Dark Coupling

Cosmological implications of interacting dark energy and dark matter fluids

Marcel van der Westhuizen & Amare Abebe

North-West University, Centre for Space Research

Contact Information:
Centre For Space Research
North-West University
Potchefstroom Campus

Phone: 084 293 8928
Email: marcelvdw007@gmail.com

Problems with ΛCDM model

The expansion of the universe has thus far been well described by the ΛCDM model, where the energy budget of the universe is divided between $\approx 5\%$ baryonic matter (standard model particles), $\approx 25\%$ non-baryonic cold dark matter (which keeps galaxies from flying apart) and $\approx 70\%$ dark energy in the form of the cosmological constant Λ (which explains late-time accelerated expansion). This model has proven to be very successful, but problems with the ΛCDM model remain, which include:

The Cosmological Constant Problem, which refers to the measured energy density of the vacuum being over a 120 orders of magnitude smaller than the theoretical prediction.

The Cosmic Coincidence Problem, which alludes to the dark matter and dark energy densities having the same order of magnitude at the present moment of cosmic history, while differing with many orders of magnitude in the past and predicted future [1].

The Hubble Tension, which concerns the 4.4σ level difference between values of the Hubble constant H_0 as measured from the Cosmic Microwave Background versus the value obtained from Type Ia Supernovae using a calibrated local distance ladder [2] .

Dark Coupling Models

These problems motivate research beyond the ΛCDM model. One possible approach is to investigate cosmological models in which there are non-gravitational interactions between the dark sectors of the universe. This allows the two dark sectors to exchange energy (and/or momentum) while dark matter and dark energy are not separately conserved, but the energy (and/or momentum) of the total dark sector is conserved. This coupling between dark matter and dark energy modifies the continuity equations into:

$$\dot{\rho}_{dm} + 3H\rho_{dm} = Q \quad ; \quad \dot{\rho}_{de} + 3H\rho_{de}(1+\omega) = -Q \tag{1}$$

Where $\rho_{dm/de}$ are the dark matter/energy densities, ω is the equation of state of dark energy and Q is the rate of energy exchange, which defines the direction of energy flow between the dark sectors such that:

$$Q = \begin{cases} > 0 & \text{Dark Energy} \to \text{Dark Matter} \\ < 0 & \text{Dark Matter} \to \text{Dark Energy} \\ = 0 & \text{No interaction } (\Lambda\text{CDM case}) \end{cases} \tag{2}$$

It should be noted that in order to avoid "fifth force" constraints, we assume that photons (rad) and baryons (b) are separately conserved and uncoupled. The behaviour of these coupled models may be understood by seeing how the interaction effects the effective equations of state, relative to the uncoupled background equations ($Q = 0$) in (1) such that:

$$\omega_{dm}^{eff} = -\frac{Q}{3H\rho_{dm}} \quad ; \quad \omega_{de}^{eff} = \omega_{de} + \frac{Q}{3H\rho_{de}} \tag{3}$$

Thus, the effects of an interaction may be understood to imply that if:

$$Q > 0 \to \begin{cases} \omega_{dm}^{eff} < 0 & \text{Dark matter redshifts } \textit{slower} \text{ (less DM in past)} \\ \omega_{de}^{eff} > \omega_{de} & \text{Dark energy has } \textit{less} \text{ accelerating pressure} \end{cases}$$

$$Q < 0 \to \begin{cases} \omega_{dm}^{eff} > 0 & \text{Dark matter redshifts } \textit{faster} \text{ (more DM in past)} \\ \omega_{de}^{eff} < \omega_{de} & \text{Dark energy has } \textit{more} \text{ accelerating pressure} \end{cases}$$

When $Q = 0$, the effective equations of state reduce back to the case for the ΛCDM model, where dark matter is pressureless ($\omega_{dm} = 0$) and dark energy has a constant negative pressure.

Since there is currently no fundamental theory for these couplings, they are purely phenomenological and must be tested against observations. We will consider two models which have interactions proportional to the Hubble parameter. Solving the conservation equations (1) for both models show how the energy densities evolves, such that:

Model 1: $Q_1 = \delta H \rho_{dm}$

$$\rho_{dm} = \rho_{dm}^{(0)} a^{(\delta-3)} \tag{4}$$

$$\rho_{de} = \rho_{de}^{(0)} a^{-3(1+\omega_{de})} + \rho_{dm}^{(0)} \frac{\delta}{\delta + 3\omega_\Lambda} \left[a^{-3\omega_\Lambda} - a^{\delta}\right] a^{-3} \tag{5}$$

with $\left(0 < \delta < -\frac{3\omega}{r_0+1}\right)$ to ensure $\rho_{dm/de} > 0$ throughout evolution.

