

Dark coupling: cosmological implications of interacting dark energy and dark matter fluids

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Abstract. In this study, cosmological models are considered where dark matter and dark energy are coupled and may have non-gravitational interactions with each other. These dark couplings are introduced to potentially alleviate both the Hubble tension and the cosmic coincidence problem (regarding the current observed ratio of dark matter to dark energy today). Assuming two different linear dark couplings, the conservation and Friedmann equations are used to predict how these couplings effect crucial events in the expansion history of the universe. These events include the big bang and cosmic acceleration, as well as the radiation-matter and matter-dark energy equality. These results are compared with the standard uncoupled Λ CDM model where dark energy is assumed to be a cosmological constant. Cosmological parameters for this study are obtained from Type-Ia Supernovae data using a previously developed Markov Chain Monte-Carlo (MCMC) simulation.

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1. Problems with the Λ CDM model

The expansion of the universe has thus far been well described by the Λ CDM model, where the energy budget of the universe is divided between $\approx 5\%$ baryonic matter (standard model particles), $\approx 25\%$ non-baryonic cold dark matter (which keeps galaxies from flying apart) and $\approx 70\%$ dark energy in the form of the cosmological constant Λ (which explains late-time accelerated expansion). This model has proven to be very successful [1], but problems with the Λ CDM model remain, which include:

| *The Cosmological Constant Problem* or vacuum catastrophe, which refers to the measured energy density of the vacuum being over ~~a~~ 120 orders of magnitude smaller than the theoretical prediction. This has been referred to as the worst prediction in the history of physics and casts doubt on dark energy being a cosmological constant, motivating research into alternative dark energy models [2].

The Cosmic Coincidence Problem, which alludes to the dark matter and dark energy densities having the same order of magnitude at the present moment of cosmic history, while differing with many orders of magnitude in the past and predicted future [3].

The Hubble Tension, which concerns the 4.4σ level difference between values of the Hubble constant H_0 as measured from the Cosmic Microwave Background versus the value obtained from Type Ia Supernovae using a calibrated local distance ladder [4].

2. Dark coupling models

| These problems motivate research beyond the Λ CDM model. One possible approach is to investigate cosmological models in which there are non-gravitational interactions between the dark sectors of the universe. This allows the two dark sectors to exchange energy (and/or momentum) while dark matter and dark energy are not separately conserved, but the energy (and/or

momentum) of the total dark sector is conserved. This coupling between dark matter and dark energy modifies the continuity equations into [4, 5, 6, 7]:

$$\dot{\rho}_{\text{dm}} + 3H\rho_{\text{dm}} = Q \quad ; \quad \dot{\rho}_{\text{de}} + 3H\rho_{\text{de}}(1 + \omega) = -Q, \quad (1)$$

where $H = (\dot{a}/a)$ is the Hubble parameter, with a the scale factor which denotes the relative size of the universe. $\rho_{\text{dm/de}}$ is the dark matter/dark energy density, ω is the equation of state of dark energy ($\omega_{\text{dm}} = 0$ since dark matter is assumed to be pressureless) and Q is the rate of energy exchange, which defines the direction of energy flow between the dark sectors such that:

$$Q = \begin{cases} > 0 & \text{Dark Energy} \rightarrow \text{Dark Matter} \\ < 0 & \text{Dark Matter} \rightarrow \text{Dark Energy} \\ = 0 & \text{No interaction (\Lambda\text{CDM case})} \end{cases} \quad (2)$$

The behaviour of coupled models may be understood by how the interaction effects the effective equations of state, relative to the uncoupled background equations ($Q = 0$) in (1) such that:

$$\omega_{\text{dm}}^{\text{eff}} = -\frac{Q}{3H\rho_{\text{dm}}} \quad ; \quad \omega_{\text{de}}^{\text{eff}} = \omega_{\text{de}} + \frac{Q}{3H\rho_{\text{de}}}. \quad (3)$$

