The 5D MSSM at two loops

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Abstract. Examining the evolution equations of the couplings and soft-terms derived from the two-loop renormalisation group equations in a five-dimensional minimal supersymmetric model compactified on an S_1/Z_2 orbifold (which yields the standard four space-time dimensions), different possibilities can be discussed. In this work we shall consider the limiting case of superfields where the Standard Model matter fields are restricted to the brane. We will compare our two-loop results to the results found at one-loop level, where in this model the power law running in five dimensions and a compactification scale in the $10-10^3$ TeV range has significant effects on the running. We also show that the gluino mass may drive a large enough A_t to reproduce the measured Higgs mass of 125 GeV and have a light stop superpartner below $\sim$1 TeV, as is preferred by the fine tuning argument for the Higgs mass.

1. Introduction

As one of the most successful results of the LHC experiment, the discovery of the Higgs boson, with a mass of about 125 GeV [1, 2] has spawned many new models beyond the Standard Model (SM) as its properties are probed more and more [3]. Amongst the researches into supersymmetry (SUSY) breaking models in the framework of the Minimal Supersymmetric SM (MSSM) recent results have been encouraging. However, the origin of SUSY breaking remains the main unknown ingredient in supersymmetric theories [4, 5]. Another provocative extension to the SM is that of extra dimensional theories, where these may provide a new mechanisms by which to break SUSY and to solve the supersymmetric flavour problem [6]. The simplest amongst these are five dimensional (5D) theories, which when viewed as an effective four dimensional (4D) field theory with a cutoff are non-renormalisable, as many parameters such as gauge couplings, Yukawa couplings etc. are sensitive to this cutoff scale [7]. 5D MSSMs, with their power law running for a sufficiently low compactification radius R, generate at low energies a large enough trilinear soft breaking term A_t to explain the observed Higgs mass [7].

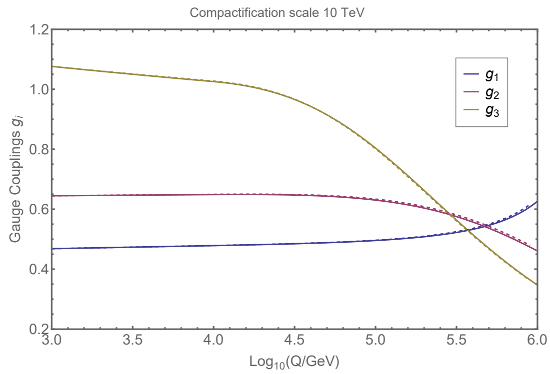

Figure 1. Running of the inverse fine structure constants $\alpha_i^{-1}(Q)$, for three different models with compactification scale 10 TeV as a function of $\text{Log}_{10}(\text{Q/GeV})$.

Using this 5D MSSM structure, one might be concerned that one-loop linear sensitivity to the cutoff, behaving as ΛR, does not result in terms of the form $(\Lambda R)^2$ at two-loop. As such it is important to confirm the results and conclusions made at one loop, that are sensitive to this scale, are still consistent and under control at two loops [8]. This is the aim of this current proceeding, to begin the calculation and analysis of the two-loop contributions in such a model. Therefore, the structure of this paper is as follows: In section 2 we outline the five dimensional theory we will study here, in section 3 we explore the model, and in section 4 we present the the calculation of the two-loop renormalisation group equations (RGEs). In section 5 we conclude.

2. The Five Dimensional MSSM

We define the 5D MSSM to be the field theory in 4D space-time times an interval of length R in which the gauge fields and the Higgses (H_u, H_d) propagate in the fifth dimension, y, whilst the matter fields in our model are restricted to the brane at $y = 0$. Therefore there are no contributions from Kaluza-Klein (KK) excited state of the fermions on the brane. The compactification of bulk fields produces towers of new particle states in the 4D theory at $Q > 1/R$.

