The QCD Equation of State in Small Systems

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Abstract. Multiparticle correlations measurements in even the smallest collision systems are consistent with predictions from viscous relativistic hydrodynamics calculations. However, these hydrodynamics calculations use a continuum extrapolated—i.e. infinite volume—equation of state. For the modest temperature probed in these small collisions, the controlling dimensionless product of the temperature and system size $T \times L \sim 400$ MeV$\times$2 fm/197 MeV fm ~ 4 is not particularly large. One should therefore investigate the small system size corrections to the equilibrium QCD equation of state used in modern viscous hydrodynamics simulations.

We present first results on just such finite system size corrections to the equation of state, trace anomaly, and speed of sound for two model systems: 1) free, massless scalar theory and 2) quenched QCD with periodic boundary conditions (PBC). We further present work-in-progress results for quenched QCD with Dirichlet boundary conditions.

We show that free, massless scalar fields, which are maximally sensitive to the finite size box, deviate enormously from their infinite volume conformal limit. Quenched QCD with PBC show corrections of $\sim 20\%$ for the trace anomaly near the phase transition. These corrections are more modest, but will have a meaningful, quantitative impact on the extracted bulk and shear viscosities in these small systems.

1. Introduction

In the theoretical and experimental study of heavy ion collisions, the interest is in understanding the non-trivial, emergent many-body dynamics of the strong nuclear force, quantum chromodynamics (QCD). What does that mean? The strong force is one of the four fundamental forces of nature: gravity, electromagnetism, the weak force, and the strong force. The particle physics of, e.g., the electromagnetic force is extremely well understood, with predictions of, e.g., the anomalous magnetic moment of the electron made out to the part per trillion level, in exact agreement with data [1]. However, when a large number of particles whose dynamics is given by the electromagnetic force interact, there are emergent properties. For example, and perhaps both most important and most famous, water is a substances whose properties are dictated by the electromagnetic force. A system with a large number of water molecules is no longer relevantly described by the position and momentum of the individual particles that make up the various water molecules. Rather, one uses quantities such as the temperature, pressure, entropy, heat capacity, conductivity, etc. to describe the system. We have a unique opportunity with the strong force to study both experimentally and theoretically the emergent many-body properties of a non-Abelian gauge theory.

Experimentally, in a heavy ion collision, a large nucleus such as ^{197}Au or ^{208}Pb, is collided with another large nucleus at nearly the speed of light. The centre-of-mass energy of a nucleon-

nucleon pair in one of these collisions is $\sim$ 200 GeV at the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory and $\sim$ 5 TeV at the Large Hadron Collider at CERN. These are macroscopically large energies; the total energy in a RHIC collision is about that of two mosquitoes colliding. Through Einstein's $E = mc^2$, $\mathcal{O}(10000)$ particles are generated in an LHC collision. These particles are measured in giant detectors such as PHENIX and STAR at RHIC and ALICE, ATLAS, and CMS at LHC.

2. Geometry and Flow

One of the crucial measurements made in heavy ion collisions at RHIC or LHC is the momentum distribution of "low" momentum particles. These are particles with momenta $\lesssim$ 2 GeV in the plane transverse to the axis defined by the collision direction of the nuclei. Experimentally, one may extract the Fourier decomposition of the momenta for these measured particles. The even moments of the decomposition dominate over the odd moments, and the second Fourier coefficient, v_2, dominates over all other moments.

One may interpret these Fourier coefficient data as follows. In a heavy ion collision, the nuclei will collide at some non-zero impact parameter. The geometrical anisotropy formed by this off-centre collision is then translated through collective motion into a momentum anisotropy. One naturally imagines that the collective motion is dictated by relativistic viscous hydrodynamics [2–5]; i.e. through energy-momentum conservation and an equation of state $e(p)$, see Eq. (2), given by the underlying theory, QCD. Detailed comparisons between the measured Fourier coefficients as a function of momentum and predictions from relativistic viscous hydrodynamics with a realistic equation of state computed from lattice QCD are in very good agreement for large collision systems [2–4].

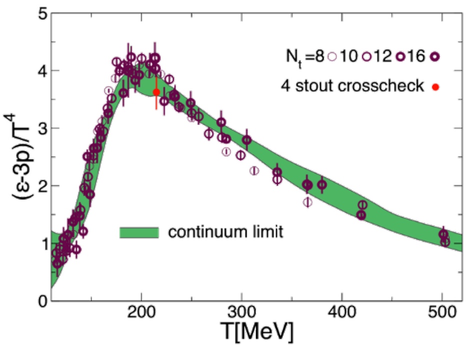

Figure 1. The trace anomaly Δ from lattice QCD, which is used to determine the equation of state $e(p)$ [6].

One can see in Eq. (2) that these hydrodynamics predictions describe data well not only for large nuclear collision systems but, shockingly, the predictions describe very well the measured collective motion even in the smallest collision systems in which protons are collided [4].