Thus, the effects of an interaction may be understood to imply that if:

$$Q > 0 \rightarrow \begin{cases} \omega_{\text{dm}}^{\text{eff}} < 0 & \text{Dark matter redshifts slower (less DM in past)} \\ \omega_{\text{de}}^{\text{eff}} > \omega_{\text{de}} & \text{Dark energy has less accelerating pressure (older universe)} \end{cases}$$

$$Q < 0 \rightarrow \begin{cases} \omega_{\text{dm}}^{\text{eff}} > 0 & \text{Dark matter redshifts faster (more DM in past)} \\ \omega_{\text{de}}^{\text{eff}} < \omega_{\text{de}} & \text{Dark energy has more accelerating pressure (younger universe)} \end{cases}$$

When $Q = 0$, the effective equations of state reduce back to the case for the ΛCDM model, where dark matter is pressureless ($\omega_{\text{dm}} = 0$) and dark energy has a constant negative pressure.

Since there is currently no fundamental theory for these couplings, they are purely phenomenological and must be tested against observations. Two models will be considered which have interactions proportional to the Hubble parameter [4, 5, 6, 7]. Solving the conservation equations (1) for both models show how the energy densities evolve, such that:

Model 1: $Q_1 = \delta H \rho_{\text{dm}}$

$$\rho_{\text{dm}} = \rho_{(\text{dm},0)} a^{(\delta-3)} \quad (4)$$

$$\rho_{\text{de}} = \rho_{(\text{de},0)} a^{-3(1+\omega_{\text{de}})} + \rho_{(\text{dm},0)} \frac{\delta}{\delta + 3\omega} \left[a^{-3\omega} - a^{\delta} \right] a^{-3}, \quad (5)$$

with $\left(0 < \delta < -\frac{3\omega}{(1+r_0)}\right)$ to ensure $\rho_{\text{dm/de}} > 0$ throughout evolution.

Model 2: $Q_2 = \delta H \rho_{\text{de}}$

$$\rho_{\text{dm}} = \rho_{(\text{dm},0)} a^{-3} + \rho_{(\text{de},0)} \frac{\delta}{\delta + 3\omega} \left[1 - a^{-(\delta+3\omega)} \right] a^{-3} \quad (6)$$

$$\rho_{\text{de}} = \rho_{(\text{de},0)} a^{-(\delta+3\omega+3)}, \quad (7)$$

with $\left(0 < \delta < -\frac{3\omega}{(1+1/r_0)}\right)$ to ensure $\rho_{\text{dm/de}} > 0$ throughout evolution.

missing subscripts 0 - and in Fig 3 $r = \rho_{\text{dm}}/\rho_{\text{de}}$, which I think is what you should have here

Here $r_0 = (\rho_{\text{de}}/\rho_{\text{dm}})$ is the ratio of dark energy to dark matter today; and δ is a dimensionless coupling constant which determines the strength of the interaction between dark matter and dark energy. Furthermore, when $Q = 0$, both models reduce to the Λ CDM case where $\rho_{\text{dm}} \propto a^{-3}$ and $\rho_{\text{de}} = \text{constant}$.

Negative couplings ($\delta < 0$) are often used in the literature for these models [4, 6, 7], which are problematic since this leads to negative energy densities in the future. Therefore it is very important to take note of these positive energy density conditions for δ , as they ensure that the energy density for both dark matter and dark energy is not only positive for the past expansion history, but for the future as well.

3. Cosmological parameters

The current cosmological parameters for these models may be obtained from a data set of 359 low and intermediate redshift Type-Ia Supernovae (obtained from the SDSSII/SNLS2 Joint Light-curve Analysis (JLA)). This data is used with a previously developed Markov Chain Monte-Carlo (MCMC) simulation for a flat FLRW universe to obtain cosmological parameters for each of these models from their corresponding Friedmann equations (15). The contribution of $\Omega_{(\text{rad},0)}$ on the expansion and the MCMC model is negligible, but has been chosen as $\Omega_{(\text{rad},0)} = 9e^{-5}$ for further calculations [1]. Details of this MCMC model may be found in [8, 9].

