Matters of the $R_h = ct$ universe

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Abstract. Decades of astronomical observations have shown that the standard model of cosmology based on General Relativity - the closest we have to a standard theory of gravitation - does not adequately describe our universe without the *ad hoc* introduction of dark matter and dark energy to late-time cosmology and inflation to early-universe cosmology. This certainly has created dilemmas in the cosmology, and the wider astronomy, community, and several alternative models of cosmology and gravitation are being considered at the moment. Here I will give a brief overview of the cosmological dynamics of the $R_h = ct$ universe in the framework of non-standard forms of matter and gravitation.

1. Introduction

Modern cosmology based on Einstein's General Relativity (GR) theory in a maximally symmetric spacetime, the Friedmann-Lemaître-Robertson-Walker (FLRW) metric, describing a universe that is homogeneous (all regions of space look alike, no preferred positions) and isotropic (no preferred directions). The recent discovery of the accelerated rate of cosmic expansion has inspired a wave of new research into the nature of gravitational physics. Whereas it is not conclusively known what caused this recent cosmic acceleration, the prevailing argument is that dark energy caused it. And new alternatives to dark energy and/or generalisations to GR abound already, such as:

- Adding extra matter fields, e.g, scalar-tensor models, Modified Newtonian Dynamics (MOND)
- Adding extra dimensions, e.g., Kaluza-Klein, Gauss-Bonnet models
- Higher-order theories, e.g., Hořava-Lifshitz, $f(R)$ gravity theories

Melia's $R_h = ct$ spacetime model [1] is one such attempt to solve existing cosmological dilemmas that ΛCDM has not resolved through a simple manipulation of the field equations in the existing GR framework, i.e., without necessarily needed any modification or generalisation of the existing gravitational action.

1.1. Matters of gravity

Relativistic physics treats gravity as a manifestation of spacetime curvature. There are three different geometrical representations of spacetime curvature [2] leading to three possible gravitational interpretations.

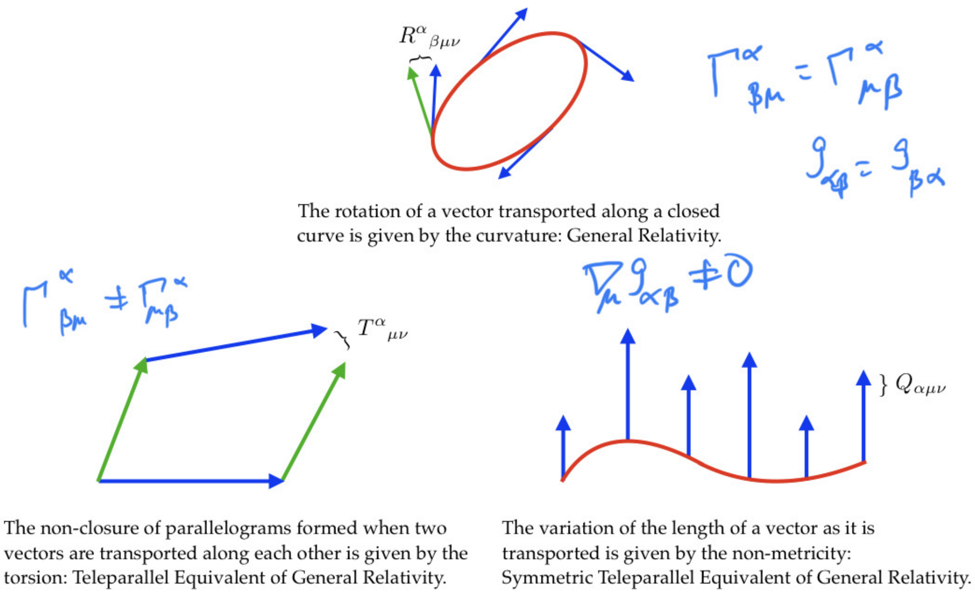

Figure 1. The geometrical meaning of curvature, torsion and non-metricity. Adapted from Jimenez et al [2].

