Black holes and nilmanifolds: quasinormal modes as fingerprints of extra dimensions

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Outline

What is the significance of these extra dimensions?

Why do we consider extra-dimensional scenarios?

What is the significance of these extra dimensions?

Historically: *A path to unification?*

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Today: *A path to physics BSM?*

- 1980s: “KK renaissance”, 1984 “superstring revolution”

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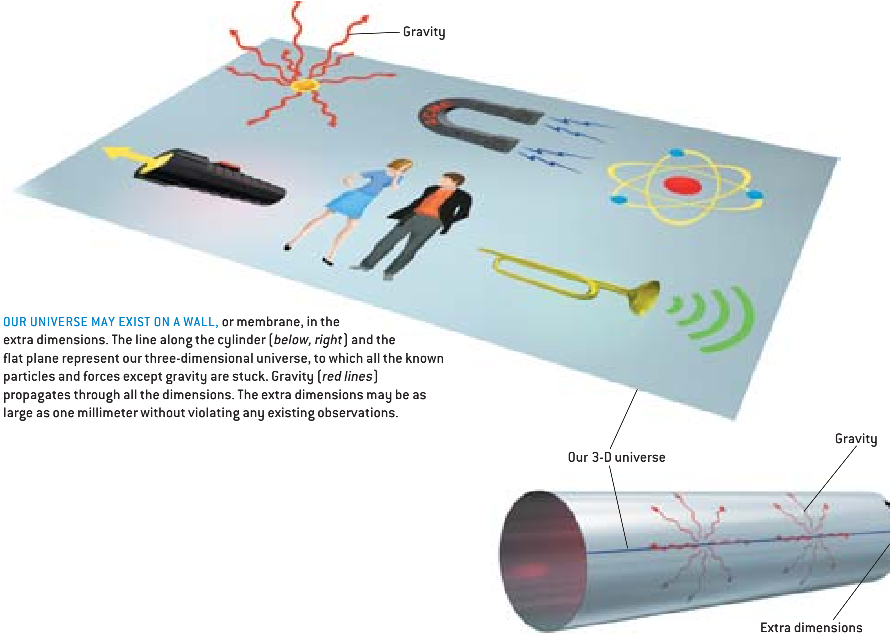
- 1998: Arkani-Hamed, Dimopoulos, Dvali: large EDs



OUR UNIVERSE MAY EXIST ON A WALL, or membrane, in the extra dimensions. The line along the cylinder (*below, right*) and the flat plane represent our three-dimensional universe, to which all the known particles and forces except gravity are stuck. Gravity (*red lines*) propagates through all the dimensions. The extra dimensions may be as large as one millimeter without violating any existing observations.

Taken from ADD's 2002 Scientific American article, "The universe's unseen dimensions"

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Today: *A path to physics BSM?*

- 1980s: “KK renaissance”, 1984 “superstring revolution”
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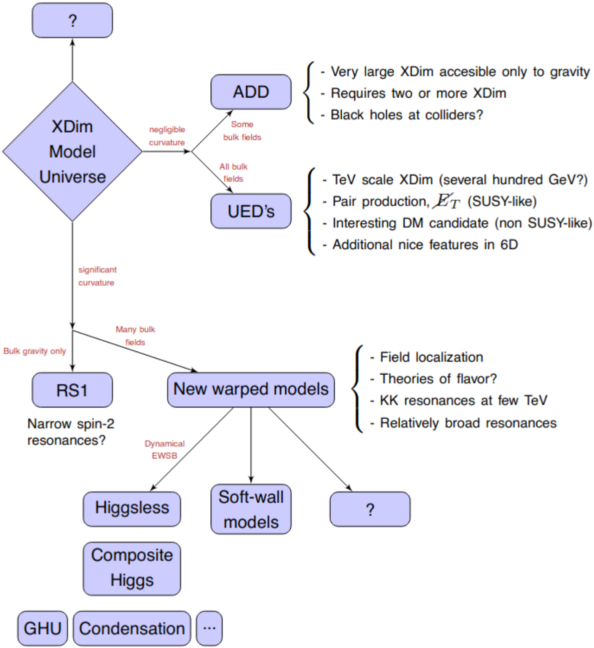
- 1999: Randall & Sundrum: warped EDs

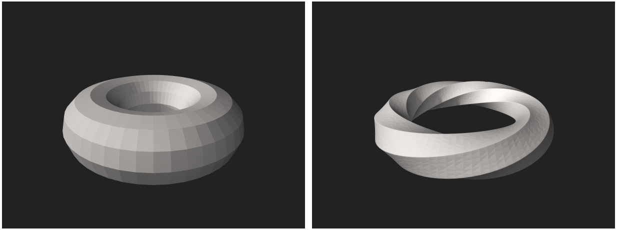
...



