# Equation of State of Neutron Stars

S.Gopal[a] , T.Motsei[b], B.Nkosi[c] ,P.Thokwane[d] , T.C Khumalo[c,e] , A. Muronga[f]

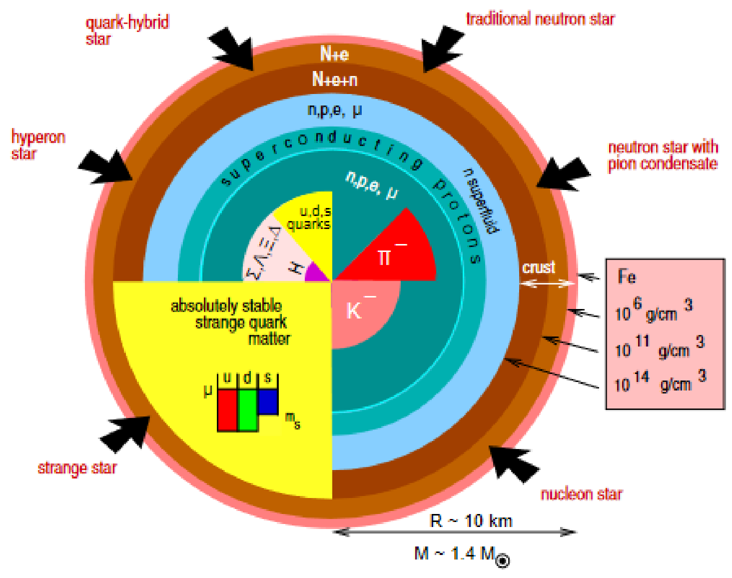
[a] University of the Western Cape,Cape Town, South Africa
[b] University of the Free State
[c] University of Witwatersrand, Johannesburg, South Africa
[d] University of Venda, Limpopo, South Africa
[e] iThemba LABS, Cape Town, South Africa
[f] Nelson Mandela University, Gqeberha, South Africa