is e the Euler constant or do you mean 9×10^{-5} ? Also later

The limit $\omega > -1$ has been imposed for all models, while applying the positive energy conditions ($0 < \delta < -\frac{3\omega}{(1+r_0)}$) and ($0 < \delta < -\frac{3\omega}{(1+1/r_0)}$) for Q_1 and Q_2 respectively. The priors of these parameters are the results from the Λ CDM case. This gives the following results:

Table 1: Cosmological parameters from type Ia Supernovae

Model	$\Omega_{(\text{dm},0)}$	$\Omega_{(\text{bm},0)}$	H_0	ω	δ
Λ CDM	$0.213^{+0.037}_{-0.037}$	$0.055^{+0.031}_{-0.030}$	$69.7^{+0.5}_{-0.5}$	$-1.000^{+0.000}_{-0.000}$	$0.000^{+0.000}_{-0.000}$
$Q_1 = \delta H \rho_{\text{dm}}$	$0.234^{+0.036}_{-0.024}$	$0.043^{+0.022}_{-0.016}$	$68.0^{+0.9}_{-0.9}$	$-0.949^{+0.057}_{-0.036}$	$0.296^{+0.146}_{-0.184}$
$Q_2 = \delta H \rho_{\text{de}}$	$0.232^{+0.031}_{-0.022}$	$0.044^{+0.021}_{-0.017}$	$69.4^{+0.5}_{-0.5}$	$-0.948^{+0.059}_{-0.037}$	$0.257^{+0.161}_{-0.167}$

with $\Omega_{(\text{de},0)} = 1 - \Omega_{(\text{dm},0)} - \Omega_{(\text{bm},0)}$ since a spatially flat universe is assumed. Here it can be seen that H_0 is slightly lower and closer to the CMB value for both Q_1 and Q_2 , which slightly alleviates the Hubble Tension [4]. It should be noted that since the limits to avoid early time instabilities have not yet been considered, these results should be taken as preliminary.

4. Evolution of energy densities

Universe models will be considered which contain radiation (rad), baryons (bm), dark matter (dm) and dark energy (de). In order to avoid fifth force constraints, it is assumed that radiation and baryons are separately conserved and uncoupled [5] such that:

$$\dot{\rho}_{\text{rad}} + 3H\rho_{\text{rad}}(1 + \omega_{\text{rad}}) = 0 \quad ; \quad \dot{\rho}_{\text{bm}} + 3H\rho_{\text{bm}}(1 + \omega_{\text{bm}}) = 0, \quad (8)$$

where $\omega_{\text{rad}} = 1/3$; $\omega_{\text{bm}} = 0$. Solving these equations, it is found that radiation evolves as $\rho_{\text{rad}} = \rho_{(\text{rad},0)}a^{-4}$ and baryonic matter as $\rho_{\text{bm}} = \rho_{(\text{bm},0)}a^{-3}$. If the density parameter $\Omega = \rho/\rho_c$ (where $\rho_c = (3H^2/8\pi G)$ and G is the universal gravitational constant) is introduced, the corresponding density parameter for each component may be obtained. This may then be

expressed in terms of redshift z through the transformation $1/a = (1+z)$. Therefore the various energy densities ρ_i are transformed into the following density parameters Ω_i :

$$\Omega_{\text{rad}} = \frac{H_0^2}{H^2} \Omega_{(\text{rad},0)} (1+z)^4 \quad (9)$$

$$\Omega_{\text{bm}} = \frac{H_0^2}{H^2} \Omega_{(\text{bm},0)} (1+z)^3 \quad (10)$$

$$Q_1 : \Omega_{\text{dm}} = \frac{H_0^2}{H^2} \Omega_{(\text{dm},0)} (1+z)^{-(\delta-3)} \quad (11)$$

$$\Omega_{\text{de}} = \frac{H_0^2}{H^2} \left[\Omega_{(\text{de},0)} (1+z)^{3(1+\omega)} + \Omega_{(\text{dm},0)} \frac{\delta}{\delta+3\omega} \left[(1+z)^{3\omega} - (1+z)^{-\delta} \right] (1+z)^3 \right] \quad (12)$$

$$Q_2 : \Omega_{\text{dm}} = \frac{H_0^2}{H^2} \left[\Omega_{(\text{dm},0)} (1+z)^3 + \Omega_{(\text{de},0)} \frac{\delta}{\delta+3\omega} \left[1 - (1+z)^{(\delta+3\omega)} \right] (1+z)^3 \right] \quad (13)$$

$$\Omega_{\text{de}} = \frac{H_0^2}{H^2} \Omega_{(\text{de},0)} (1+z)^{(\delta+3\omega+3)}. \quad (14)$$

