# The effects of ion beams on slow and fast ion-acoustic solitons in space plasmas with two-temperature electrons

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# Earth's magnetosphere and plasma

- Area of space surrounding the Earth where ionised gas of charged particles interacts with the Earth's magnetic field.

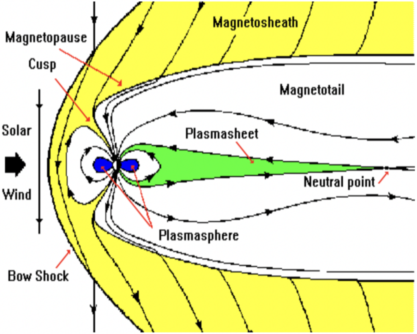
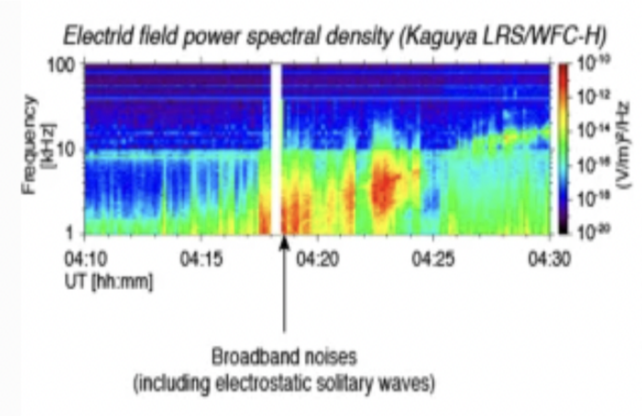



Figure: A schematic presentation of the Earth's magnetosphere [1].

- This ionised gas is known as plasma.

# Broadband Electrostatic Noise (BEN)

- Ion beams provide free energy




- The free energy generates electrostatic wave emissions called Broadband Electrostatic Noise (BEN)
- Frequency range: 10 Hz to 10 kHz

# Importance and Aim

- Studying these linear and non-linear waves is important for the understanding of space weather
- The theoretical studies of these waves in this study can help model the different regions of the magnetosphere
- Aim: Investigate the effect of ion beams on linear and non-linear waves in space plasmas

# Model and governing equations

- We first consider a three-component plasma of hot Boltzmann electrons and two warm counter-steaming ion beams.
- The motion of the particle species is governed by the continuity, momentum, and equation of state of each $j^{\text{th}}(=$ he, ce, i1, i2) species, respectively, [2]

$$\frac{\partial n_j}{\partial t} + \frac{\partial n_j v_j}{\partial x} = 0 \tag{1}$$

$$\frac{\partial v_j}{\partial t} + v_j \frac{\partial v_j}{\partial x} + \frac{1}{n_j}\frac{\partial P_j}{\partial x} + \frac{\partial \phi}{\partial x} = 0 \tag{2}$$

$$\frac{\partial P_j}{\partial t} + v_j \frac{\partial P_j}{\partial x} + 3P_j \frac{\partial v_j}{\partial x} = 0 \tag{3}$$

# Linear dispersion relation

- The plasma system is coupled by Poisson's equation,

$$\frac{\partial^2 \phi}{\partial x^2} = n_{\mathrm{he}} + n_{\mathrm{ce}} - n_{i1} - n_{i2} \tag{4}$$

- Linearising the set of fluid equations (1) to (4) will lead to the linear dispersion relation equation [2]

$$1 + \frac{1}{k^2 \lambda_{\mathrm{de}}^2} = \frac{\omega_{p1}^2}{(\omega - kV_1)^2 - 3k^2 v_{t1}^2} + \frac{\omega_{p2}^2}{(\omega - kV_2)^2 - 3k^2 v_{t2}^2} \tag{5}$$