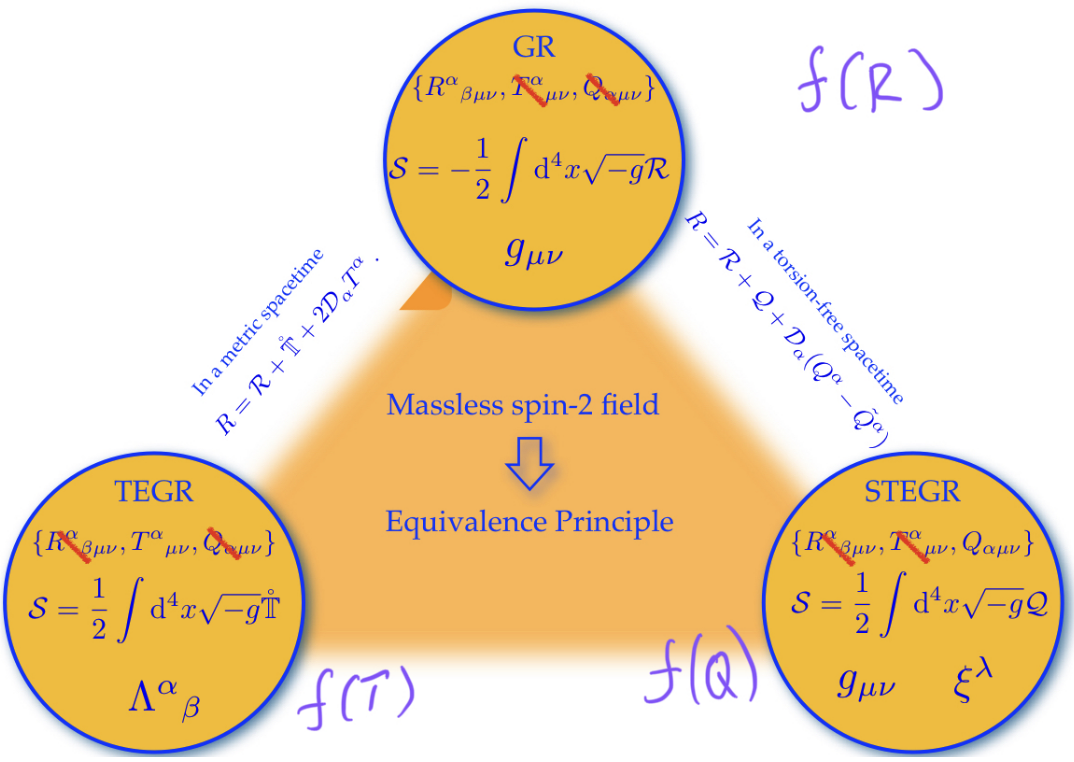

Figure 2. Three alternative gravitational descriptions defined in terms of the Ricci scalar R, torsion scalar T and non-metricity scalar Q, and their modified $f(R)$, $f(T)$ and $f(Q)$ generalisations. Adapted from Jimenez et al [2].

1.2. Standard cosmology

The standard (or "concordance") cosmological model is based on the GR description of gravitation the action of which is given by [1]:

$$S_{GR} = \frac{1}{2}\int d^4x\sqrt{-g}\left[R + 2\left(L_m - \Lambda\right)\right] . \tag{1}$$

The Corresponding Einstein's field equations:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu} , \tag{2}$$

with the first (geometric) term represented by the Einstein tensor, and the RHS of the equation representing the energy-momentum tensor of matter fluid forms. The other two descriptions of gravity (TEGR and STEGR) describe similar background expansion history provided:

- In TEGR theory
$$T = -6H^2 , \tag{3}$$
- In the STEGR theory
$$Q = 6H^2 . \tag{4}$$

Flat FLRW in GR has $R = 6H^2$. The basic evolution equation for the scale factor $a(t)$ of the universe, obtained from the above field equations, is given by the Raychaudhuri equation

$$3\frac{\ddot{a}}{a} = -\frac{1}{2}(\rho + 3p) , \tag{5}$$

where ρ and p are the matter energy density and isotropic pressure, respectively. For ordinary matter, the "active gravitational mass" (AGM)

$$\rho + 3p > 0 . \tag{6}$$

This means that ordinary matter will tend to cause the universe to decelerate, *i.e.,* $\ddot{a} < 0$. But astronomical observations show that cosmic expansion is accelerating in recent epoch, *i.e.,* $\ddot{a} > 0$. A positive cosmological constant causes an accelerated expansion, so Λ could be a quick fix provided that it dominates in the right-hand side of the following equation:

$$3\frac{\ddot{a}}{a} = -\frac{1}{2}(\rho + 3p) + \Lambda . \tag{7}$$

Some serious problems associated with the cosmological constant, among them the eponymous *cosmological constant problem* [3] and the *coincidence problem* [4] challenge the choice of Λ as a dark energy candidate.