The Λ CDM equations for Ω_{dm} , Ω_{de} are obtained by setting the interaction strength $\delta = 0$ in equations (11) - (14). The evolution of these energy densities may be seen in Figure 1:

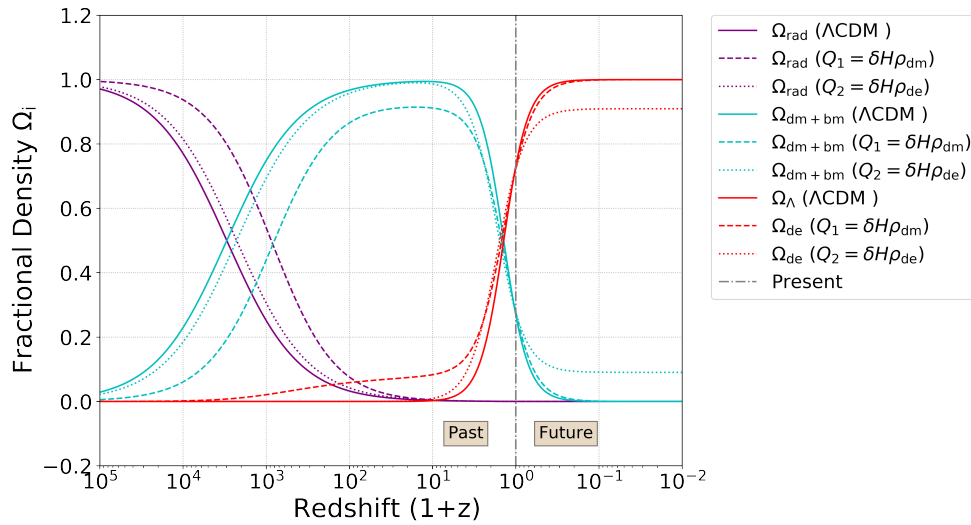


Figure 1: Fractional energy densities vs. redshift.

In all cases, there is an early time radiation domination followed by matter domination, which finally gives way to the the current era of dark energy domination. Here it may be seen that since $\delta > 0 \rightarrow Q > 0$ for both coupled models, that there is less dark matter is the past and that the matter-radiation equality therefore happened later (smaller redshift) in cosmic evolution, while the matter-dark energy equality happens earlier (larger redshift). The energy densities ρ_i (in Joule.m^{-3}) throughout cosmic evolution can be seen in Figure 2. These results are summarised in Tables 2, 3 and 4.

The previously mentioned cosmic coincidence problem may now be addressed by considering how the ratio of dark energy to dark matter $r = (\rho_{\text{dm}}/\rho_{\text{de}})$ evolves with redshift z in Figure 3. Here it can clearly be seen that for the Λ CDM case, the current value of $r \approx (\frac{3}{7})$ seems fine tuned and coincidental in comparison to Q_1 and Q_2 , where r converges and becomes constant in the past and the future respectively. Thus, alleviating the cosmic coincidence problem [3].

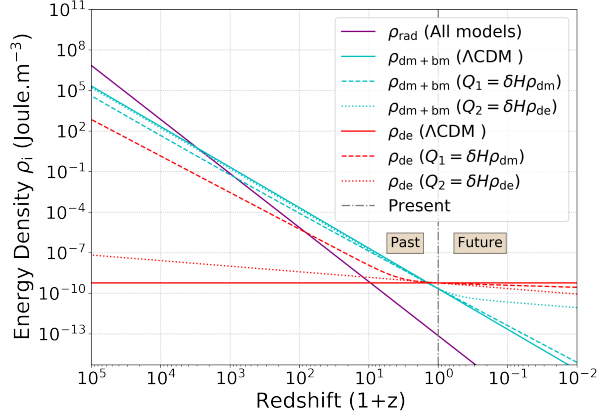


Figure 2: Energy densities vs redshift.

