A dynamical systems analysis of interacting dark energy models

Vuyani MOLOSI, Amare ABEBE
North-West University, Centre for Space Research, Potchefstroom
32695306@nwu.ac.za

Abstract

We investigate, using dynamical system analysis, the impacts of various interaction models whereby dark energy is coupled with dark matter. Phase space analysis of each interaction is conducted where we obtain the cosmological consequence of each choice of interaction with all components of the universe considered, namely, the radiation, matter, and dark energy dominated universes. We show that linear models breakdown at the early stages of the universe thus introducing product-like models to resolve for the breakdown. Thorough analysis on the nature of critical points was conducted, from which we found the existance of unstable radiation epoch; unstable dark matter epoch; and stable dark energy epoch. An upper limit on the coubling cosntant for interactions between dark matter and dark energy was found. This limit was crucial for cosmologically acceptable results of the matter dominated epoch, that is, instability and deceleration of this epoch.

Introduction

Different observations from various sources have proposed that our universe is currently experiencing a phase of accelerated expansion [4, 5]. This expansion has caused an imbalance in the governing Frieedmann equations and thus it is a challenging issue for standard cosmology today. There have been attempts to counteract the imbalances, either by changing the governig equations or by introducing new source terms in the Friedmann equations. In the framework of standard cosmology, this source is called dark energy (DE) [1]. The cosmological constant Λ is the simplest candidate for DE which give a vacuum energy backround responsible for the current acceleration. Though we know that DE has a negative pressure, its fundamental nature is still unknown in modern cosmology.
Dark matter (DM) is one other component of the universe which its fundamental nature is yet to be known. DM has so far only been detected gravitationally, of which its candidates include black holes, massive halo objects, and plenty other non-baryonic particle models [2, 3]. Since it is known that DM and DE exist out there in the universe, they must interact with each other, and since there is no underlying theory that prohibits their interaction. We then attempt to find answers to the natural question; what is the form of interaction term between the two components, if they indeed interact.

Dynamical systems in cosmology

For a flat Universe filled with radiation, matter and dark energy, we give the Friedmann equation as,

$$H^2 = \frac{8\pi G}{3}\left(\rho_r + \rho_m + \rho_\Lambda\right) \quad (1)$$
$$1 = \frac{8\pi G}{3H^2}\rho_r + \frac{8\pi G}{3H^2}\rho_m + \frac{8\pi G}{3H^2}\rho_\Lambda \quad (2)$$
$$= \Omega_r + \Omega_m + \Omega_\Lambda \quad (3)$$

where

$$\Omega_r = \frac{8\pi G}{3H^2}\rho_r \quad ; \quad \Omega_m = \frac{8\pi G}{3H^2}\rho_m \quad ; \quad \Omega_\Lambda = \frac{8\pi G}{3H^2}\rho_\Lambda \quad (4)$$

are the fractional energy densities of radiation, matter and dark energy, respectively. We then define new dimensionless variables;

$$x^2 = \Omega_m \quad ; \quad y^2 = \frac{8\pi G\alpha}{3H} \quad ; \quad m^2 = \frac{8\pi G\beta}{3} \quad ; \quad y^2 + m^2 = \Omega_\Lambda \quad (5)$$

where α and β are constants with dimensions $(mass)^3$ and $(mass)^2$, respectively.

Continuity equations

We incorporate the interaction term Q between dark matter and the dark energy components into the continuity equations;

$$\dot{\rho_r} + 4H\rho_r = 0 \quad (6)$$
$$\dot{\rho_m} + 3H\rho_m = Q \quad (7)$$
$$\dot{\rho_\Lambda} + 3H(1+w_\Lambda)\rho_\Lambda = -Q \quad (8)$$

- $Q > 0$ implies transition of energy content from DE to DM
- $Q < 0$ implies transition of energy content from DM to DE

Due to the complex nature of our Friedmann equations, we switch to the newly defined dimensionless variables, the dynamical equations then take the form;

$$x' = \frac{x^2(2m^2 + 2x^2 + 5y^2 - 2) + f(x,y)(2m^2 + 2x^2 + y^2 - 2)}{2x(2m^2 + y^2 - 2)},$$
$$y' = \frac{y[(4m^2 + x^2 + 4y^2 - 4) + f(x,y)]}{4m^2 + 2y^2 - 4} \quad (9)$$

which are generalizations of the Friedmann equations. The function f is given as, $f(x,y) = \frac{8\pi G}{3H^3}Q$.