Even though gauge mediation is preferred for supersymmetry breaking (and some recent work on gauge mediated supersymmetry breaking (GMSB) in a five-dimensional context can be found in Refs. [4], the universality of squark massses in GMSB ultimately means that even though the gaugino mediated limit [9] might allow for light squarks (and 5D RGE evolution allows for a large A_t and the observed Higgs mass). We assume here that SUSY breaking occurs at the unification scale, which is found by determining the scale at which $g_1 = g_2 = g_3$, which is lower when compared to the 4D MSSM due to the effects of compactification, as seen in figure 1. As a result, we will be rather agnostic about the precise details of how SUSY is broken in this proceeding, and our conclusions will be quite general. We do, however, have some minimum requirements:

i) We use the Yukawa and gauge couplings at the SUSY scale (1 TeV) as inputs.
ii) We specify the value of the gluino mass, M_3 at 1 TeV.
iii) We take the trilinear soft breaking terms, $A_{u/d/e}$, to vanish at the unification scale.
iv) We specified μ, B_μ and the value of the sfermion masses at 1 TeV.

An interesting feature of the 5D MSSM is the approximate unification of gauge couplings [7, 10], which are here calculated to two-loop and presented in figure 1.

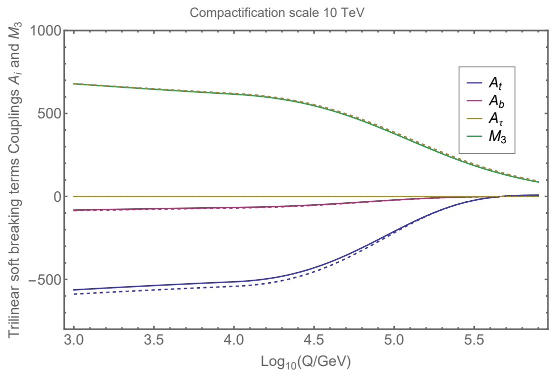

Figure 2. Running of the trilinear couplings $A_i(Q)$, for two different models with compactification scale 10 TeV, as a function of $\text{Log}_{10}(\text{Q/GeV})$.

3. The Higgs mass at two-loop

To allow for the correct Higgs mass $m_h = 125$ GeV, the electroweak parameters should be in the range

$$\tan\beta \subset (5, 30), \quad \mu \leq 1\text{TeV}, \tag{3.1}$$

as presented in figure 3. We do not expect $\tan\beta$ to be much larger, due to $B_s \to X_s\gamma$ flavour constraints and μ being bounded by naturalness considerations of the renormalisation group effects on the Higgs tadpole equations (minimisation of the scalar potential). As such, one may achieve the correct Higgs mass with a lower stop mass scale. Using the (MSSM) two-loop Higgs mass formula given in Ref. [11] figure 3 is generated.

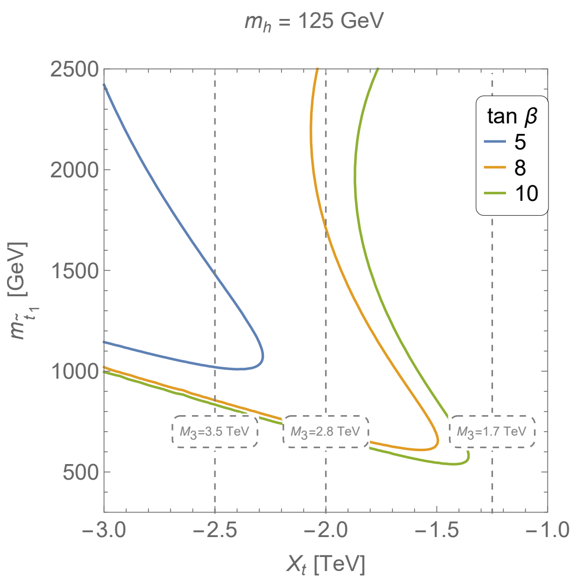

Figure 3. Contour plot of the Higgs mass at two-loops as a function of X_t.