Taken from E. Pontón's 2011 TASI lectures, "TeV scale EDs"

Any Lie group G of dimension d can be understood as a d-dimensional differentiable manifold. To make G compact, we quotient by the lattice Γ. For nilpotent groups, there is always a Γ.

$\Rightarrow$ compact ED model from solvable Lie groups: $\mathcal{M}_{4+d} = \mathcal{M}_4 \times \mathcal{H}_d$



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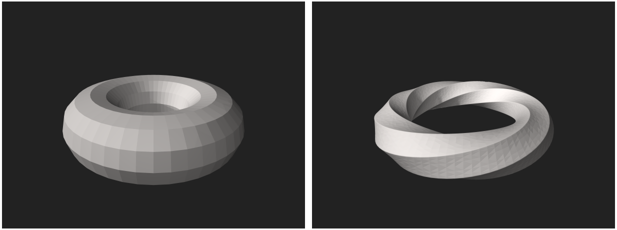
Heisenberg algebra...

$$[Z_1, Z_2] = -\mathrm{f}Z_3 \ , \quad [Z_1, Z_3] = [Z_2, Z_3] = 0$$



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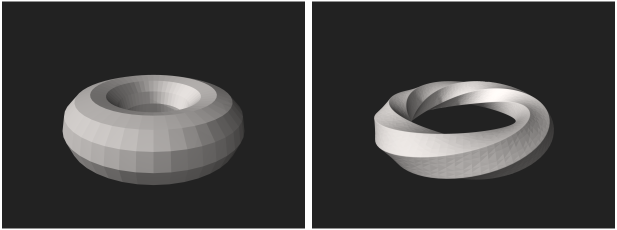
$$\mathrm{d}e = \mathtt{f} e^1 \wedge e^2 \ , \quad \mathrm{d}e^1 = 0 \ , \quad \mathrm{d}e^2 = 0$$

$$e^1 = r^1 dy^1 \ , \quad e^2 = r^2 dy^2 \ , \quad e^3 = r^3 (dy^3 + N r^1 dy^2) \ , \quad N = \frac{r^1 r^2}{r^3} \mathtt{f}$$

...gives us the metric $ds^2_{\mathcal{H}} = g^{\mathcal{H}}_{ij} dx^i dx^j$



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Why the negativity?

Negatively-curved EDs: a BSM landscape of untapped potential?

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Negatively-curved EDs: a BSM landscape of untapped potential?

Phenomenological implications:

- natural resolution to the hierarchy problem

  $\rightarrow$ volume grows exponentially with ℓ_G/ℓ_c

  $\rightarrow$ RSI-like KK mass spectrum w/o light KK modes

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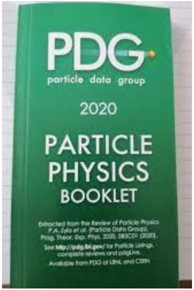
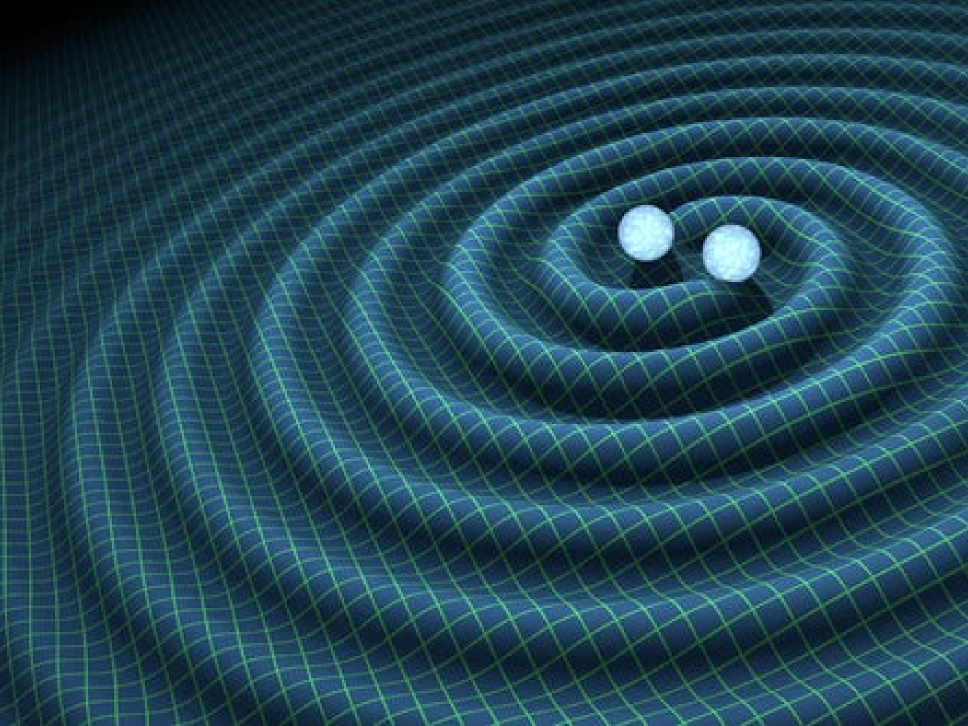
  $\rightarrow$ RSI-like KK mass spectrum w/o light KK modes
- zero modes of Dirac operator emerges w/o gauge breaking
- enables homogeneity & flatness of observed universe