One then naturally is led to ask: what are the QCD predictions for the corrections to various thermodynamic quantities due to finite system sizes?

3. Finite System Size Corrections

Since QCD is a notoriously complicated theory, it's valuable to investigate the importance of finite system size corrections in simpler systems first. One may, for example, consider a massless scalar field theory that has certain dimensions constrained to lie between perfectly reflecting walls [7]. Interestingly, then, many of the thermodynamic variables as a function of system size

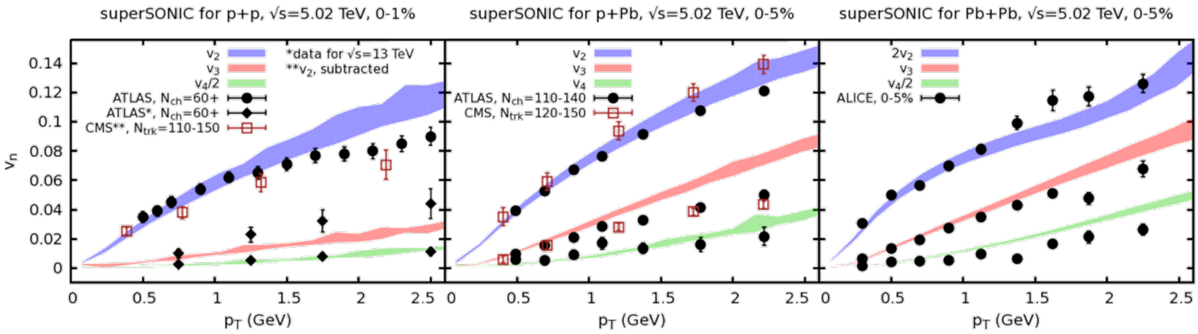

Figure 2. The measured Fourier coefficients v_n as a function of transverse momentum p_T compared to predictions from a viscous relativistic hydrodynamics calculation [4].

appear to mimic the usual temperature dependence of the phase transition predicted by lattice QCD in infinite volume systems [6]; see Eq. (3). One may compare the analytic results derived using scalar field theory to a full lattice QCD calculation that tested the sensitivity to system asymmetry [8]. One sees in Eq. (3) that the full lattice simulations show a qualitatively similar behavior as that predicted by the scalar theory.

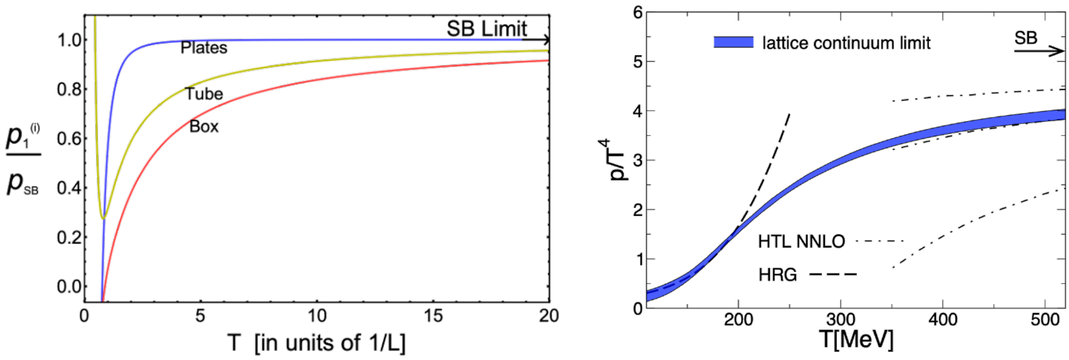

Figure 3. (Left) The pressure as a function of the dimensionless variable $T \times L$ for a massless scalar field confined between two infinite parallel plates divided by the infinite volume pressure, inside a symmetric tube of infinite length, or a symmetric box [7]. (Right) The dimensionless pressure divided by temperature to the fourth power as a function of the temperature as computed in lattice QCD [6].

One may then attempt to determine the sensitivity to the trace anomaly, which is the driving quantity needed for the equation of state. It turns out that the trace of the energy momentum tensor for a massless scalar field is identically zero, even when conformal invariance is broken by the system's boundary conditions. Therefore, analytically, the finite system size corrections to the trace anomaly can only come from finite system size corrections to the running coupling. In [9], an estimate of the finite system size corrections to the thermodynamic properties derived using thermal field theory in QCD [10] was found using the lattice running coupling for a scalar field [11]. We show the result of making such an estimate in Eq. (3). One may see that the trace anomaly $\Delta \equiv T^\mu_\mu$ decreases with system size. We may again compare the analytics to a full, quenched lattice QCD calculation [9]; we show a preliminary result of such a calculation in Eq. (3). The full numerics agree with our expectations: the trace anomaly Δ decreases with