The evolution of IDE models

Different forms of the interaction term Q were investigated. Some of the interaction terms studied are of the form;

(A) $Q = 0$ *(no interaction)*
(B) $Q = 3b^2 H\rho_{tot}$
(C) $Q = 3b^2 H\frac{\rho_\Lambda\rho_m}{\rho_{tot}}$
(D) $Q = 3b^2 H\frac{\rho_m^2}{\rho_{tot}}$

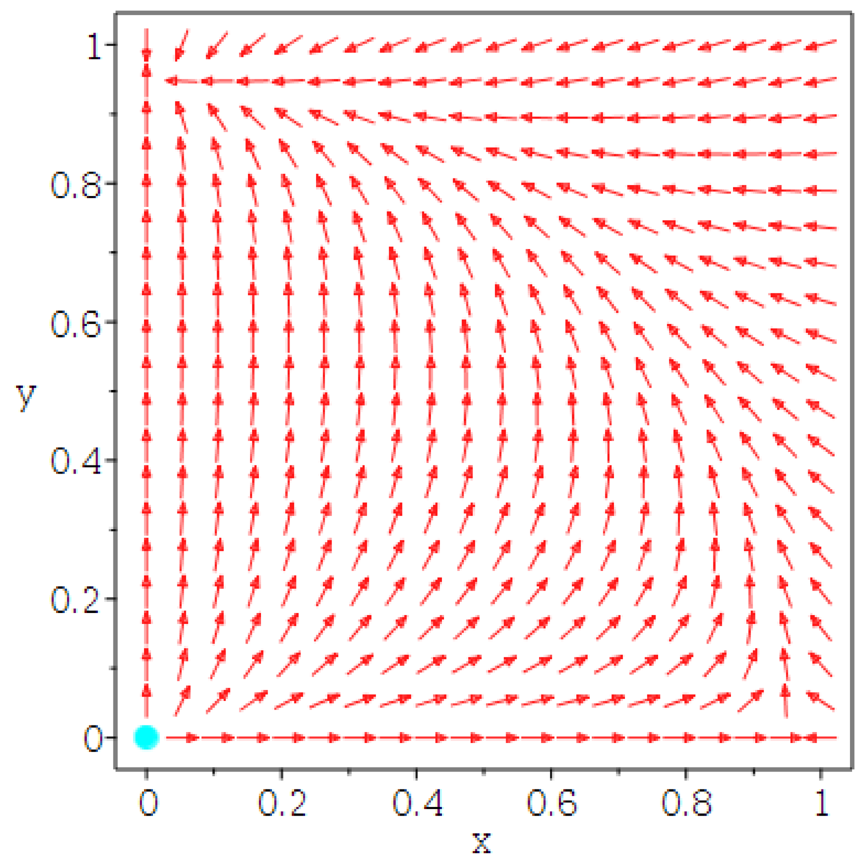

Figure 1: The depiction of the evolution of phase space for the non-interaction case, $Q = 0$. Coupling constant is chosen as $b = 0.2$.

Analysis of the critical points shows existance of;

- **Unstable** radiation dominated epoch at $(x, y) = (0, 0)$.
- **Unstable** matter dominated epoch at $(x, y) = (1, 0)$.
- **Stable** dark energy dominated epoch at $(x, y) = (0, 1)$.