Constraining EDs

- Limits on R from Deviations in Gravitational Force Law
- Limits on R from On-Shell Production of Gravitons: $\delta = 2$
- Mass Limits on M_{TT}
- Limits on $1/R = M_c$
- Limits on Kaluza-Klein Gravitons in Warped Extra Dimensions
- Limits on Kaluza-Klein Gluons in Warped Extra Dimensions
- Black Hole Production Limits
  – Semiclassical Black Holes
  – Quantum Black Holes

ATLAS, CMS, DELPHI, ALEPH, CDF, D0, OPAL, etc.





Constraining the number of spacetime dimensions from GWTC-3 binary black hole mergers #1

Ignacio Magana Hernandez (Wisconsin U., Milwaukee) (Dec 14, 2021)

e-Print: 2112.07650 [astro-ph.HE]

pdf cite 1 citation

Constraining cosmological extra dimensions with gravitational wave standard sirens: from theory to current and future multi-messenger observations #2

Maxence Corman (Perimeter Inst. Theor. Phys. and Waterloo U.), Abhirup Ghosh (Potsdam, Max Planck Inst.), Celia Escamilla-Rivera (Mexico U., ICN), Martin A. Hendry (Glasgow U.), Sylvain Marsat (APC, Paris) et al. (Sep 17, 2021)

e-Print: 2109.08748 [gr-qc]

pdf cite 0 citations

Constraining extra dimensions on cosmological scales with LISA future gravitational wave siren data #3

Maxence Corman (Perimeter Inst. Theor. Phys.), Celia Escamilla-Rivera (UNAM, Mexico), M.A. Hendry (SUPA, UK) (Apr 3, 2020)

Published in: *JCAP* 02 (2021) 005 • e-Print: 2004.04009 [astro-ph.CO]

pdf DOI cite 12 citations

Theoretical Physics Implications of the Binary Black-Hole Mergers GW150914 and GW151226 #4

Nicolas Yunes (Montana State U.), Kent Yagi (Princeton U.), Frans Pretorius (Princeton U.) (Mar 29, 2016)

Published in: *Phys.Rev.D* 94 (2016) 8, 084002 • e-Print: 1603.08955 [gr-qc]

pdf DOI cite 477 citations

Gravitational wave source counts at high redshift and in models with extra dimensions #5

Juan García-Bellido (Madrid, IFT), Savvas Nesseris (Madrid, IFT), Manuel Trashorras (Madrid, IFT) (Mar 17, 2016)

Published in: *JCAP* 07 (2016) 021 • e-Print: 1603.05616 [astro-ph.CO]

pdf DOI cite 10 citations

Size of Shell Universe in Light of Fermi GBM Transient Associated with GW150914 #6

