Statistical Thermal Models for Particle Production in Heavy-Ion Collisions

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Abstract The number of various particles reported in relativistic heavy-ion collision research is consistent with the notion that they attain thermal equilibrium at temperatures substantially higher than those at which they kinetically freeze-out, which is a remarkable conclusion. This study attempts to explain this phenomenon by using statistical thermal models based on statistical mechanics' theories to simulate the behaviour, properties, and distribution of matter at extreme temperatures of microscopic matter. Additionally, the focus of the study is to apply statistical thermal models to determine how particle ratios and densities are influenced by temperature for particles produced in heavy-ion collisions. Statistical thermodynamics models are applied in the last stage of heavy ion collision, which is hypothesized to be in thermal equilibrium. The reason for this is that, as the temperature rises beyond 200 MeV, the quark-gluon plasma begins to form, and after the quark-gluon plasma forms, hadronization occurs, resulting in the production of elementary particles. The ratios of these elementary particles, kaons, pions, anti-protons and protons, were calculated and found to be in good agreement with the experimental results obtained from other studies. In conclusion, the study obtained the $\bar{P}/P$, K/π ratio, and u-quark and gluon densities plots as a function of temperature.

1. Introduction

Particle production in heavy-ion collisions is used to understand everything from nature's microstructure to its macrostructure, including how the universe was created. Due to the change in temperature, there is more matter than antimatter in the universe today. The temperature has decreased since the early universe, hence a decrease in antimatter [1]. One of the most difficult problems in physics is determining what happened to antimatter and why there is an asymmetry between matter and antimatter.

The theory of quantum chromodynamics (QCD) is the main theory that explains the interaction of matter in extreme conditions. The QCD phase diagram can be studied to understand the properties of the different phases of matter by studying the systems formed in relativistic heavy-ion collisions [2]. In particular, QCD predicts that in extreme conditions, such as high temperature or high baryonic density, a new phase of matter known as quark-gluon plasma (QGP) will form, in which degrees of freedom can be observed in a volume larger than the size of a single hadron [3, 4]

Lattice QCD calculations [5, 6] predict a phase transition to a quark-gluon plasma at around T = 170 MeV, which is equivalent to 1012 K. This extreme condition is thought to be similar to the early stages of the Universe's evolution following the Big Bang. Thus, studying the QGP's characteristics and how it evolves will allow us to probe the various stages of the Universe's expansion [7]. Statistical mechanics offers a more suitable theoretical framework for working with matter producing a large number of particles [8, 9]. Moreover, it provides a convenient way to relate macroscopic and microscopic worlds through computer simulations. The properties of strongly interacting nuclear matter under extreme conditions are important for understanding the early universe when it was just atoms right after the big bang.

This study uses statistical mechanics principles [10] and derives statistical thermal model equations for application in systems that are in thermal equilibrium. The Fermi-Dirac and Bose-Einstein statistical distribution equations are derived to perform detailed statistical thermal model calculations on matter and antimatter to evaluate particle production under extreme conditions in heavy ion collisions and to further understand the thermodynamic properties of macroscopic particles through the microscopic world [11,12]. The statistical thermal models together with statistical distributions are applied in the freeze-out stage of heavy ion collisions.

2. Methodology

In order to evaluate the properties of particles produced under extreme conditions in relativistic heavy-ion collisions, this study implements statistical thermal models based on statistical mechanics' theories. Since the heavy-ion collisions are relativistic systems, we derived the expression to evaluate the particle density of relativistic particles using Fermi-Dirac and Bose-Einstein statistics.

The number density of Fermi or Bose gas is given by:

$$dN = \frac{g d\tau}{e^{(\epsilon-\mu)/T} \mp 1}, \tag{1}$$

where g is the degeneracy of the particle, which is g = 2s + 1. The integration is over the phase space so

$$d\tau = \frac{dV}{(2\pi h)^3} dp_x dp_y dp_z. \tag{2}$$

To evaluate this integral, it is best to convert it to spherical coordinates and obtain the momentum particle density.

$$dN_p = \frac{1}{(2\pi\hbar)^3} \int_0^{2\pi} \int_0^{2\pi} \frac{gV}{e^{(\epsilon-\mu)/T} \mp 1} sin\theta d\theta d\varphi dp \tag{3}$$

$$= \frac{4\pi g V}{(2\pi\hbar)^3} \frac{p^2}{e^{(\epsilon-\mu)/T} \pm 1} dp \tag{4}$$

$$= \frac{1}{2\pi^2 \hbar^3} \frac{gVp^2}{e^{(\epsilon-\mu)/T} \pm 1} dp. \tag{5}$$