# Electron number densities

- The number densities of the cool and hot Boltzmann electrons are, respectively, given by:[3]

$$n_{he} = n_{he0} \exp\left(\frac{e\phi}{k_B T_{he}}\right) \tag{6}$$

$$n_{ce} = n_{ce0} \exp\left(\frac{e\phi}{k_B T_{ce}}\right) \tag{7}$$

# Ion beam densities

- Using the set of fluid equations, (1)-(4), the number densities of the ion beams, $n_1$ and $n_2$, can be written, in Ghosh form, as

$$n_1 = \frac{n_1^0}{2\sqrt{3\sigma_1}}\{[(M - U_1^0 + \sqrt{3\sigma_1})^2 - 2\Phi]^{1/2} \pm [(M - U_1^0 - \sqrt{3\sigma_1})^2 - 2\Phi]^{1/2}]\} \tag{8}$$

$$n_2 = \frac{n_2^0}{2\sqrt{3\sigma_2}}\{[(M - U_2^0 + \sqrt{3\sigma_2})^2 - 2\Phi]^{1/2} \pm [(M - U_2^0 - \sqrt{3\sigma_2})^2 - 2\Phi]^{1/2}]\} \tag{9}$$

# Sagdeev potential

- Substituting the number density expressions for all the charged species into Poisson's equation, leads to the energy equation

$$\frac{1}{2}\left(\frac{d\Phi}{d\zeta}\right)^2 + S(\Phi, M) = 0 \tag{10}$$

$$\begin{aligned} S(\Phi, M) = [1 - \exp(\Phi)] + \frac{n_1^0}{6\sqrt{3\sigma_1}} \{ & (M - U_1 + \sqrt{3\sigma_1})^3 - [(M - U_1 + \sqrt{3\sigma_1})^2 - 2\Phi]^{3/2} \pm \\ & (M - U_1 - \sqrt{3\sigma_1})^3 \mp [(M - U_1 - \sqrt{3\sigma_1}) - 2\Phi]^{3/2} \} + \\ \frac{n_2^0}{6\sqrt{3\sigma_2}} \{ & (M - U_2 + \sqrt{3\sigma_2})^3 - [(M - U_2 + \sqrt{3\sigma_2})^2 - 2\Phi]^{3/2} \pm \\ & (M - U_2 - \sqrt{3\sigma_2})^3 \mp [(M - U_2 - \sqrt{3\sigma_2}) - 2\Phi]^{3/2} \} \end{aligned} \tag{11}$$

# The linear dispersion plots

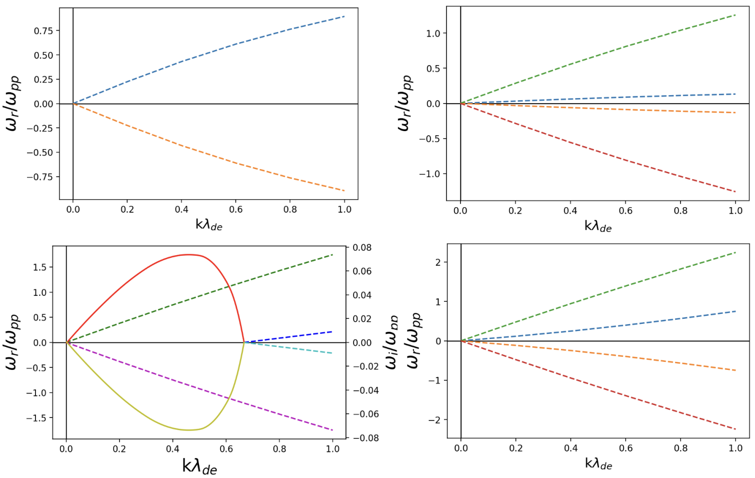



Figure: Dispersion plots showing the frequency and the wave number $k\lambda_{de}$. (a) $U_0 = 0.0$, (b) $U_0 = 0.5$, (c) $U_0 = 1.0$, (d) $U_0 = 1.5$

# Ion-acoustic modes: One beam

- For solitons solutions to exist, we must have $F(M) = S''(0, M) < 0$ and $F(M) = 0 \iff M = M_0 \in \mathbb{R}$