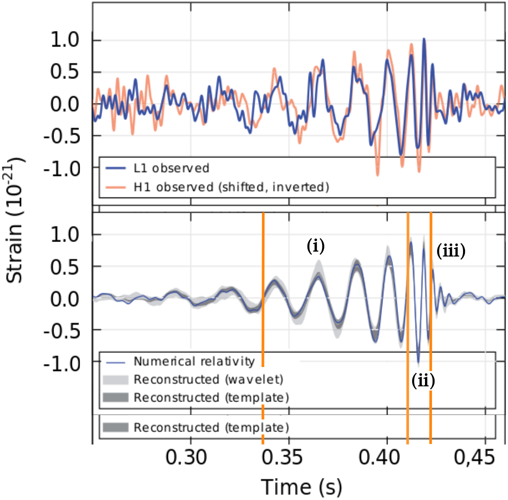
Merab Gogberashvili (Javakhishvili State U. and Tbilisi, Inst. Phys.), Alexander Sakharov (CERN and Manhattan Coll., Riverdale and New York U.), Edward Sarkisyan-Grinbaum (CERN and Texas U., Arlington) (Feb 21, 2016)

Published in: *Phys.Lett.B* 763 (2016) 397-400 • e-Print: 1602.06526 [hep-ph]

Objective

Using new techniques to probe underexplored BSM landscapes...

Quasinormal mode: "ringdown"



(i) inspiral
(ii) merger
(iii) ringdown

B. P. Abbott *et al.*, PRL **116**, 061102 (2016).

Quasinormal mode and frequency

$$\Psi(x^\mu) = \sum_{n=0}^{\infty} \sum_{\ell,m} \frac{\psi_{sn\ell}(r)}{r^{(d-2)/2}} e^{-i\omega t} Y_{\ell m}(\theta_i) \ , \qquad \omega_{sn\ell} = \omega_R - in\omega_I$$

- $\mathbb{R}e\{\omega\}$ = physical oscillation frequency
- $\mathbb{I}m\{\omega\}$ = damping $\rightarrow$ dissipative, "quasi"

Quasinormal mode and frequency

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- s: spin of perturbing field
- m: azimuthal number for spherical harmonic decomposition in θ_i
- ℓ: angular/multipolar number for spherical harmonic decomposition in θ_i
- n: overtone number labels QNMs by a monotonically increasing $|\mathbb{I}m\{\omega\}|$

The 7D metric

$$ds^2_{7D} = g^{BH}_{\mu\nu} dx^\mu dx^\nu + g^{\mathcal{H}}_{ij} dx^i dx^j$$

$$\Psi^s_{n\ell m}(t, r, \theta, \phi, y^1, y^2, y^3) = \sum_{n=0}^{\infty} \sum_{\ell,m} R^s_{n\ell}(r) Y^s_{m\ell}(\theta, \phi) Z(y^1, y^2, y^3) e^{-i\omega t}$$

$$ds^2_{BH} = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2(\sin^2 d\theta^2 + d\phi^2)$$

$$f(r) = 1 - 2M/r$$

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Laplacian of a product space is the sum of its parts

$$\left(\nabla^2_{BH} + \nabla^2_{\mathcal{H}}\right) \sum \Phi(x) Z_k(y) = 0 \ ,$$

$$\nabla^2 Y^s_{m\ell}(\theta, \phi) = \tfrac{-\ell(\ell+1)}{r^2} Y^s_{m\ell}(\theta, \phi)$$

$$\nabla^2 Z_k(y) = -\mu_k^2 Z_k(y)$$

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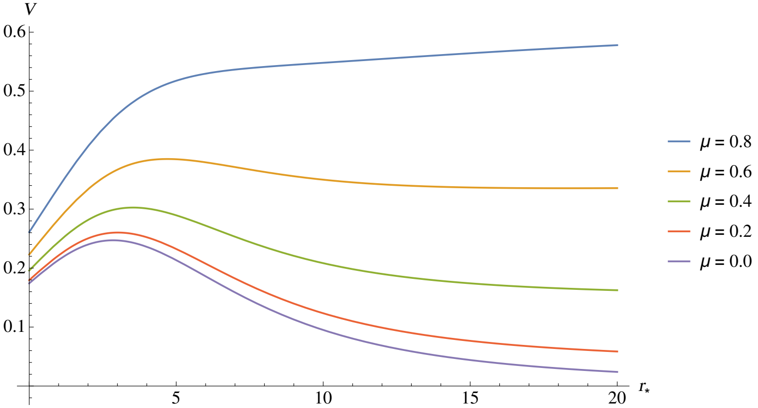
$$\text{KG:} \quad \frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g} g^{\mu\nu} \partial_\nu \Psi \right) - \mu^2 \Psi = 0$$

The wavelike equation

$$\frac{d^2\psi}{dr_*^2} + \left(\omega^2 - V(r)\right)\psi = 0$$

$$V(r) = \left(1 - \frac{2M}{r}\right)\left(\frac{\ell(\ell+1)}{r^2} + \frac{2M}{r^3} + \mu^2\right)$$