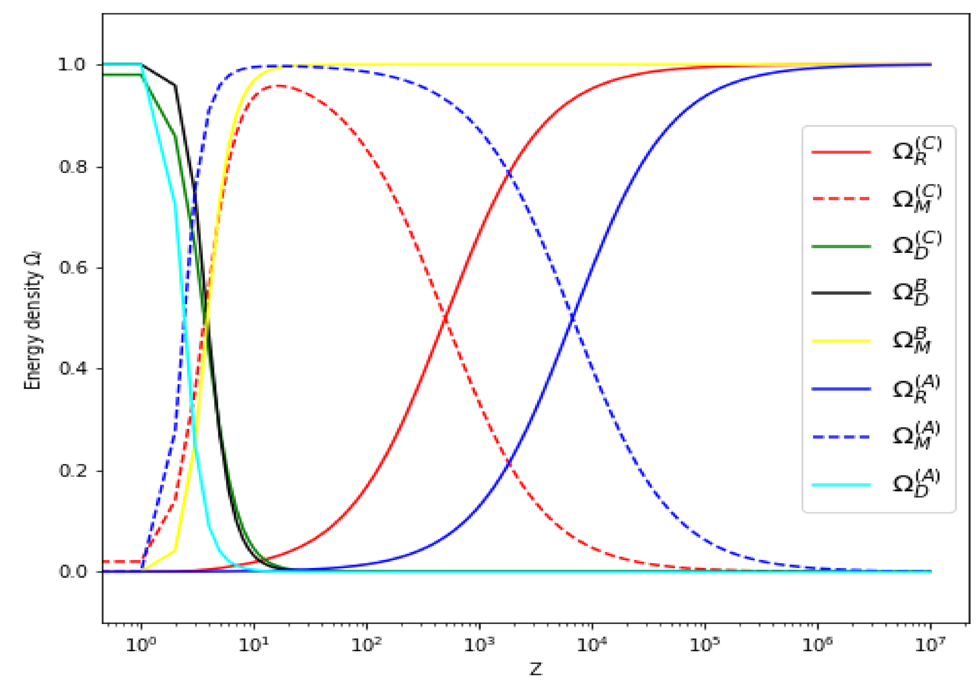

Figure 2: The cosmic evolution of Ω_i is plotted as a function of redshift, for various interactions Q.

- Non-interacting case, $Q = 0$, corresponds to the ΛCDM model
- Initial conditions were chosen at present as $\Omega_{0,r} = 4.5 \times 10^{-5}$, $\Omega_{0,m} = 0.3$ and $\Omega_{0,\Lambda} = 0.7$.