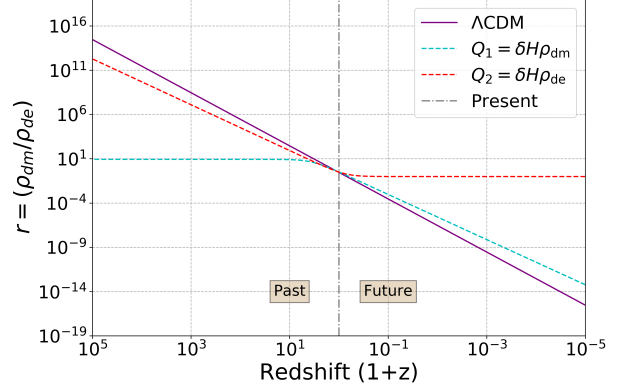


Figure 3: Cosmic Coincidence Problem.

5. Expansion history of universe models

The expansion of these universe models may be described by the Friedmann equation for a flat FLRW universe (15) and the deceleration parameter q (16), with the constituents (9)-(14):

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} (\rho_{\text{rad}} + \rho_{\text{bm}} + \rho_{\text{dm}} + \rho_{\text{de}}) , \quad (15)$$

$$q = \Omega_{\text{rad}} + \frac{1}{2} (\Omega_{\text{dm}} + \Omega_{\text{bm}}) + \frac{1}{2} \Omega_{\text{de}} (1 + 3\omega) . \quad (16)$$

The Friedmann equation may be numerically integrated, which alongside the deceleration parameter q yields the total expansion histories of the universe models:

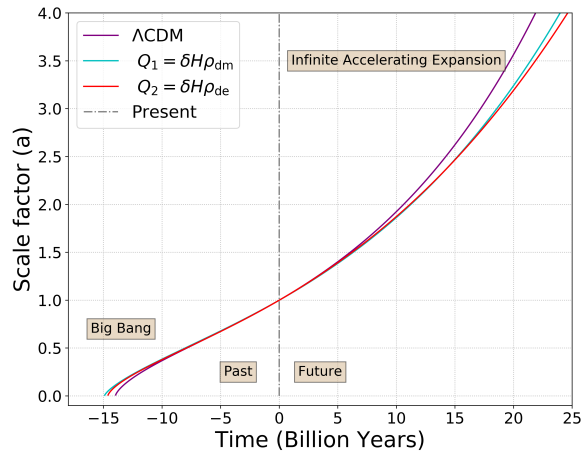


Figure 4: Expansion history of universe models.

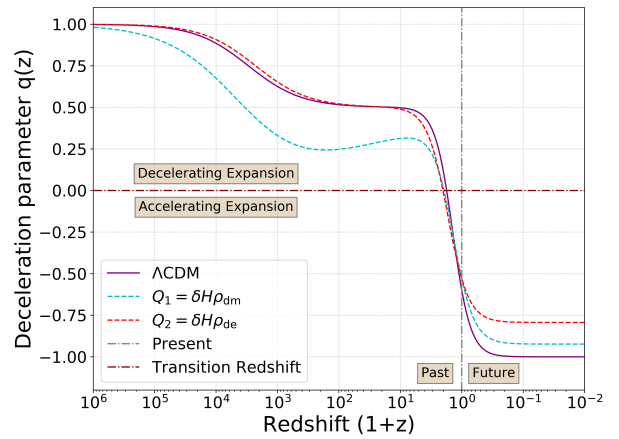


Figure 5: Deceleration parameter vs redshift.