The fundamental mode: $n = 0$, $\ell = 2$



The fundamental mode: $n = 0$, $\ell = 2$

μ	ω [WKB: $\mathcal{O}(V^{(6)})$]	ω [DO: $\mathcal{O}(L^{-6})$]
0.0	$0.48364 - 0.09677i$	$0.48364 - 0.09676i$
0.1	$0.48680 - 0.09568i$	$0.48680 - 0.09567i$
0.2	$0.49632 - 0.09240i$	$0.49633 - 0.09239i$
0.3	$0.51234 - 0.08680i$	$0.51237 - 0.08679i$
0.4	$0.53510 - 0.07868i$	$0.53520 - 0.07866i$
0.5	$0.56493 - 0.06763i$	$0.56526 - 0.06763i$
0.6	$0.60224 - 0.05284i$	$0.60320 - 0.05315i$
0.7	$0.13961 + 0.27633i$	$0.65000 - 0.03433i$

The QNM spectrum

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In agreement with massive scalar QNFs of S. Dolan, Phys. Rev. D 76 (2007) 084001

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$\Rightarrow$ An upper bound on our QNM probe (“sensitivity range cutoff")

Bounds from LVC?

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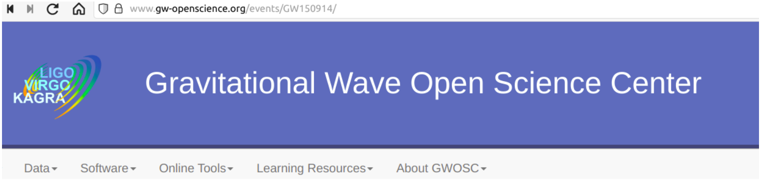
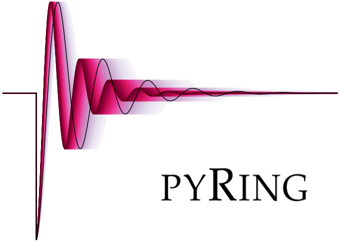




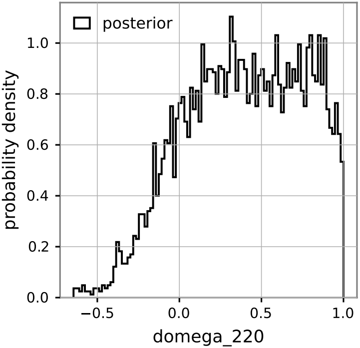
$$\delta\omega = \omega^{GR}(1+\delta\omega)$$

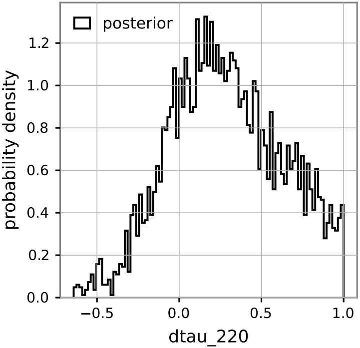
$$\delta\tau = \tau^{GR}(1+\delta\tau)$$

The fundamental mode: $n = 0$, $\ell = 2$

μ	$\delta\omega$	$\delta\tau$
0.0	0.0000	0.0000
0.1	0.0065	0.0113
0.2	0.0262	0.0473
0.3	0.0594	0.1149
0.4	0.1066	0.2302
0.5	0.1687	0.4306
0.6	0.2472	0.8206
0.7	0.3440	1.8181