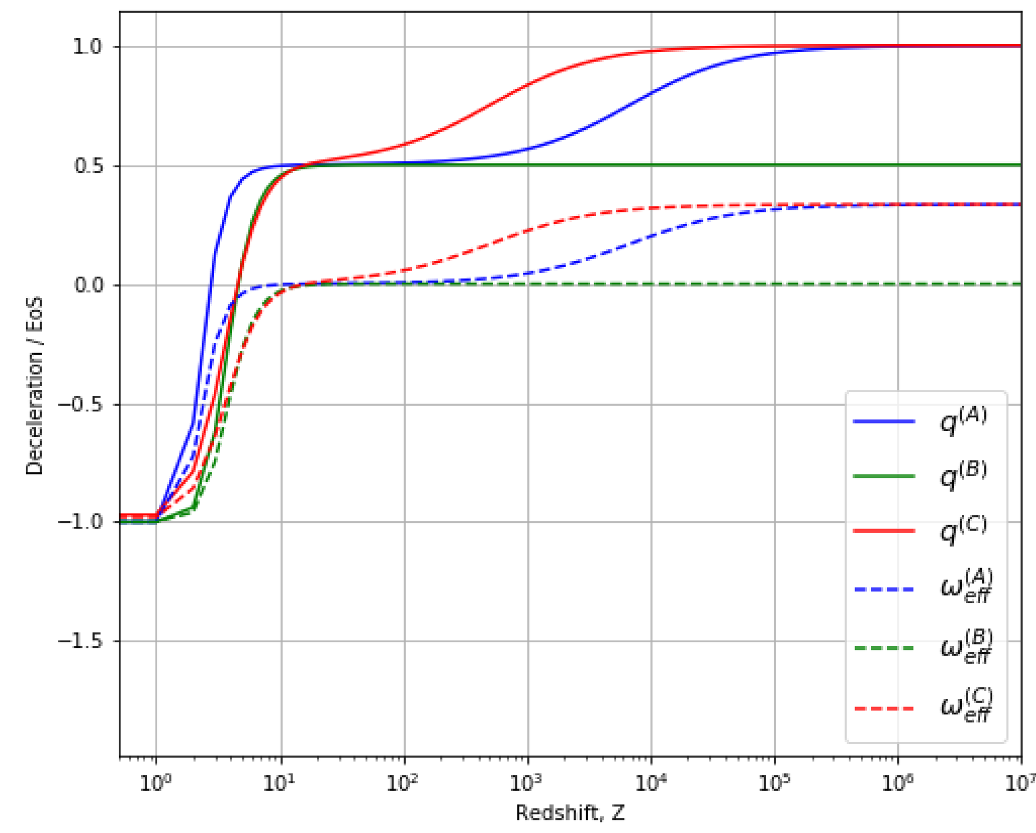

Figure 3: The evolution of deceleration parameter and equation of state are plotted as a function of redshift, for various interaction.

- The **Radiation** epoch has $w_{eff} = 1/3$ and $q = 1$, while
- The **Matter** dominated epoch has $w_{eff} = 0$ and $q = 1/2$, and
- The **Dark energy** epoch showed $w_{eff} < 0$ and $q = -1$.

Observational constraints

The radiation epoch ended at $t = 50,953$ years when its density matched with that of the matter at a redshift of $z = 3408.27$.

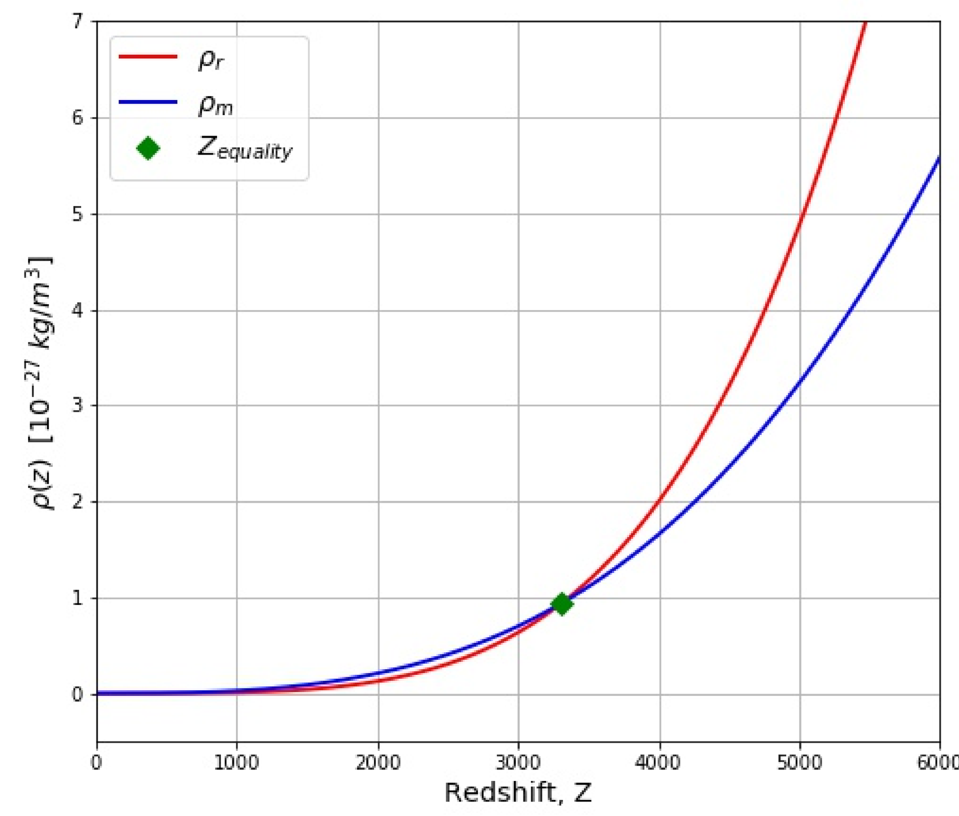

Figure 4: The radiation density (red) and matter density (blue) are ploted as function of redshift. The green diamond point shows the equality redshift at 3408.27

