides better performance than un-normalised beam sizes, this is likely to only be the case for short transmission distances. The beams with a normalised second moment radii will diverge to a greater extent than the beams with un-normalised radii as they have been reduced in size [19]. Additionally, beams with different M^2 will diverge at different rates. Since turbulence has a larger effect on larger beams and the turbulence will no longer affect all the normalised beams equally, at larger distances there is likely to no longer be an advantage to using beams with the same radii.

5. Conclusion

Mode averaging can mitigate the effects of turbulence on the total received power if the best mode combinations are used. When using OAM modes with un-normalised beam sizes, modes with a higher M^2 should be combined with a Gaussian beam. On the other hand, modes with a lower M^2 should be combined with its adjacent beam (adjacent ℓ values) at lower SRs and with Gaussian beams at higher SRs. When using OAM modes with normalised beam sizes, modes should always be combined with adjacent modes. There has been much research into the use of OAM modes in mode averaging, however, the work presented in this proceeding, is only preliminary work before studying the full LG basis.

References

[1] Trichili A, Cox M A, Ooi B S and Alouini M S 2020 *Journal of the Optical Society of America B* **37** A184–A201

[2] Allen L, Beijersbergen M W, Spreeuw R J C and Woerdman J P 1992 *Physical Review A* **45** 8185–8189

[3] Willner A E, Huang H, Yan Y, Ren Y, Ahmed N, Xie G, Bao C, Li L, Cao Y, Zhao Z and et al 2015 *Advances in Optics and Photonics* **7** 66

[4] Berdagué S and Facq P 1982 *Applied Optics* **21** 1950

[5] Cox M A, Mphuthi N, Nape I, Mashaba N P, Cheng L and Forbes A 2020 *IEEE Journal of Selected Topics in Quantum Electronics* **27** 1–21

[6] Anguita J A, Neifeld M A and Vasic B V 2008 *Applied Optics* **47** 2414

[7] Li S, Chen S, Gao C, Willner A E and Wang J 2018 *Optics Communications* **408** 68–81

[8] Weyrauch T and Vorontsov M A 2008 *Free-space laser communications with adaptive optics: Atmospheric compensation experiments* vol 2 (Springer) p 247–271

[9] Navidpour S, Uysal M and Kavehrad M 2007 *IEEE Transactions on Wireless Communications* **6** 2813–2819

[10] Wang A, Zhu L, Deng M, Lu B and Guo X 2021 *Optics Express* **29** 13171

[11] Cox M A, Cheng L, Rosales-Guzmán C and Forbes A 2018 *Physical Review Applied* **10**

[12] Song H, Li L, Pang K, Zhang R, Zou K, Zhao Z, Du J, Song H, Liu C, Cao Y, Willner A N, Bock R, Lynn B, Tur M and Willner A E 2019 *Optics InfoBase Conference Papers* **Part F160-OFC 2019** 10–12

[13] Song H, Li L, Pang K and Zhang R 2020 *Optics Letters* **45** 3042–3045

[14] Li L, Song H, Zhang R, Zhao Z, Liu C, Pang K, Song H, Du J, Willner A N, Almaiman A and et al 2021 *Optics Communications* **480** 126488

[15] Li L, Song H, Zhang R, Zhao Z, Liu C, Pang K, Song H, Du J, Willner A N, Almaiman A, Lynn B, Bock R, Tur M and Willner A E 2019 Demonstration of both mode and space diversity in a 100-Gbit/s QPSK free-space optical link to increase system tolerance to turbulence *Optics InfoBase Conference Papers* vol Part F160 pp 4–6 ISBN 9781943580538

[16] Trichili A, Rosales-Guzmán C, Dudley A, Ndagano B, Salem A B, Zghal M and Forbes A 2016 *Scientific Reports* **6** 27674

[17] Huang S, Mehrpoor G R and Safari M 2018 *IEEE Transactions on Communications* **66** 2079–2092

[18] Saghafi S and Sheppard C 1998 *Optics Communications* **153** 207–210

[19] Phillips R L and Andrews L C 1983 *Applied Optics* **22** 643

[20] Lane R G, Glindemann A and Dainty J C 1992 *Waves in Random Media* **2** 209–224

[21] Pinnell J, Nape I, Sephton B, Cox M A, Rodríguez-Fajardo V and Forbes A 2020 *Journal of the Optical Society of America A* **37**

[22] Mehrpoor G R, Safari M and Schmauss B 2015 *2015 4th International Workshop on Optical Wireless Communications, IWOW 2015* 78–82