Obliquely propagating ion-acoustic supersolitons in a collisionless magnetized three-component plasma consisting of adiabatic ions and two-temperature electrons

Dr. Shivani Singh
Large amplitude nonlinear soliton and supersoliton ion-acoustic waves are investigated in a three-component magnetized plasma model, consisting of warm inertial adiabatic ions and two-temperature electrons.

The direction of wave propagation is oblique to the magnetic field.

The isothermal cool electrons are Boltzmann distributed.

The non-thermal hot electrons are Cairns distributed.
The existence of nonlinear solitary structures, which are propagating obliquely relative to the ambient magnetic field are are determined by using the Sagdeev pseudopotential formalism, under the assumption of quasineutrality.

The plasma composition and parameter range is tested to establish whether the system supports the existence of supersolitons.

The plasma model supports positive and negative potential solitons, coexistence, negative potential double layers and positive potential supersolitons.
The Parameters

The plasma system is enclosed by the following parameters.

- $\omega$ – the ion cyclotron frequency
- $n_c$ – the density of the isothermal cold electrons
- $n_h$ – the density of the non-thermal hot electrons
- $n$ – the density of the ions
- $T_i$ – the average temperature of the ions
- $n_{c0}$ – density of the undisturbed isothermal cold electrons
- $n_{h0}$ – density of the undisturbed non-thermal hot electrons
- $T_c$ – the average temperature of the isothermal cold electrons
- $T_h$ – the average temperature of the non-thermal hot electrons
- $\beta = \frac{4\alpha}{1 + 3\alpha}$, for $\alpha \geq 0$ – Cairns non-thermal parameter
- $\gamma = \frac{5}{3}$ – heat ratio
The Parameters

- $\theta$ – angle between the magnetic field and the direction of propagation
- $M$ – Mach number
- $\phi$ – electrostatic potential

\[
\frac{n_{c0} + n_{h0}}{T_{ef}} = \frac{n_{c0}}{T_c} + \frac{n_{h0}}{T_h}
\]

\[
\sigma_c = \frac{T_{ef}}{T_c}, \quad \sigma_h = \frac{T_{ef}}{T_h}, \quad n_{ch} = \frac{n_{c0}}{n_{h0}}, \quad \sigma_{ch} = \frac{T_c}{T_h} \quad \text{and} \quad \sigma = \frac{T_i}{T_{ef}}
\]

\[
n_{h0} + n_{c0} = n_0, \quad \overline{n_{h0}} = \frac{n_{h0}}{n_0} \quad \text{and} \quad \overline{n_{c0}} = \frac{n_{c0}}{n_0}
\]

\[
\overline{n_c}\sigma_c + \overline{n_h}\sigma_h = 1 \quad \text{and} \quad \overline{n_c} + \overline{n_h} = 1
\]

\[
n = \overline{n_h}(1 - \beta\sigma_h\phi + \beta\sigma_h^2\phi^2)\exp(\sigma_h\phi) + \overline{n_c}\exp(\sigma_c\phi)
\]
\[ S(\phi) = \phi + \frac{\sigma \gamma}{\gamma - 1} n^{\gamma - 1} + \frac{M^2}{2n^2} \] (6)

\[ G(\phi) = \sigma (n^\gamma - 1) + \frac{n_c}{\sigma_c} \left[ \exp(\sigma_c \phi) - 1 \right] \] (7)

\[ + \frac{n_h}{\sigma_h} [(1 - 3\beta - 3\beta \sigma_h \phi + \beta \sigma_h^2 \phi^2) \exp(\sigma_h \phi) - (1 + 3\beta)] \] (8)

\[ F(\phi) = n - 1 - \frac{\cos^2 \theta}{M^2} nG \] (9)

\[ V(\phi, M) = -\omega^2 \frac{\int_0^\phi S'(\phi)F(\phi)d\phi}{[S'(\phi)]^2} \] (10)
Figure 1: Sagdeev potentials for $\beta = 0.5$, $\gamma = \frac{5}{3}$, $\cos \theta = 0.7$, $\sigma = 0.01$, $\omega_c = 0.2$, $\sigma_{ch} = 0.1$ and $n_{ch} = 0.1$. The color coding is as follows. The red curve corresponds to $M_{\text{min}} = 0.8133$. The black curve is $M = 0.92$, the blue curve is $M = 0.98$ and the green figure is $M_{\text{di}} = 1.009$. 
Figure 2: Upper panel: The electric potential profile for the (red) soliton. The electric field signature of the soliton has a symmetrical bell shape. Lower Panel: The electric potential profile for the (blue) supersoliton. It is wiggled in appearance.
Figure 3: The singularity that occurs when
\( \sigma = 0.001, \gamma = \frac{5}{3}, \cos \theta = 0.7, \omega_c = 0.2, \sigma_{ch} = 0.1 \) and \( M = 1.1. \)
The three critical values of $n_{ch}$ for which $V'''(0, M_{min})$ changes polarity from negative to positive are as follows:

1. for $\sigma = 0.1$, $V'''(0, M_{min}) > 0$ for all $n_{ch} > 0.2836$. For $n_{ch} \leq 0.2835$, $V'''(0, M_{min}) < 0$

2. for $\sigma = 0.01$, $V'''(0, M_{min}) > 0$ for all $n_{ch} > 0.4033$. For $n_{ch} \leq 0.4033$, $V'''(0, M_{min}) < 0$

3. for $\sigma = 0.001$, $V'''(0, M_{min}) > 0$ for all $n_{ch} > 0.4179$. For $n_{ch} \leq 0.4179$, $V'''(0, M_{min}) < 0$
Figure 4: Sagdeev potentials for $\sigma = 0.001$, $\gamma = \frac{5}{3}$, $\cos \theta = 0.7$, $\omega_c = 0.2$, $M = 0.98$ and $\sigma_{ch} = 0.1$. The red dashed curve corresponds to $n_{ch} = 0.0005$, blue is for $n_{ch} = 0.001$, green is for $n_{ch} = 0.002$ and black is for $n_{ch} = 0.003$. 
Figure 5: Sagdeev potentials for
\( \sigma = 0.001, \gamma = \frac{5}{3}, \cos \theta = 0.7, \omega_c = 0.2, M = 0.98 \) and \( \sigma_{ch} = 0.1 \). The red dashed curve corresponds to \( n_{ch} = 0.23 \), black is for \( n_{ch} = 0.11 \), blue is for \( n_{ch} = 0.09 \) and green is for \( n_{ch} = 0.08 \).
Figure 6: Sagdeev potentials for $\sigma = 0.001$, $\gamma = \frac{5}{3}$, $\cos \theta = 0.7$, $\omega_c = 0.2$, $M = 0.98$, $\sigma_{ch} = 0.1$ and $n_{ch} = 0.1$. The red dashed curve is $\beta = 0$, the black dashed curve is $\beta = 0.1$, the blue dashed curve is $\beta = 0.2$ and the green dashed curve is $\beta = 0.3$. 
Figure 7: Sagdeev potentials for $\sigma = 0.001$, $\gamma = \frac{5}{3}$, $\cos \theta = 0.7$, $\omega_c = 0.2$, $M = 0.98$, $\sigma_{ch} = 0.1$ and $n_{ch} = 0.1$. The red dashed curve is $\beta = 0.405$, the black dashed curve is $\beta = 0.45$, the green dashed curve is $\beta = 0.5$ and the blue dashed figure is $\beta = 0.76$. 

References


Thank you