Model 2: $Q_2 = \delta H \rho_{de}$

$$\rho_{dm} = \rho_{dm}^{(0)} a^{-3} + \rho_{de}^{(0)} \frac{\delta}{\delta + 3\omega} \left[1 - a^{-(\delta+3\omega)}\right] a^{-3} \tag{6}$$

$$\rho_{de} = \rho_{de}^{(0)} a^{-(\delta+3\omega+3)} \tag{7}$$

with $\left(0 < \delta < -\frac{3\omega}{1/r_0+1}\right)$ to ensure $\rho_{dm/de} > 0$ throughout evolution.

Here $r_0 = \frac{\rho_{de}}{\rho_{dm}}$ is the ratio of dark energy to dark matter today; and δ is a dimensionless coupling constant which determines the strength of the interaction between dark matter and dark energy. Furthermore, when $Q = 0$, both models reduce to the ΛCDM case where $\rho_{dm} \propto a^{-3}$ and $\rho_{de} =$ constant.

Cosmological Parameters

The current cosmological parameters for these models may be obtained from Type-Ia Supernovae data from a previously developed Markov Chain Monte-Carlo (MCMC) simulation for a flat FRLW universe. This gives the following results:

Model	Ω_{dm}	Ω_{bm}	H_0	ω	δ
ΛCDM	$0.213^{+0.037}_{-0.037}$	$0.055^{+0.031}_{-0.030}$	$69.7^{+0.5}_{-0.5}$	$-1.000^{+0.000}_{-0.000}$	$0.000^{+0.000}_{-0.000}$
Q_1	$0.234^{+0.036}_{-0.024}$	$0.043^{+0.022}_{-0.016}$	$68.0^{+0.9}_{-0.9}$	$-0.949^{+0.057}_{-0.036}$	$0.296^{+0.146}_{-0.184}$
Q_2	$0.232^{+0.031}_{-0.022}$	$0.044^{+0.021}_{-0.017}$	$69.4^{+0.5}_{-0.5}$	$-0.948^{+0.059}_{-0.037}$	$0.257^{+0.161}_{-0.167}$

with $\Omega_{de} = 1 - \Omega_{dm} - \Omega_{bm}$. Here we can see that H_0 is slightly lower and closer to the CMB value for both Q_1 and Q_2, which slightly alleviates the *Hubble Tension* [2] .

Evolution of Energy Densities

The evolution of the energy densities of the various constituents of the universe may now be considered. The dark matter and dark energy densities evolve according equations (4)-(7), while radiation evolves as $\rho_{rad} \propto a^{-4}$ and baryonic matter as $\rho_{bm} \propto a^{-3}$. Thus the evolution of the energy densities may be seen below:

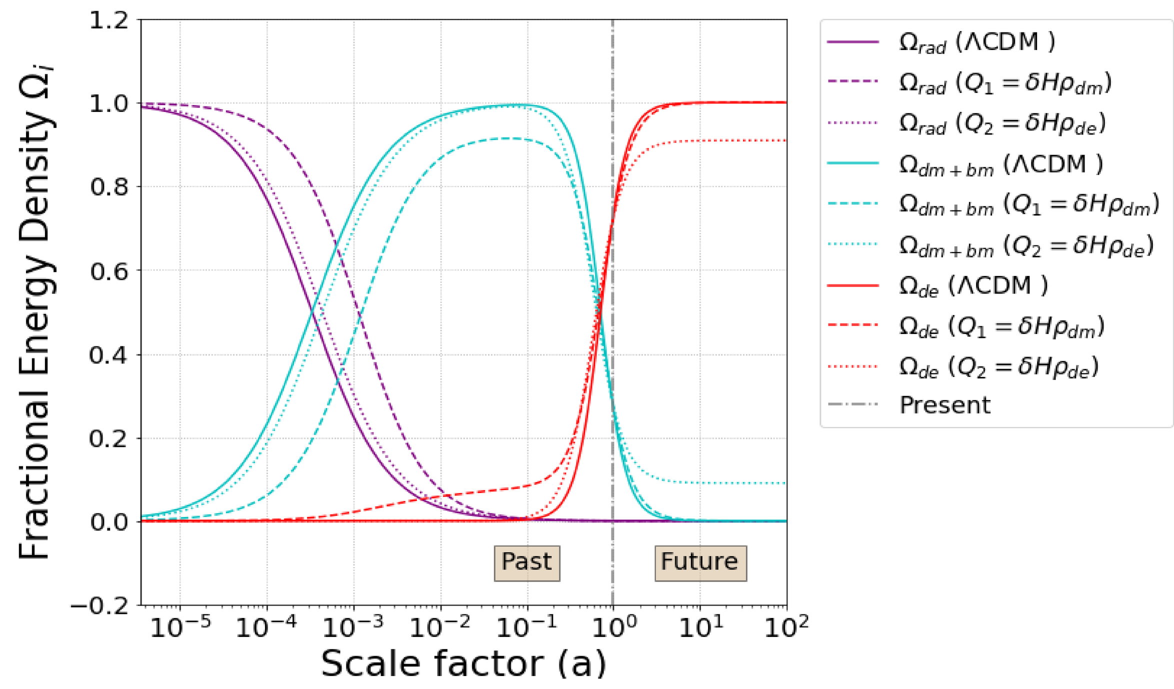

Figure 1: Fractional Energy Densities vs. Scale Factor

In all cases, there is an early time radiation domination followed by matter domination and finally giving way to the current era of dark energy domination. Here we may see that since $\delta > 0 \to Q > 0$ for both coupled models, that there is *less* dark matter is the past and that the matter-radiation equality therefore happened *later* in cosmic evolution.

The previously mentioned *Cosmic Coincidence Problem* may now be addressed by considering how the ratio of dark energy to dark matter $r = \frac{\rho_{dm}}{\rho_{de}}$ evolves with scale factor:

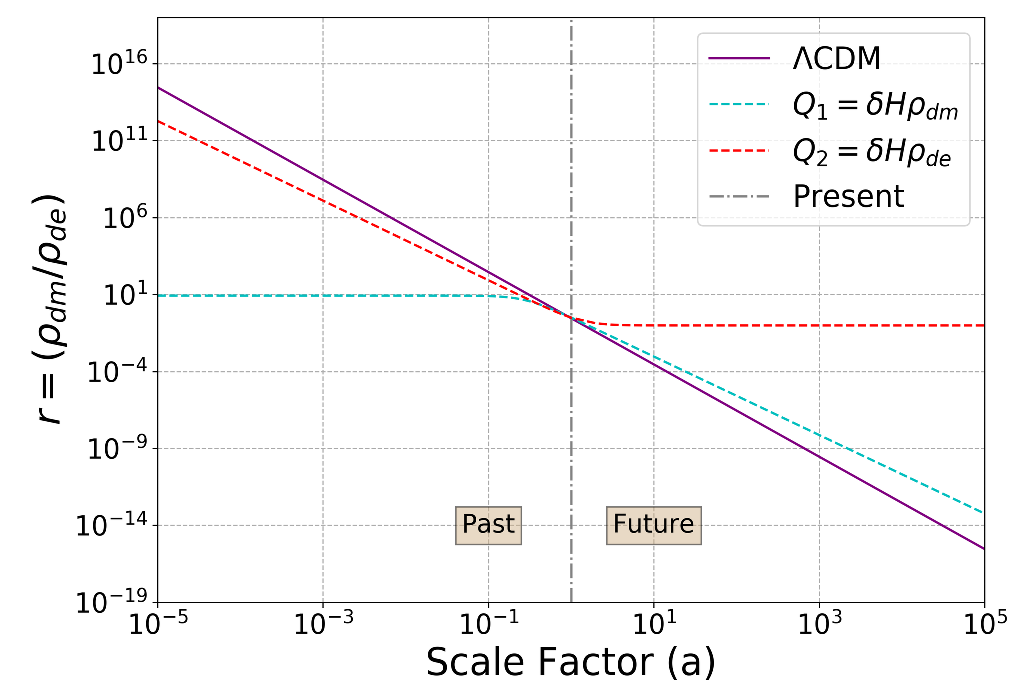

Figure 2: Fractional Densities $r = \frac{\rho_{dm}}{\rho_{de}}$ vs. Scale Factor (Coincidence Problem)

In Figure 2 it can clearly be seen that for the ΛCDM case, the current value of $r \approx \left(\frac{3}{7}\right)$ seems fine tuned and coincidental in comparison to Q_1 and Q_2, where r converges in the *past* and the *future* respectively. [1]