NITheP National Institute for Theoretical Physics

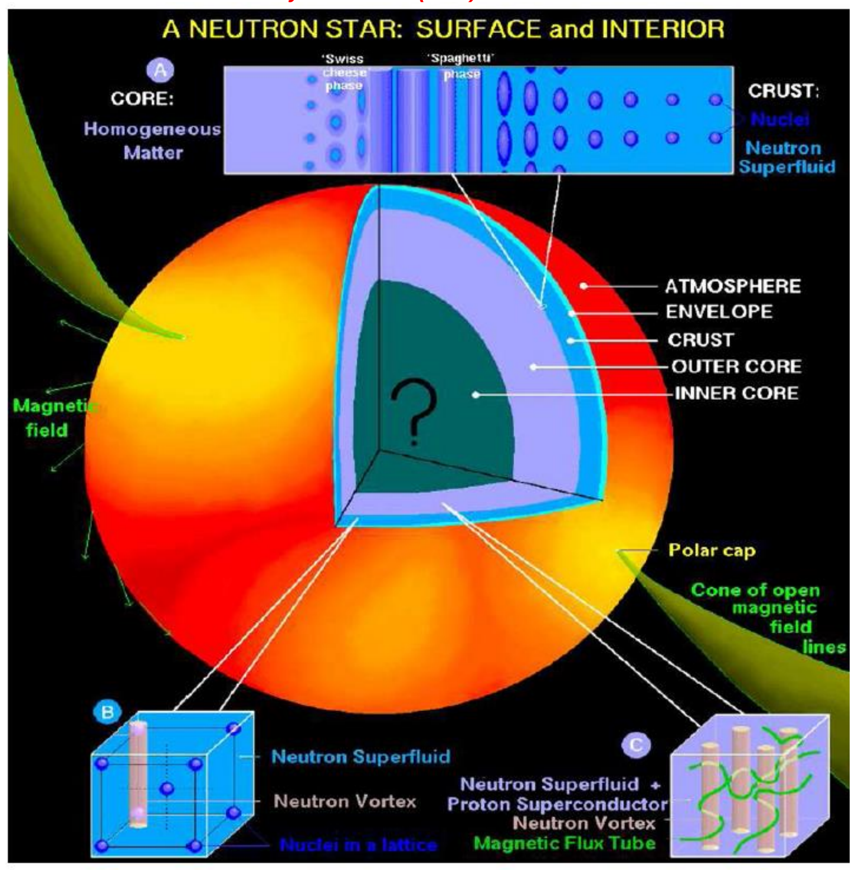
NELSON MANDELA UNIVERSITY

## SCIENTIFIC MOTIVATION

- To understand the properties of nuclear matter under extreme conditions in heavy ion collisions and astrophysics.
- Neutron stars are stellar laboratories of nuclear matter under extreme conditions, thus, understanding the structure of neutron stars will broaden our knowledge on the behaviour of matter in the quark-gluon plasma (QGP) formed in heavy-ion collisions.
- For this work, we do this by understanding the dependence of the mass-radius and pressure-radius relation on the relevant equation of state (EoS).




J.M. Lattimer New Astronomy Reviews 54 (2010) 101 -109




### INTRODUCTION TO NEUTRON STARS & EQUATION OF STATE

- The equation of state is important as it can be used to describe the behavior of the matter in a star under extreme conditions.
- Recent detection of gravitational waves from a neutron star merger by LIGO and VIRGO, in 2017 and the recent merger of a black hole and neutron star provide parameters to constrain the Equation of State

Properties of neutron star

- Mass- Ranges between 1.18 -2 (Solar Mass)
- Radius- Range between 10- 20 km
- Density- ~$10^{17}$ kg/m$^3$
- Number of neutrons~$10^{57}$ neutrons

## EQUATION OF STATE OF COMPACT STELLAR OBJECTS: WHITE DWARF & NEUTRON STAR CASES

- Stellar objects structure equations were derived from classical mechanics
- Spherical geometry was assumed

$$\frac{dm}{dr} = 4\pi r^2 \rho(r)$$ Mass

$$\frac{dp}{dr} = -\frac{Gm(r)\rho(r)}{r^2}$$ Pressure

- Special relativity included so as to account for the contributions of the interaction energy of the particles that make up the stellar object

$$\epsilon(r) = \rho(r)c^2$$ Energy density-mass density relationship from special relativity

$$\frac{dm}{dr} = \frac{4\pi r^2 \epsilon(r)}{c^2}$$ Mass

$$\frac{dp}{dr} = -\frac{Gm(r)\epsilon(r)}{r^2c^2}$$ Pressure

White Dwarf Case

$$\frac{d\overline{M}(r)}{dr} = \beta r^2 \overline{p}(r)^{1/\gamma}$$

$\overline{M}(r) = \frac{M(r)}{M_O}$ Normalized Mass

$p = \epsilon_0 \overline{p}$ Normalized Pressure

$$\frac{d\overline{p}(r)}{dr} = -R_0 \frac{\alpha \overline{p}(r)^{1/\gamma}\overline{M}(r)}{r^2}$$

$\epsilon = \epsilon_0 \overline{\epsilon}$ Normalized Energy density

$\overline{p} = \overline{K}\epsilon^{\gamma}$ Normalized Polytropic EoS

$$\alpha = R_0/\overline{K}^{1/\gamma} \qquad \beta = \frac{4\pi\epsilon_0}{M_\odot c^2 (K\epsilon_0^{\gamma-1})^{1/\gamma}}$$

Neutron Star Case

$$\frac{dm}{dr} = 4\pi r^2 \rho(r)$$

Special relativity — GR corrections

$$\frac{dp}{dr} = -\frac{G\epsilon(r)M(r)}{c^2r^2}\left[1+\frac{p(r)}{\epsilon(r)}\right]\left[1+\frac{4\pi r^2 p(r)}{M(r)c^2}\right]\left[1-\frac{2GM(r)}{c^2 r}\right]^{-1}$$

### EQUATION OF STATE

Derived from the:
- Fermi gas model of electrons for the white dwarf
- Fermi gas model of neutrons for the pure neutron star
- Polytrope Equation of state obtained from the Fermi gas model

White Dwarf Case:

$$p = K\epsilon^{\gamma}$$

Relativistic $\gamma = \frac{4}{3}$

$$K_{rel} = \frac{\hbar c}{12\pi^2}\left(\frac{3\pi^2\rho}{m_N c^2}\frac{Z}{A}\right)^{4/3}$$

Non-relativistic $\gamma = \frac{5}{3}$

$$K_{non-rel} = \frac{\hbar^2}{15\pi^2 m_e}\left(\frac{3\pi^2\rho}{m_N c^2}\frac{Z}{A}\right)^{5/3}$$