From here we only need to substitute the appropriate expression for the energy and integrate overall possible energy values. In the relativistic case we should use:

$$E^2 = p^2 c^2 + m^2 c^4. \tag{6}$$

Our expression becomes

$$dN_\epsilon = \frac{gV}{2\pi^2 (\hbar c)^3} \frac{\sqrt{\epsilon^2 + c^4 m^2} \epsilon d\epsilon}{e^{(\epsilon-\mu)/T} \pm 1} dp \tag{7}$$

Finally, to obtain the particle density we integrate the above equation, note that we have to start integrating from the rest mass energy since the energy was gained during its acceleration and this maintains the kinetic energy unless the speed changes.

$$\frac{N}{V} = \frac{g}{2\pi^2 (\hbar c)^3} \int_{mc^2}^{\infty} \frac{\sqrt{\epsilon^2 + c^4 m^2}}{e^{(\epsilon-\mu)/T} \pm 1} d\epsilon \tag{8}$$

We further simplified the particle density expression of relativistic particles into a dimensionless form, this was so that we can be able to evaluate the particle density of massless up-quark and massless gluon (which are ultra-relativistic particles). The expressions for the particle density of ultra-relativistic (Dimensionless) particles for the Fermi-Dirac is;

$$\frac{N}{V} = 1.808 \frac{gT^3}{\pi^2 (\hbar c)^3} \tag{9}$$

For the Bose-Einstein, the expressions for the particle density of ultra-relativistic (Dimensionless) particles is

$$\frac{N}{V} = 0.244\,\frac{gT^3}{\pi^2(\hbar c)^3} \tag{10}$$

Considering that derived expressions are not always analytically solvable, we used a computational approach using Python as the programming language since it is good for prototyping and has quick implementation code.

3. Results and Discussions

3.1 Particle ratio calculations

The quantity of various particles reported in relativistic heavy-ion collision research is consistent with the notion that they attain thermal equilibrium at temperatures substantially higher than those at which they kinetically freeze out, which is a remarkable conclusion. The analyses of particle production in heavy-ion collisions are used to understand everything from nature's microstructure to its macrostructure, including how the universe was created. Table 1 below, shows the calculated particle ratios of matter and antimatter at a temperature T = 174 MeV and the baryon chemical potential $\mu = 46$ MeV. The particle ratios are calculated with each particle's respective mass. The antiproton and proton have the same mass of 938.27 MeV, the pion negative has the same mass as the pion positive of 139.57 MeV, and the kaon negative and the kaon positive have the same mass of 493.68 MeV. The antiproton/proton obey Pauli-principle, and they have a degeneracy of two (2), which implies that they are fermion. Thus, the particle ratio of antiproton/proton was calculated using the Fermi-Dirac equation. The antiproton/pion negative, kaon negative/pion negative, and the kaon positive/pion positive use the Bose-Einstein equation since the particles do not obey Pauli-principle, they are bosons with a degeneracy of 1.

From table 3.1 below, the antiproton/proton particle ratio is higher compared to that of kaon/pion, this is because the kaon and pion are light particles and in most cases, the transverse flow parameter values of lighter particles are found to be smaller than those of the heavier particles (proton and antiproton). Unlike the kaon mesons, the pions are the lightest mesons and hadrons, and when in collision with the antiproton it produces the smallest particle ratio. The measured kaon/pion ratios show about a factor of two enhancement of particle production compared to antiproton/pion negative collisions at similar energies. It is clear that the kaon/pion ratio increases rapidly with an increase in strangeness and temperature. Comparing theoretical results from this study with experimental results from other studies, the slight difference in the antiproton/proton particle ratio results could be due to the method used since some parameters like volume were neglected. The kaon negative/pion negative particle ratio results from this study are higher than that obtained from other studies. This implies that at higher energies heavier mesons are produced more abundantly and, therefore, one also has to take into account the strangeness in other meson-meson collisions.