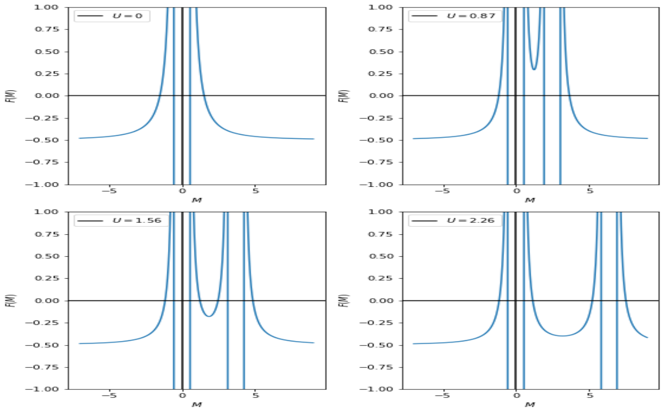



Figure: Second derivative of $S(\Phi, M)$, $F(M)$, at zero potential for various values of $U$ (asymmetric)

# Ion-acoustic modes: Two beams

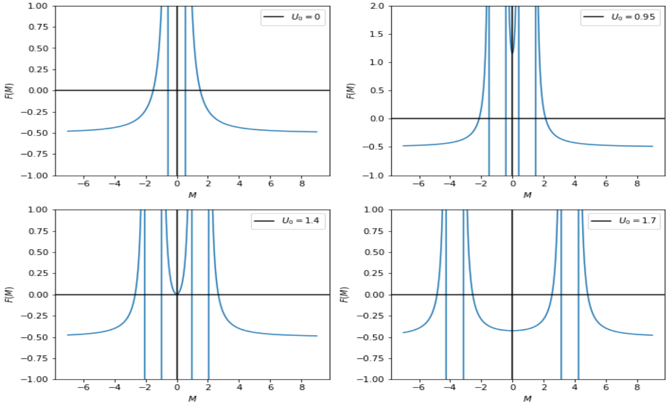



Figure: Second derivative of $S(\Phi, M)$, $F(M)$, at zero potential for various values of $U_0$ (symmetric)

# Potential plots: Fast ion-acoustic solitons

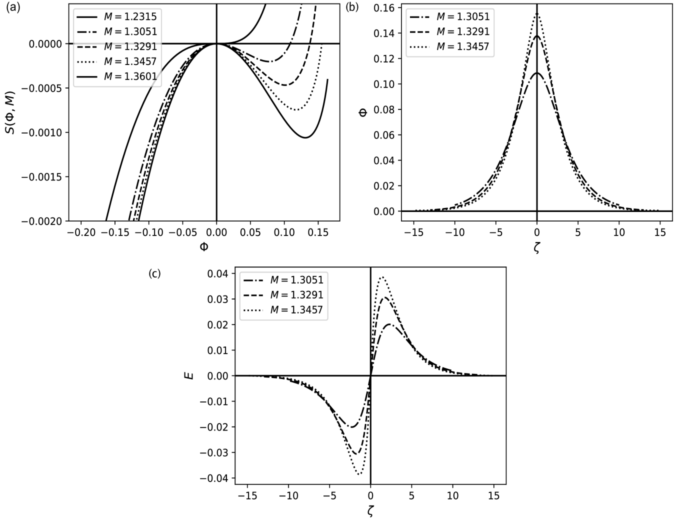



Figure: Potential profiles. (a) Sagdeev profile for U = 2, (b) Soliton profile for U = 2, (c) Electric field profile for U = 2