4. The two-loop RGEs

The dominant two-loop correction to the gauge couplings in our model are proportional to $(2\mu/M_c)^2$, and these exclusively come from two-point diagrams with heavy KK mode interchanges [8]. However, we have

$$P_H^H = P_{H'}^{H'} = 0, \text{ and } P_{\sum^B}^{\sum^A} = 2g^2[T(H) + C(\Sigma)]\delta_{AB} = g^2 Q\delta_{AB}, \tag{4.1}$$

where $r = \delta_{AA}$ is the dimension of the group, and the leading contribution to $\beta_{\tilde{g}}^{(2)}$ vanishes. This cancellation reflects the fact that a model with $N = 2$ SUSY has to be renormalised only at the one-loop level, i.e., the one-loop counter-terms render the theory finite at any order of perturbation.

The sub leading two-loop correction, proportional to $2\mu/M_c$, where this correction does not vanish because the underlying $N = 2$ SUSY is broken by the the interaction of the field in the loops, can be written as [8]:

$$\begin{aligned}
\beta_{\tilde{g}}^{(2)} &= 2\tilde{g}^5 C(V)\{3[T(H) + T(H')] + T(f) + C(\Sigma) - 9C(V)\} \\
&\quad -2g^3 r^{-1} C(H)\Big[(4-6)g^2 C(H)d(H) + \frac{1}{2}y^{Hff'}y_{Hff'} + (H \leftrightarrow H)\Big] \\
&\quad -2g^3 C(\Sigma)2g^2[2T(H) - C(\Sigma)] - 2g^3 r^{-1} C(f)\Big[\frac{1}{2}y^{fkl}y_{fkl} - 2g^2 C(f)d(f)\Big],
\end{aligned} \tag{4.2}$$

where $d(\Phi)$ is the dimension of the Φ representation and f stands for the chiral fermions. The terms in the first line in equation (4.3) come from the possible Feynman diagrams. Diagrams with exchanges of (H, V) and (H', V) and V fields have a factor of 3, since there are three different ways to distribute the momentum along the fifth dimension (i.e., the KK modes) among the fields in the internal loops. For diagrams with (f, V) and $(\sum, V)$ in the loops, there is only one possible configuration, corresponding to the exchange of lower of V and $\sum$ fields, respectively. The contribution in diagrams with a chiral fermion f, the generic expression in equation (4.2) becomes

$$\gamma_f^{(2)f} = -\tilde{h}_f^2 P_{f^c}^{f^c} - 2g^2 C(f)P_f^f + 2g^4 C(f) \times [2T(H) + C(\Sigma) - 3C(V)]. \tag{4.3}$$

The full set of gauge couplings in 5D at two-loop are given by:

$$\beta_{g_1}^{(2)} = g_1^3\Big(\frac{199}{25}g_1^2 + \frac{27}{5}g_2^2 + \frac{88}{5}g_3^2 - \frac{26}{5}Y_u^2 - \frac{14}{5}Y_d^2 - \frac{18}{5}Y_e^2\Big), \tag{4.4}$$

$$\beta_{g_2}^{(2)} = g_2^3\Big(\frac{3}{2}g_1^2 + g_2^2 + 24g_3^2 - 6Y_u^2 - 6Y_d^2 - 2Y_e^2\Big), \tag{4.5}$$

$$\beta_{g_3}^{(2)} = g_3^3\Big(\frac{11}{5}g_1^2 + 9g_2^2 - 40g_3^2 - 4Y_u^2 - 4Y_d^2\Big). \tag{4.6}$$

The gaugino mass Parameters in 5D at two-loop can be written as:

$$\begin{aligned}
\beta_{M_1}^{(2)} &= 2g_1^2\Big(\frac{398}{25}g_1^2 M_1 + \frac{27}{5}g_2^2 M_1 + \frac{88}{5}g_3^2 M_1 + \frac{27}{5}g_2^2 M_2 + \frac{88}{5}g_3^2 M_3 - \frac{26}{5}Y_u(-A_u + M_1 Y_u) \\
&\quad -\frac{14}{5}Y_d(-A_d + M_1 Y_d) - \frac{18}{5}Y_e(-A_e + M_1 Y_e)\Big),
\end{aligned} \tag{4.7}$$