Table 3.1: The calculated particle ratios at T = 174 MeV and $\mu_B = 46$ MeV

Particle ratio	Theoretical data	Experimental [13]
$\bar{P}/P$	0.59	0.65 ± 0.07
$\bar{P}/\pi^-$	0.09	0.08 ± 0.01
π^-/π^+	1.00	1.0 ± 0.02
K^-/K^+	1.00	0.88 ± 0.05

K^-/π^-	0.23	0.149 ± 0.02

In the universe today there is more matter than antimatter, this is due to the change in temperature. Figure 3.1 shows the particle ratio of $\bar{P}/P$, which takes us back to the early universe when antimatter and matter existed in equal quantity. From the plots, it is observed that the $\bar{P}/P$ particle production increases with an increase in temperature. This is because protons and antiprotons usually produce other particles when they annihilate at rest, however their total kinetic and rest mass energies add up to twice the proton's rest mass energy. The plot of the $\bar{P}/P$ particle ratio shows high production compared to the $\bar{P}/\pi^-$ ratio, this is because the pions are light mesons, and when in collision with heavy baryons like the antiproton it produces the smallest particle ratio.

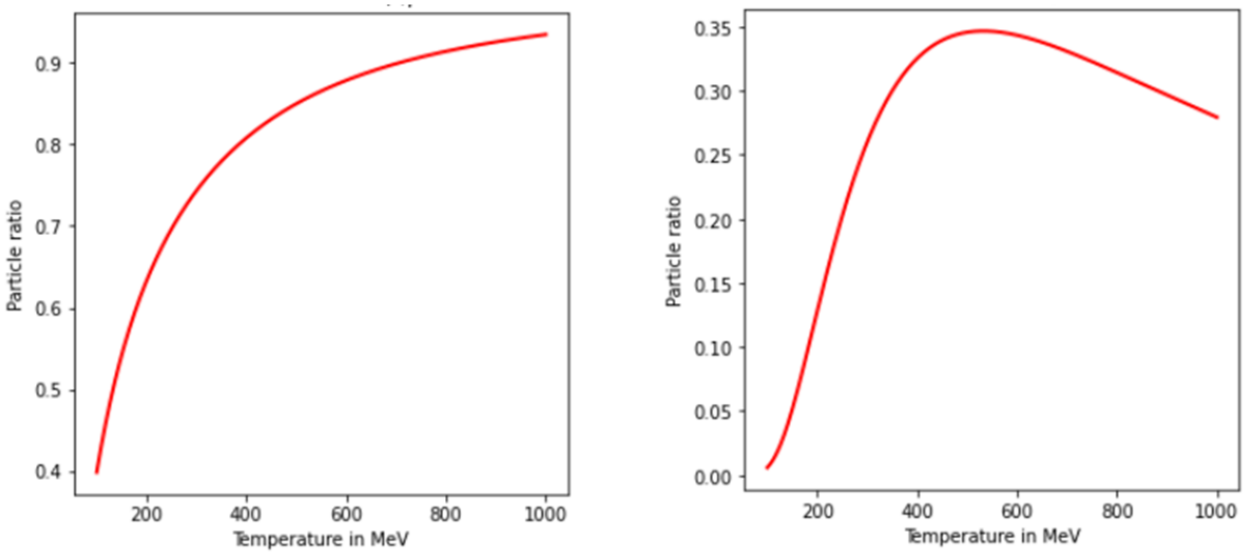

Figure 3.1: The particle ratio of (a) $\bar{P}/P$ and (b) $\bar{P}/\pi^-$ as a function of temperature.

In order to evaluate the strangeness production, the particle ratio of kaons and pions was calculated. The particle ratio plots of (a) K^-/π^- and (b) K^+/π^+ show similar behaviour, this is because the kaons and pions are fermions which implies that the particles are distributed similarly using Fermi-Dirac statistics. The plots show a horn-like strangeness production with an increase in temperature, which is consistent with results obtained from other studies [14].