# Potential plots: Slow ion-acoustic solitons

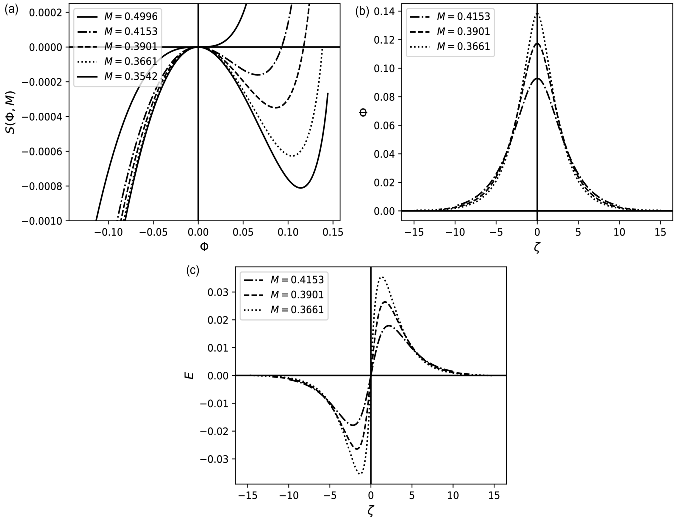



Figure: Potential profiles. (a) Sagdeev profile for U = 0.5, (b) Soliton profile for U = 0.5, (c) Electric field profile for U = 0.5

# Co-existence: Fast solitons

- Co-existence of fast ion-acoustic solitons

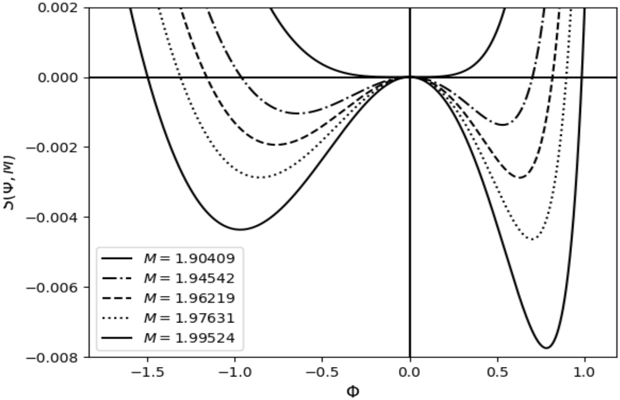



Figure: Sagdeev potential profile for $n_{i1} = 0.5$, $n_{i2} = 0.5$, $n_{ce} = 0.7$, $n_{he} = 0.3$, and $U_0 = 2$

# Satellite observations

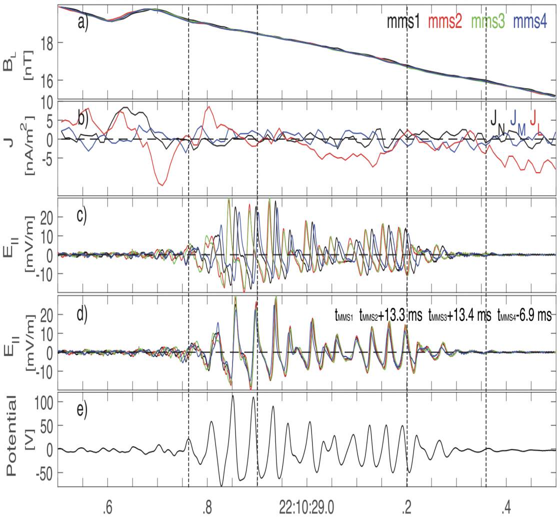



Figure: MMS satellite observations [4]

# Conclusion

In conclusion, it was found that:

- Solitons can exist below $M_0$.
- For two ion beams, both backward and forward propagating solitons exist
- For one ion beam, only forward propagating slow solitons exist.
- For two-temperature electrons, we report on the co-existence of fast solitons

# References


[1] G.S.Lakhina, S.V.Singh, and R.Rubia, Advances in Space Research 68, 1864-1875 (2021)

[2] G.S.Lakhina, S.V.Singh, and R.Rubia, Physica Scripta 95, 1402-4896 (2020)

[3] R.Rubia, S.V.Singh, and G.S.Lakhina, Journal of Geophysical Research: Space Physics 122, 9134-9147 (2017)

[4] C.M.Liu, A.Vaivads, D.B.Graham, Yu V.Khotyaintsev, H.S. Fu1, A.Johlander, M.André, and B.L.Giles, Geophysical Research Letters, 702–12 (2019)