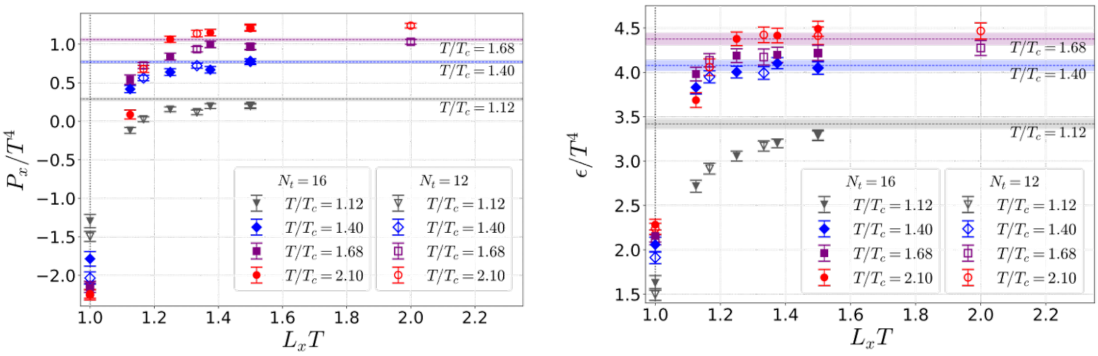

Figure 4. (Left) The pressure in the short direction divided by temperature to the fourth power as a function of the dimensionless variable $T \times L$ for quenched lattice QCD [8]. (Right) The dimensionless pressure divided by temperature to the fourth power as a function of the dimensionless variable $T \times L$ as computed in quenched lattice QCD [8].

system size, and, further, the phase transition is washed out by a decreasing system size.

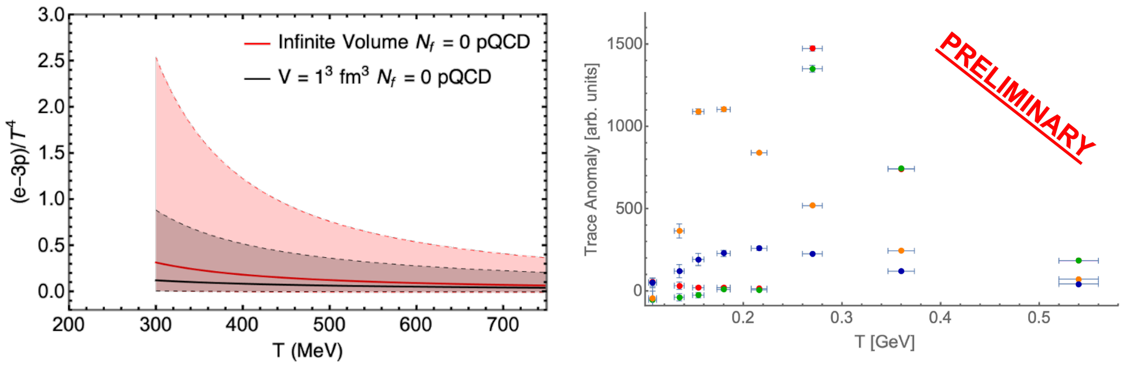

Figure 5. (Left) The trace anomaly from using a running coupling from lattice scalar field theory [11] in thermal QCD [9, 10]. (Right) A preliminary calculation of the trace anomaly in a finite-sized system in quenched lattice QCD [9].

4. Conclusions

We are interested in the fundamental research in the emergent dynamics of QCD. We live in an especially interesting time for such research, as the community is able to simultaneously investigate this physics theoretically and experimentally. Detailed comparison of data with viscous relativistic hydrodynamics predictions imply that there is near perfect fluidity in the aftermath of large nuclear collisions. Surprisingly, this near perfect fluidity is also seen in the smallest hadronic collision systems. Since $T \times L$ is not small in these smallest systems, we must ask: what are the finite system size corrections to the inputs into the hydrodynamic simulations that are being compared to data?

We have shown in this proceedings that finite size effects can, in fact, be large for the thermodynamic properties of quantum field theories at sizes relevant for hadronic collisions at RHIC and LHC. In particular, there is likely a non-trivial impact on the extraction of the QCD equation of state from data and/or conclusions for the viscosity to entropy density ratio determined by comparing experimental measurements to relativistic hydrodynamics predictions.

In future work, we look forward to explicitly computing the finite size effects in the QCD running coupling, refining finite size lattice calculations with periodic boundary conditions, and implementing more realistic Dirichlet boundary conditions in lattice calculations. In all of these future works, maintaining gauge invariance will be a challenge. As a first step towards computing the finite size corrections to the running coupling, a calculation of the next-to-leading order contribution to $2 \to 2$ scattering in ϕ^4 theory has been carried out [12]. In the higher momentum regime, parton energy loss provides another tool for investigating the properties of the quark-gluon plasma generated in heavy ion collisions [13]. Recent work computed the finite system size corrections to energy loss [14]; it will be interesting to see these finite system size effects propagated through to a full energy loss model.

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