$$\begin{aligned}
\beta_{M_2}^{(2)} &= 2g_2^2\Big(\frac{3}{2}g_1^2 M_2 + 2g_2^2 M_2 + 24g_3^2 M_2 + \frac{3}{2}g_1^2 M_1 + 24g_3^2 M_3 - 6Y_u(-A_u + M_2 Y_u) \\
&\quad -6Y_d(-A_d + M_2 Y_d) - 2Y_e(-A_e + M_2 Y_e)\Big),
\end{aligned} \tag{4.8}$$

$$\begin{aligned}
\beta_{M_3}^{(2)} &= 2g_3^2\Big(\frac{11}{5}g_1^2 M_3 + 9g_2^2 M_3 - 80g_3^2 M_3 + \frac{11}{5}g_1^2 M_1 + 9g_2^2 M_2 \\
&\quad -4Y_u(-A_u + M_3 Y_u) - 4Y_d(-A_d + M_3 Y_d),\Big)
\end{aligned} \tag{4.9}$$

$$
\begin{aligned}
\beta_{Y_d}^{(2)} &= Y_d\Big(\frac{167}{900}g_1^4 - \frac{9}{4}g_2^4 - \frac{160}{9}g_3^4 + \frac{1}{10}g_1^2g_2^2 + \frac{8}{9}g_1^2g_3^2 + 8g_2^2g_3^2 \\
&\quad + \frac{1}{2}g_1^2Y_d^\dagger Y_d - \frac{3}{2}g_2^2Y_d^\dagger Y_d - 4Y_d^\dagger Y_dY_d^\dagger Y_d - 2Y_u^\dagger Y_uY_u^\dagger Y_u \\
&\quad - \frac{1}{10}g_1^2Y_d^\dagger Y_d + \frac{3}{2}g_2^2Y_d^\dagger Y_d - 2Y_d^\dagger Y_dY_u^\dagger Y_u\Big), \qquad (4.10) \\
\beta_{Y_u}^{(2)} &= Y_u\Big(\frac{767}{900}g_1^4 - \frac{9}{4}g_2^4 - \frac{160}{9}g_3^4 + \frac{1}{10}g_1^2g_2^2 + \frac{136}{45}g_1^2g_3^2 + 8g_2^2g_3^2 \\
&\quad - \frac{1}{2}g_1^2Y_u^\dagger Y_u + \frac{3}{2}g_2^2Y_u^\dagger Y_u - 4Y_u^\dagger Y_uY_u^\dagger Y_u - 2Y_d^\dagger Y_dY_d^\dagger Y_d \\
&\quad + \frac{1}{10}g_1^2Y_d^\dagger Y_d - \frac{3}{2}g_2^2Y_d^\dagger Y_d - 2Y_d^\dagger Y_dY_u^\dagger Y_u\Big), \qquad (4.11) \\
\beta_{Y_e}^{(2)} &= Y_e\Big(\frac{303}{100}g_1^4 - \frac{9}{4}g_2^4 + \frac{9}{10}g_1^2g_2^2 + \frac{9}{10}g_1^2Y_e^\dagger Y_e + \frac{3}{2}g_2^2Y_e^\dagger Y_e - 4Y_e^\dagger Y_eY_e^\dagger Y_e\Big) \qquad (4.12)
\end{aligned}
$$