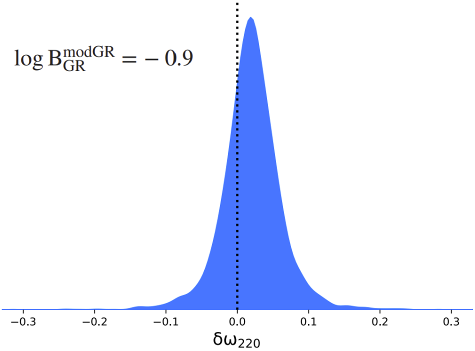
The fundamental mode: $n = 0, \ell = 2$

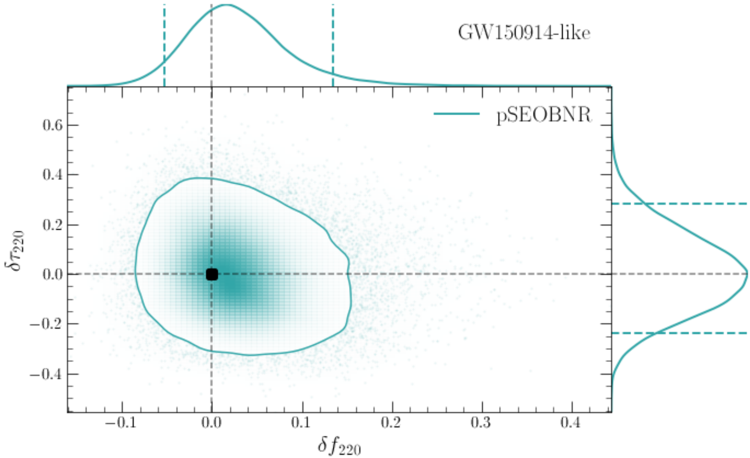




Tests of GR with GWTC-3 [2112.06861]

$$\delta\omega_{O3} = 0.02^{+0.07}_{-0.07}$$
$$\delta\tau_{O3} = 0.13^{+0.21}_{-0.22}$$





Ghosh, Brito, Buonnano [2104.01906] using `pSEOBNR` *simulated waveform model*

Thank you

Thank you

And a warm thanks to

Suppose we place a 4D Schwarzschild black hole within a 7D spacetime, perturbed by a 7D scalar test field of mass μ:

$$\text{KG:}\ \ \frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\Psi\right) - \mu^2\Psi = 0\ ,$$

$$g_{\mu\nu}dx^\mu dx^\nu = g_{ab}(x)dx^a dx^b + g_{ij}(y)dx^i dx^j\ ,$$

$$g_{\mu\nu} = \begin{bmatrix} -f(r) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & f(r)^{-1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & +1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & r_1^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & r_2^2 + r_3^2N^2y_1^2 & r_3^2Ny_1 \\ 0 & 0 & 0 & 0 & 0 & r_3^2Ny_1 & r_3^2 \end{bmatrix},$$

where $f(r) = 1 - 2M/r$

Variable-separable QNM solution:

$$\Psi^s_{n\ell m\mu}(t,r,\theta,\phi,y_1,y_2,y_3) = \sum_{n=0}^{\infty}\sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell} R^s_{n\ell\mu}(r)Y^s_{m\ell}(\theta,\phi)Z_\mu(y_1,y_2,y_3)e^{i\omega t} \ .$$

Laplacian of a product space is the sum of its parts

$$\left(\nabla^2_{BH} + \nabla^2_{nil}\right)\sum \Phi(x)Z_k(y) = 0 \ ,$$

- $\nabla^2 Y^s_{m\ell}(\theta,\phi) = \frac{-\ell(\ell+1)}{r^2} Y^s_{m\ell}(\theta,\phi)$
- $\nabla^2 Z_k(y) = -\mu_k^2 Z_k(y)$

$$\mu^2_{k,j,m} = \frac{4\pi^2 k^2}{(r_3)^2}\left[1 + \frac{(2m+1)r_3}{2\pi|k|}|\mathfrak{f}|\right]$$

Gravitational perturbations

Table I. Stabilities of generalised static black holes. In this table, "d" represents the spacetime dimension, $n+2$. The results for tensor perturbations apply only for maximally symmetric black holes, while those for vector and scalar perturbations are valid for black holes with generic Einstein horizons, except in the case with $K=1, Q=0, \lambda>0$ and $d=6$.

		Tensor		Vector		Scalar	
		$Q=0$	$Q\neq 0$	$Q=0$	$Q\neq 0$	$Q=0$	$Q\neq 0$
$K=1$	$\lambda=0$	OK	OK	OK	OK	OK	$d=4,5$ OK $d\geq 6$?
	$\lambda>0$	OK	OK	OK	OK	$d\leq 6$ OK $d\geq 7$?	$d=4,5$ OK $d\geq 6$?
	$\lambda<0$	OK	OK	OK	OK	$d=4$ OK $d\geq 5$?	$d=4$ OK $d\geq 5$?
$K=0$	$\lambda<0$	OK	OK	OK	OK	$d=4$ OK $d\geq 5$?	$d=4$ OK $d\geq 5$?
$K=-1$	$\lambda<0$	OK	OK	OK	OK	$d=4$ OK $d\geq 5$?	$d=4$ OK $d\geq 5$?

$$\mathcal{R}_{ED} = (d-3)K\gamma_{ij}$$