Constants used

| Relativistic | Non-relativistic |
|---|---|
| $\alpha = R_0 = 1.473km$ | $\alpha = 0.05km$ |
| $\epsilon_0 = 4.17 M_\odot c^2/km^3$ | $\epsilon_0 = 0.01392 M_\odot c^2/km^3$ |
| $\beta = 52.46/km^3$ | $\beta = 0.005924/km^3$ |
| $K_{rel} = 1.116\times10^{-13} m^{1/3}.s^{2/3}.kg^{-1/3}$ | $K_{non-rel} = 1.55\times10^{-22} m^{2/3}.s^{4/3}.kg^{-2/3}$ |

R.R. Silbar & S. Reddy Am. J. Phys. Vol. 72, No. 7, July 2004

Neutron Star Case:

Newtonian :

$$\overline{\epsilon}(r) = A_{NR}\overline{p}^{3/5} + A_R\overline{p}$$

$\overline{M}(r) = \frac{M(r)}{M_O}$ Normalized Mass

$$\frac{d\overline{M}(r)}{dr} = \beta r^2 (A_{NR}\overline{p}^{3/5} + A_R\overline{p})$$

$p = \epsilon_0 \overline{p}$ Normalized Pressure

$$\frac{d\overline{p}(r)}{dr} = -R_0\frac{(A_{NR}\overline{p}^{3/5} + A_R\overline{p})\overline{M}(r)}{r^2}$$

General Relativistic (GR)corrections

$$\frac{d\overline{M}(r)}{dr} = \beta r^2 (A_{NR}\overline{p}^{3/5} + A_R\overline{p})$$

$$\frac{dp}{dr} = -\frac{G\epsilon(r)M(r)}{c^2r^2}\left[1+\frac{p(r)}{\epsilon(r)}\right]\left[1+\frac{4\pi r^2 p(r)}{M(r)c^2}\right]\left[1-\frac{2GM(r)}{c^2 r}\right]^{-1}$$

$$T_1 = -\frac{G(A_{NR}\overline{p}^{3/5} + A_R\overline{p})M(r)M_\odot}{c^2r^2}$$

$$T_2 = \left[1+\frac{\overline{p}(r)}{(A_{NR}\overline{p}^{3/5} + A_R\overline{p})}\right]$$

$$T_3 = \left[1+\frac{4\pi r^2\epsilon_0\overline{p}(r)}{M(r)M_\odot c^2}\right]$$

$$T_4 = \left[1-\frac{2GM_\odot M(r)}{c^2 r}\right]^{-1}$$

$$\frac{d\overline{p}}{dr} = T_1\times T_2\times T_3\times T_4$$

Constants used: Newtonian and GR corrections

$A_{NR} = 2.4216 A_R = 2.8663$

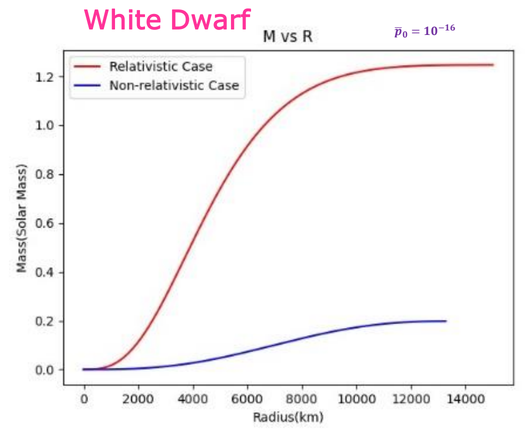
$\epsilon_0 = 0.003006 M_\odot c^2/km^3$