and

$$
\begin{aligned}
\beta_{T_d}^{(2)} &= T_d\Big(\frac{167}{900}g_1^4 - \frac{9}{4}g_2^4 - \frac{160}{9}g_3^4 + \frac{1}{10}g_1^2g_2^2 + \frac{8}{9}g_1^2g_3^2 + 8g_2^2g_3^2\Big) \\
&\quad - Y_d\Big(\frac{334}{900}g_1^4M_1 - \frac{18}{4}g_2^4M_2 - \frac{320}{9}g_3^4M_3 + \frac{1}{5}g_1^2g_2^2(M_1+M_2) \\
&\quad + \frac{16}{9}g_1^2g_3^2(M_1+M_3) + 16g_2^2g_3^2(M_2+M_3)\Big) + \frac{1}{2}g_1^2T_dY_u^\dagger Y_u - \frac{1}{2}g_1^2M_1Y_dY_u^\dagger Y_u \\
&\quad - \frac{3}{2}g_2^2T_dY_u^\dagger Y_u + \frac{3}{2}g_2^2M_2Y_dY_u^\dagger Y_u - \frac{3}{10}g_1^2T_dY_d^\dagger Y_d + g_1^2Y_dT_uY_u \\
&\quad + \frac{1}{10}g_1^2M_1Y_dY_d^\dagger Y_d + \frac{9}{2}g_2^2T_dY_d^\dagger Y_d - \frac{3}{2}g_2^2M_2Y_dY_d^\dagger Y_d - 3g_2^2Y_dT_dY_d - 20T_dY_d^\dagger Y_dY_d^\dagger Y_d \\
&\quad - 2T_dY_u^\dagger Y_uY_u^\dagger Y_u - 8Y_dT_uY_uY_u^\dagger Y_u - 6T_dY_d^\dagger Y_dY_u^\dagger Y_u - 4T_uY_uY_dY_d^\dagger Y_d, \qquad (4.13) \\
\beta_{T_u}^{(2)} &= T_u\Big(\frac{767}{900}g_1^4 - \frac{9}{4}g_2^4 - \frac{160}{9}g_3^4 + \frac{1}{10}g_1^2g_2^2 + \frac{136}{45}g_1^2g_3^2 + 8g_2^2g_3^2\Big) \\
&\quad - Y_u\Big(\frac{1352}{900}g_1^4M_1 - \frac{18}{4}g_2^4M_2 - \frac{320}{9}g_3^4M_3 + \frac{1}{5}g_1^2g_2^2(M_1+M_2) \\
&\quad + \frac{272}{45}g_1^2g_3^2(M_1+M_3) + 16g_2^2g_3^2(M_2+M_3)\Big) - \frac{3}{2}g_1^2T_uY_u^\dagger Y_u + \frac{1}{2}g_1^2M_1Y_uY_u^\dagger Y_u \\
&\quad + \frac{9}{2}g_2^2T_uY_u^\dagger Y_u - \frac{3}{2}g_2^2M_2Y_uY_u^\dagger Y_u + \frac{1}{10}g_1^2T_uY_d^\dagger Y_d + \frac{1}{5}g_1^2Y_uT_dY_d \\
&\quad - \frac{1}{10}g_1^2M_1Y_uY_d^\dagger Y_d - \frac{3}{2}g_2^2T_uY_d^\dagger Y_d + \frac{3}{2}g_2^2Y_uY_d^\dagger Y_d - 3g_2^2Y_uT_dY_d - 20T_uY_u^\dagger Y_uY_u^\dagger Y_u \\
&\quad - 2T_uY_d^\dagger Y_dY_d^\dagger Y_d - 8Y_uT_dY_dY_d^\dagger Y_d - 6T_uY_u^\dagger Y_uY_d^\dagger Y_d - 4T_dY_dY_uY_u^\dagger Y_u, \qquad (4.14) \\
\beta_{T_e}^{(2)} &= T_e\Big(\frac{303}{100}g_1^4 - \frac{9}{4}g_2^4 + \frac{9}{10}g_1^2g_2^2\Big) - Y_e\Big(\frac{606}{100}g_1^4M_1 - \frac{18}{4}g_2^4M_2 + \frac{18}{10}g_1^2g_2^2(M_1+M_2)\Big) \\
&\quad + \frac{27}{10}g_1^2T_eY_e^\dagger Y_e - \frac{9}{10}g_1^2M_1Y_eY_e^\dagger Y_e + \frac{9}{2}g_2^2T_eY_e^\dagger Y_e - \frac{3}{2}g_2^2M_2Y_eY_e^\dagger Y_e - 20T_eY_e^\dagger Y_eY_e^\dagger Y_e. \\
&\qquad (4.15)
\end{aligned}
$$

5. Conclusions

In this proceeding we have explored the 5D MSSM at two-loop. We computed the full two loop RGEs for all the supersymmetric and soft term parameters and compared these results to those found at one-loop level in Ref. [7]. In our plots we have confirmed that a large A_t may be generated at low energies from a UV boundary condition, where A_t is vanishing, through power-law running.

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