Expansion History of the Universe

The evolution of these universe models may be described by the following Friedmann equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left(\rho_{rad} + \rho_b + \rho_{dm} + \rho_{de}\right) - \kappa\frac{c^2}{a^2} \tag{8}$$

This equation may be numerically integrated and yields the total expansion history of the universe models:

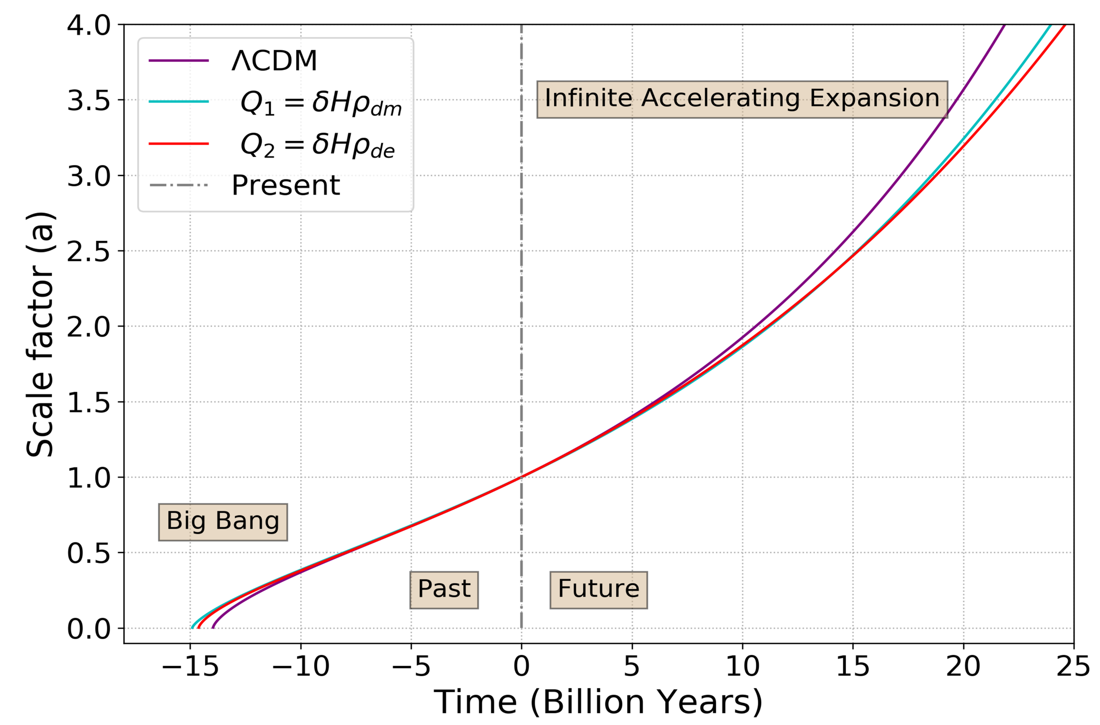

Figure 3: Expansion History of Universe Models

These models all start with a *Big Bang* singularity which leads to a period of decelerating expansion (matter domination) followed by an infinite *accelerating expansion* (dark energy domination). The following table shows at which redshift z, as well as how many billion years ago these crucial events in cosmic history occurred:

Model	Age of Universe	Accelerating Expansion	$de = (dm + bm)$
ΛCDM	13.96 Gyr	z = 0.76 (6.76 Gyr)	z = 0.40 (4.32 Gyr)
Q_1	14.90 Gyr	z = 0.94 (7.79 Gyr)	z = 0.48 (5.07 Gyr)
Q_2	14.61 Gyr	z = 1.02 (8.05 Gyr)	z = 0.57 (5.60 Gyr)

Conclusions

We have explored some implications of a coupling in the dark sectors of the universe and have seen that these models may possibly help alleviate both the *Coincidence Problem* as well as the *Hubble Tension*. Since these problems and the *Cosmological Constant Problem* cast reasonable doubt on the ΛCDM model, alternative dark energy models and their implications and limitations should be further investigated.

References

[1] Campo et al. Interacting models may be key to solve the cosmic coincidence problem. *Journal of Cosmology and Astroparticle Physics*, 2009(01):020020, Jan 2009.

[2] Di Valentino et al. Interacting dark energy in the early 2020s: A promising solution to the h0 and cosmic shear tensions. *Physics of the Dark Universe*, 30:100666, Dec 2020.