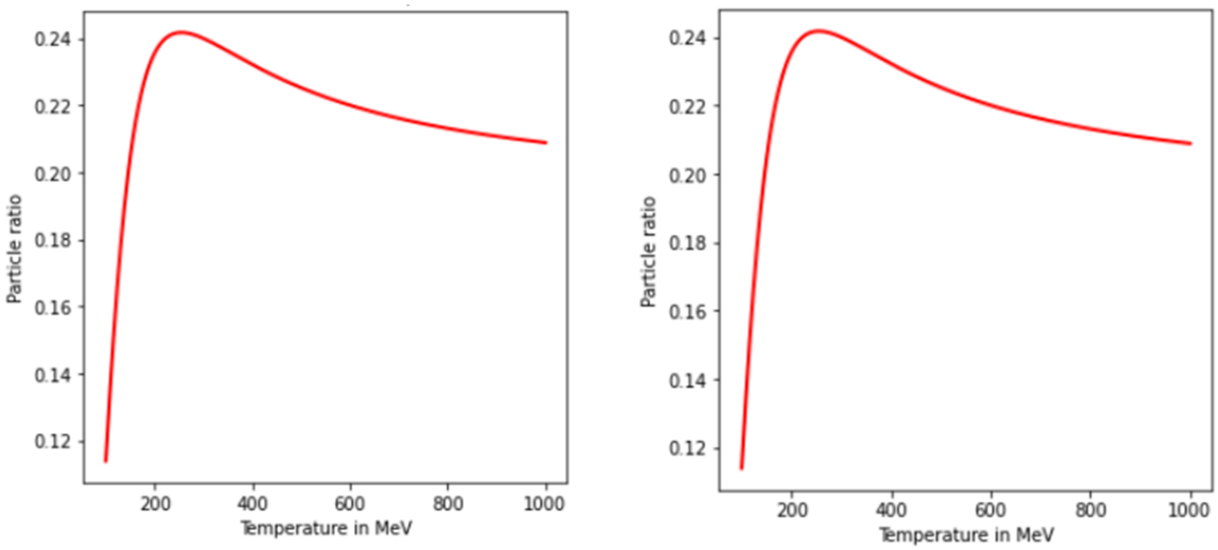

Figure 3.2: The particle ratio of (a) K^-/π^- and (b) K^+/π^+ as a function of temperature.

To determine the particle production of the QGP, the particle density of up-quark and gluon as a function of temperature were calculated. From the plots, the dotted lines indicate the early stage of production when the quarks have not yet formed when it was just kaon and proton for the gluon particle density plot. Hence, the density is very small on both plots. It is observed that the quark-gluon plasma starts to form as the temperature rises above 200MeV, whereby the hadronization occurs and elementary particles like protons and neutrons are formed.

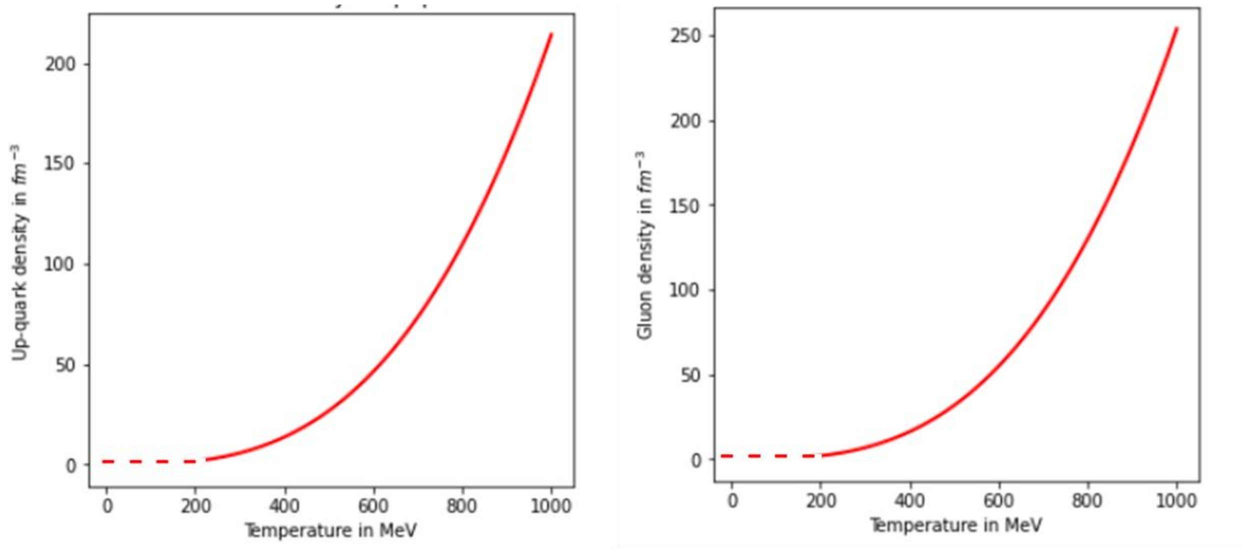

Figure 3.3: The particle density of (a) up-quark and (b) gluon as a function of temperature.