The Big Bang model is by far the most successful theory in predicting many cosmological features confirmed observationally expansion of space, origin of the cosmic microwave background (CMB), Big Bang nucleosynthesis, galactic evolution and distribution, and the formation of large-scale structures. However, there are still, broadly speaking, two serious puzzles that remain unanswered in the standard Big Bang-based cosmology:

- Early-universe problems: horizon, flatness, structure/smoothness/homogeneity, and magnetic-monopole problems. The suggested solution here is cosmic inflation, an early-epoch cosmic acceleration.

[1] We have used $8\pi G = 1 = c$

- Late-time (large-scale) problems: rotational curves of galaxies plus large-scale structure formation, and late-time cosmic acceleration. This is usually where the dark stuff comes in. The former aspect is generally thought to be solved by introducing dark matter, and the latter by dark energy (and often with the cosmological constant as the candidate for the dark energy).

One other puzzling, but not as widely explored, problem is the $R_h(t_0) = ct_0$ coincidence (often referred to as the synchronicity problem) [2]: the observed equality of our gravitational horizon $R_h(t_0)$ with the distance ct_0 light has travelled since the Big Bang in ΛCDM cosmology.

Some potential new frontiers to address the above issues include

- Inhomogeneous cosmological models, such as LTB and Szkeres cosmologies
- Anisotropic cosmological models, such as Bianchi cosmologies
- Changing [fundamental] 'constants', such as evolving Λ and G cosmologies
- Modification/generalization of GR, such as $f(R)$, $f(T)$, and $f(Q)$ models of gravitation, just to mention a few.

In addition, the $R_h = ct$ model has recently joined the fray as a potential cosmological model to address the synchronicity problem.

2. The $R_h = ct$ universe

First proposed by Melia, the controversial $R_h = ct$ model has the following characteristics [1, 5, 6]:

$$R_h = \frac{c}{H} = ct \text{ , gravitational radius=Hubble radius} \tag{8}$$

$$a = t/t_0 \tag{9}$$

$$H = 1/t \Rightarrow Ht = 1 \quad \forall t \tag{10}$$

This follows from the imposition of a vanishing total AGM

$$\rho + 3p = 0 \tag{11}$$

in the gravitational theory. No cosmological constant Λ as the source of dark energy is needed in this model.

Among other things, this model resolves the so-called synchronicity problem: why is it true that today, the dimensionless age of the universe

$$H_0 t_0 \sim 1? \tag{12}$$

Problem: for standard forms of matter, $\rho + 3p > 0$. So how can one get a vanishing AGM? This is where modified models of gravitation come in as they naturally provide the extra "curvature", "torsion", etc. . . fluid terms that contribute to the total AGM.

2.1. Solutions from $f(R)$ gravity

$f(R)$ models are a sub-class of *fourth-order* theories of gravitation:

$$S_{f(R)} = \frac{1}{2} \int d^4x \sqrt{-g} \left[f(R) + 2L_m \right] \ , \tag{13}$$

[2] Here t_0 denotes the age of the universe since the Big Bang.

with corresponding generalized Einstein field equations:

$$f'G_{\mu\nu} = T^m_{\mu\nu} + \frac{1}{2}(f - Rf')g_{\mu\nu} + \nabla_\nu\nabla_\mu f' - g_{\mu\nu}\nabla_\gamma\nabla^\gamma f' \ . \tag{14}$$

Here and in the following, f , f' , etc... are shorthands for the arbitrary function $f(R)$ of the Ricci scalar R and its first, etc... derivatives with respect to R. The sub/superscripts m and R stand for the standard matter and curvature components, respectively. The background curvature- fluid thermodynamics can be represented as:

$$\rho_R = \frac{1}{f'}\left[\frac{1}{2}(Rf' - f) - \Theta f''\dot{R}\right] \ ,$$
$$p_R = \frac{1}{f'}\left[\frac{1}{2}(f - Rf') + f''\ddot{R} + f'''\dot{R}^2 + \frac{2}{3}\Theta f''\dot{R}\right] \ ,$$

whereas the total energy density and pressure terms can be given by

$$\rho \equiv \frac{\rho_m}{f'} + \rho_R \ , \quad p \equiv \frac{p_m}{f'} + p_R \ . \tag{15}$$