$\beta = 0.03778/km^3$

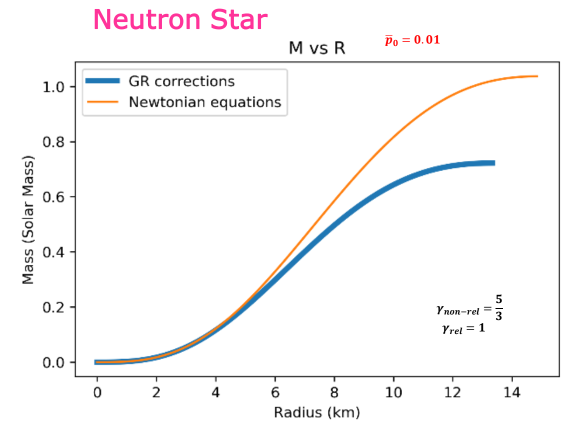
$\alpha = R_0 = 1.476km$

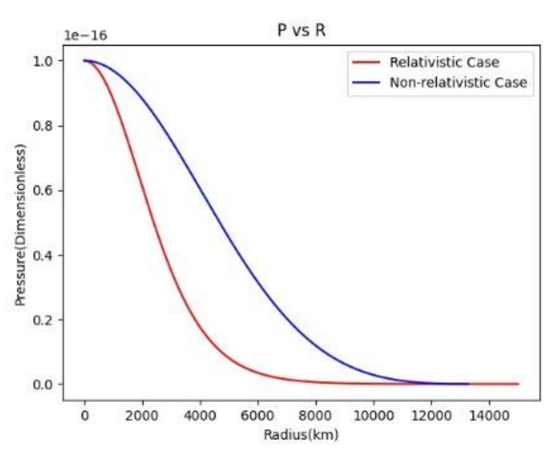
## RESULTS &OBSERVATIONS

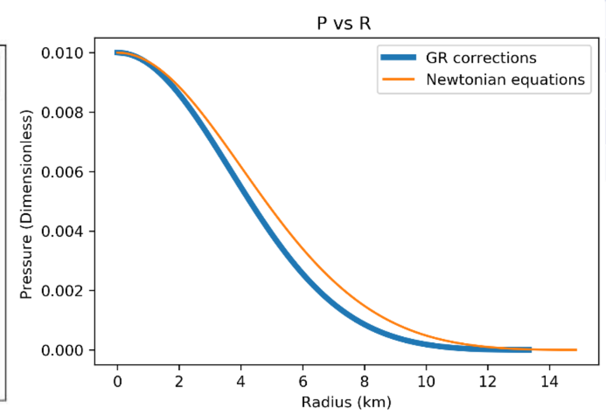
White Dwarf




Neutron Star




Mass-radius and Pressure –radius relationship

| | This work | | Literature |
|---|---|---|---|
| White dwarf | | | |
| | Non-relativistic | Relativistic | |
| Mass ($M_\odot$) | 0.19758 | 1.2694 | $0.535^{+0.012}_{-0.012}$ [1] |
| Radius (km) | 13 266.34 | 15 000 | $14679^{+139}_{-139}$ [1] |
| Compactness | $4.55\times10^4$ | $8.16\times10^3$ | $10^3 - 10^4$ [2] |
| | | | |
| Neutron Star | | | |
| | Newtonian | GR corrections | |
| Mass ($M_\odot$) | 1.03698 | 0.72212 | $1.44^{+0.15}_{-0.14}$ |
| Radius (km) | 14.8484 | 13.3334 | $13.02^{+0.15}_{-0.14}$ |
| Mass density (kg/m$^3$) | 1.5041$\times10^{17}$ | 2.0773$\times10^{17}$ | ~2.6$\times10^{17}$ [3] |
| Number or neutrons | 1.2314$\times10^{57}$ | 8.5752$\times10^{57}$ | $10^{57}$ |

[1] S.G. Parsons et .al Mon. Not. R. Astron. Soc. 402, 2591-2608 (2010)
[2] Jose PS Lemos et .al, The European Physical Journal C, 75(2):76, 2015.
[3] I.Bombaci and D.Logoteta, A&A, A128, (2018), doi:http://dx.doi.org/10.1051/0004-6361/201731604

## OUTLOOK

- The task of this project was to understand the mass-radius and pressure-radius dependence on various relevant EoS for neutron stars.
- We managed to derive the relativistic and non-relativistic EoS and applied them to white dwarfs before extending them to neutron stars.
- The mass density of the neutron star; that we calculated, is compatible to that of the nucleus.
- Provided with sufficient time, one can extend our model to a non-pure neutron star where we take into account nucleon-nucleon interaction, inputs from nuclear structure physics so as to derive more realistic EoS.

## ACKNOWLEGDEMENTS

- We would like to thank National Institute for Theoretical Physics (NITheP) for funding the internship.
- Prof. Azwinndini Muronga for the mentorship and hosting the internship.
- The Statistical and Thermal models for heavy ion collisions and astrophysics topical group members.
- Mrs Rene Kotze for logistical arrangements during the internship.