These models all start with a big bang singularity (at $a = 0$) which leads to a period of decelerating expansion during the radiation and matter dominating epochs (the latter starting when $\rho_{\text{rad}} = \rho_{\text{dm+bm}}$), followed by an infinite accelerating expansion (starting at $q = 0$ when $\rho_{\text{dm+bm}} \approx 2\rho_{\text{de}}$) and a final era of dark energy domination (starting when $\rho_{\text{dm+bm}} = \rho_{\text{de}}$). Tables 2, 3 and 4 show the redshift z , time and energy densities (ρ_{rad} , $\rho_{\text{dm+bm}}$ and ρ_{de}) at the start of these crucial events in cosmic history for each of the models. It should be noted that cosmological parameters from Table 1 are used for the Λ CDM model instead of Planck CMB parameters [1].

Table 2: Λ CDM (Supernovae data from [8])

Event	Redshift z	Time (Gyr)	ρ_{rad}	$\rho_{\text{dm+bm}}$	ρ_{de} (J/m ³)
Big bang singularity	∞	13.96	∞	∞	∞
Radiation-matter equality	2976	13.63	5.7	5.7	$5.9e^{-10}$
Cosmic acceleration ($q = 0$)	0.76	6.76	$7.0e^{-13}$	$1.2e^{-9}$	$5.9e^{-10}$
Matter-dark energy equality	0.40	4.32	$2.8e^{-13}$	$5.9e^{-10}$	$5.9e^{-10}$

Table 3: $Q_1 = \delta H \rho_{\text{dm}}$

Event	Redshift z	Time (Gyr)	ρ_{rad}	$\rho_{\text{dm+bm}}$	ρ_{de} (J/m ³)
Big bang singularity	∞	14.90	∞	∞	∞
Radiation-matter equality	831.97	14.90	$3.5e^{-2}$	$3.5e^{-2}$	$1.7e^{-3}$
Cosmic acceleration ($q = 0$)	0.94	7.79	$1.0e^{-12}$	$1.4e^{-9}$	$7.6e^{-10}$
Matter-dark energy equality	0.48	5.07	$3.5e^{-13}$	$6.6e^{-10}$	$6.6e^{-10}$

Table 4: $Q_2 = \delta H \rho_{\text{de}}$

Event	Redshift z	Time (Gyr)	ρ_{rad}	$\rho_{\text{dm+bm}}$	ρ_{de} (J/m ³)
Big bang singularity	∞	14.61	∞	∞	∞
Radiation-matter equality	2266	14.61	1.9	1.9	$1.4e^{-8}$
Cosmic acceleration ($q = 0$)	1.02	8.05	$1.2e^{-12}$	$1.4e^{-9}$	$7.8e^{-10}$
Matter-dark energy equality	0.57	5.60	$4.4e^{-13}$	$7.0e^{-10}$	$7.0e^{-10}$

6. Conclusions

Some implications of a coupling in the dark sectors of the universe have been explored (Tables 2, 3 and 4) and it has been found that these models may possibly help alleviate the coincidence problem as well as the Hubble tension. Since these problems and the cosmological constant problem cast reasonable doubt on the Λ CDM model, alternative dark energy models and their implications and limitations should be further investigated. Finally, since early time instabilities have not yet been considered, these results should be taken as preliminary.

Also - you have not considered crucial constraints from large-scale structure - especially BAO and RSD. You need to comment on that omission and cite some references.

Acknowledgements

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References

- [1] Planck Collaboration 2018 *Astron. Astrophys.* **641** A6
- [2] Weinberg S 1989 *Rev. Mod. Phys.* **61** 1-23
- [3] del Campo S, Herrera R and Pavon D 2009 *J. Cosmol. Astropart. Phys.* **2009** 020
- [4] Di Valentino E, Melchiorri A, Mena O and Vagnozzi S 2020 *Phys. Dark Universe* **30** 100666
- [5] Valiviita J, Majerotto E and Maartens R 2009 *J. Cosmol. Astropart. Phys.* **2008** 020
- [6] Gavela M B, Hernandez D et al. 2009 *J. Cosmol. Astropart. Phys.* **07** 034
- [7] Lucca M, Hooper D C 2020 *Phys. Rev. D* **102** 123502
- [8] Hough R T 2020 *Constraining modified gravity models with cosmological data* Master's thesis North-West University
- [9] Hough R T, Abebe A and Ferreira S E S 2020 *Eur. Phys. J. C* **80** 787