Imposing the vanishing AGM condition in $f(R)$ theories with energy and pressure terms as defined above, together with the flat-FLRW relation $R = 6H^2$ and matter solution for the $R_h = ct$ case

$$\rho_m = \frac{\rho_{m0}}{a^{3(1+w)}} = 3H_0^2\Omega_{m0}\left(t_0/t\right)^{3(1+w)} = \alpha R^{3(1+w)/2} \ , \tag{16}$$

where $\alpha \equiv 3t_0^{1+3w}\Omega_{m0}6^{-3(1+w)/2}$, we get the following second-order ordinary differential equation (ode) in f:

$$2R^2 f'' - f + 2\alpha R^{3(1+w)/2} = 0 \ . \tag{17}$$

Here we have used the fact that $\dot{R} = -\frac{2}{3}R\Theta = -\sqrt{\frac{2}{3}}R^{3/2}$. Solving (17) gives

$$f(R) = c_1 R^{\frac{1+\sqrt{3}}{2}} + c_2 R^{\frac{1-\sqrt{3}}{2}} - \frac{4\alpha}{1 + 12w + 9w^2}R^{\frac{3(1+w)}{2}} \ . \tag{18}$$

This same solution was obtained in [7]. This exact $f(R)$ solution was found [8] as a counter-example of the shear-free conjecture [9], i.e., describes a universe with simultaneous expansion and rotation. This then implies that the $R_h = ct$ universe is a simultaneously expanding and rotating spacetime solution! If one compares this result with the GR general condition [10], the $R_h = ct$ universe with $\rho + 3p = 0 \Rightarrow p = -\frac{\rho}{3} \Rightarrow c_s^2 = -1/3$, which is not an allowed solution for standard matter forms. This is an apparent contradiction and needs further investigation.

2.2. Solutions from $f(T)$ gravity

In the torsion (TEGR) interpretation of gravitation:

$$S_{f(T)} = \frac{1}{2}\int d^4x\sqrt{-g}\left[f(T) + 2L_m\right] \ , \tag{19}$$

and the background curvature and total-fluid thermodynamics:

$$\rho_T = \frac{1}{f'}\left[\frac{1}{2}(Tf' - f)\right] \ ,$$
$$p_T = \frac{1}{f'}\left[\frac{1}{2}(f - Tf') + 2Hf''\dot{T}\right] \ ,$$

$$\rho \equiv \frac{\rho_m}{f'} + \rho_T = \frac{\rho_m}{f'} + \frac{1}{2f'}\left(Tf' - f\right) \ , \quad p \equiv \frac{p_m}{f'} + p_T = \frac{p_m}{f'} + \frac{1}{2f'}(f - Tf') + \frac{2Hf''\dot{T}}{f'} \ . \quad (20)$$

The vanishing AGM condition leads to the ode

$$2T^2 f'' - Tf' + f + (1+3w)\alpha(-T)^{\frac{3(1+w)}{2}} = 0 \ , \quad (21)$$

the solution of which is given by

$$f(T) = c_3 T + c_4\sqrt{T} - \frac{2\alpha}{2+3w}(-T)^{\frac{3(1+w)}{2}} \ . \quad (22)$$

2.3. Solutions from $f(Q)$ gravity

In the STEGR interpretation of gravitation:

$$S_{f(Q)} = \frac{1}{2}\int d^4x\sqrt{-g}\left[f(Q) + 2L_m\right] \ . \quad (23)$$

The background non-metric and total-fluid thermodynamics are given by

$$\rho_Q = 3H^2 f' - f/2 = \frac{1}{2f'}(Qf' - f)$$

$$p_Q = f/2 - \left(3H^2 + \frac{f''H\dot{Q}}{f'}\right)f' = \frac{1}{f'}\left[\frac{1}{2}(f - Qf') + 2Hf''\dot{Q}\right] \ ,$$

$$\rho \equiv \frac{\rho_m}{f'} + \rho_Q \ , \quad p \equiv \frac{p_m}{f'} + p_Q \quad (24)$$

The vanishing AGM condition leads to the ode

$$2Q^2 f'' - Qf' + f + (1+3w)\alpha Q^{\frac{3(1+w)}{2}} = 0 \ , \quad (25)$$

the solution of which is given by

$$f(Q) = c_5 Q + c_6\sqrt{Q} - \frac{2\alpha}{2+3w}Q^{\frac{3(1+w)}{2}} \ . \quad (26)$$

3. Conclusion and outlook

The $R_h = ct$ universe is a possible cosmological alternative to ΛCDM in explaining certain issues such as the synchronicity problem. However, there are no sources of normal matter that satisfy the vanishing AGM $\rho + 3p = 0$ condition within the standard GR framework. We have demonstrated that his condition can be obtained from $f(R)$, $f(T)$ and $f(Q)$ gravity models provided that the gravity Lagrangians are as given by Eqs. 18, 22 and 26. In particular, we have shown that the $f(R)$ solution that satisfies vanishing AGM corresponds to the shear-free solution that allows a simultaneously expanding and rotating universe. Further work on other cosmological aspects, such as perturbations, needs to be done to scrutinise these solutions for cosmological viability

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