3.2 Particle density calculations

This section discusses the particle density of quarks and antiquarks particles as a function of baryon chemical potential at temperatures T = 200 MeV and T = 400 MeV. Figure 3.3 shows the plot of the particle density of the antiquark and quark particles. From the plots, it is observed that the particle density values at T = 400 MeV are much higher than at T = 200 MeV. This is because when the temperature is increased, the average velocity of the particles is increased. The average kinetic energy of these particles is also increased. The result is that the particles will collide more frequently because the particles move around faster and will encounter more reactant particles.

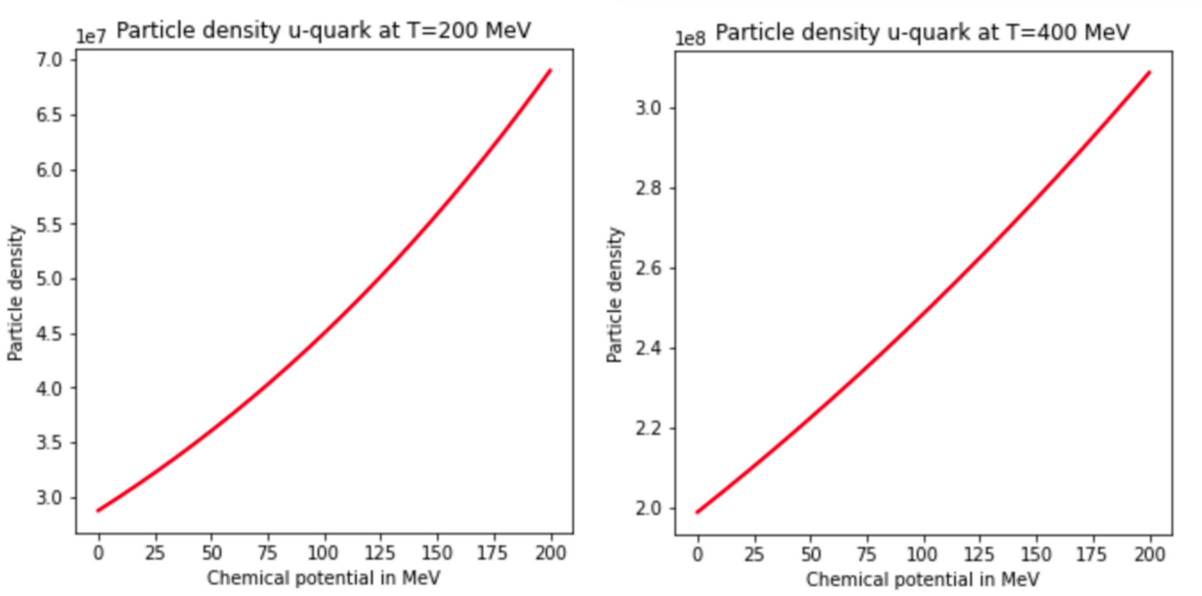

Figure 3.4: The plot of up-quark particle density as a function of the chemical potentials at T = 200 MeV and T = 400 MeV.

4. Summary and Conclusion

In the present study, we have performed detailed statistical thermal model calculations of matter and antimatter particles to evaluate the properties of the particles produced under extreme conditions in heavy ion collisions. The statistical distributions were derived and applied to matter and antimatter particles using thermal and statistical models. The K/π ratios are often used to study strangeness production enhancement. For heavy ion collisions, both the K^+/π^+ and K^-/π^- ratios are found to increase steadily with the collision energy. This behaviour is consistent with the increasing pair production rate as the collision energy increases. The measured K/π ratios at RHIC show a two-factor enhancement of particle production during collisions. The particle ratio calculations of K^+/π^+ and $\bar{p}$/p were found to be in good agreement with the experimental results from other studies. The quark-gluon plasma formed as the temperature rise beyond 200 MeV, and hadronization occurred which resulted in the production of elementary particles. The particle density of the antiquarks quarks particle was determined at temperatures T = 200MeV and T = 400MeV, as a function of baryon chemical potential. The findings provide a solid foundation for further evaluation of the particle density of antimatter and matter using more parameters like volume and pressure for a better understanding of the effect of temperature on the system as a function of chemical potential.

Conflicts of interest

The authors declare that they have no conflict of interest.

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