The matter dominated era ended at a time $t = 10.19$ billion years when matter and DE densities became equal, at a redshift $z = 0.3079$.

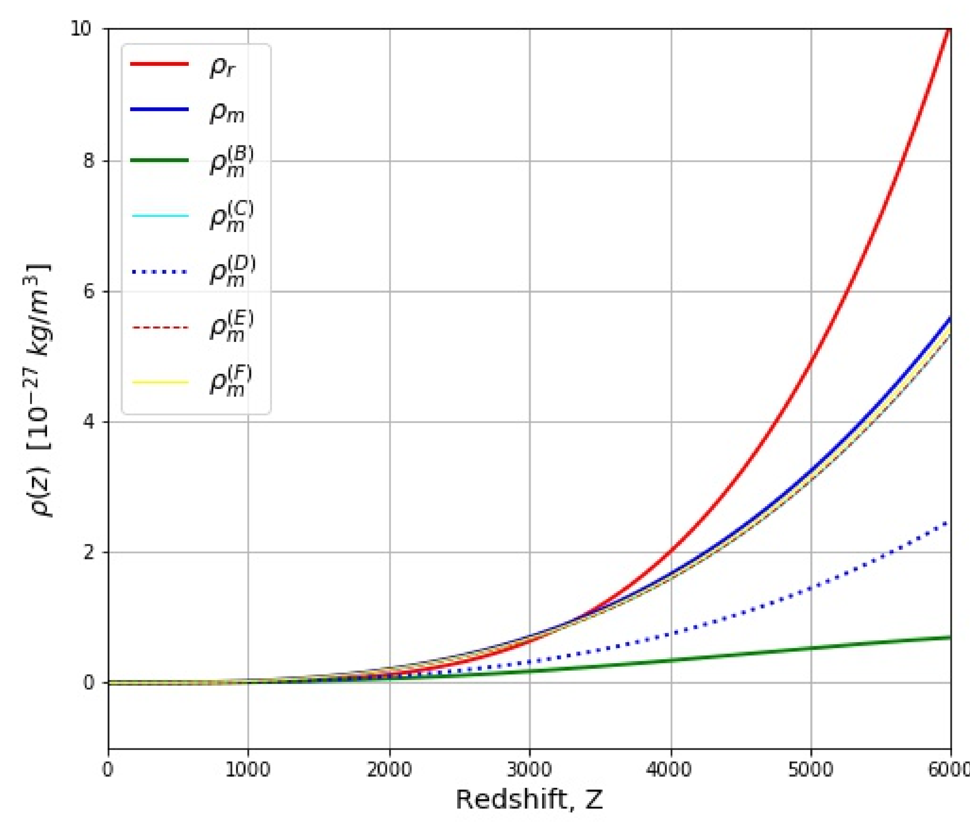

Figure 5: The radiation density and matter density of various interactions are ploted as function of redshift.

Conclusions

Analysis of linear and non-linear forms of the interaction term Q have shown that linear interactions suffer from lack of the radiation epoch. However, non-linear terms showed cosmologically acceptable results, showing all three epochs, namely, the unstable radiation dominated, unstable matter dominated, and stable dark energy dominated epoch. The radiation era ended at a redshift $z = 3408.27$ from which then the matter dominated epoch began to dominate. We also found that at $z = 0.3079$ the matter dominated era ended which was around $13.8 - 10.19 = 3.61$ billion years ago. With interactions considered, we found various redshifts for radiation-matter, and matter-dark energy equalities.

References

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