Statistical and Thermal Models for Heavy Ion Collisions and Astrophysics

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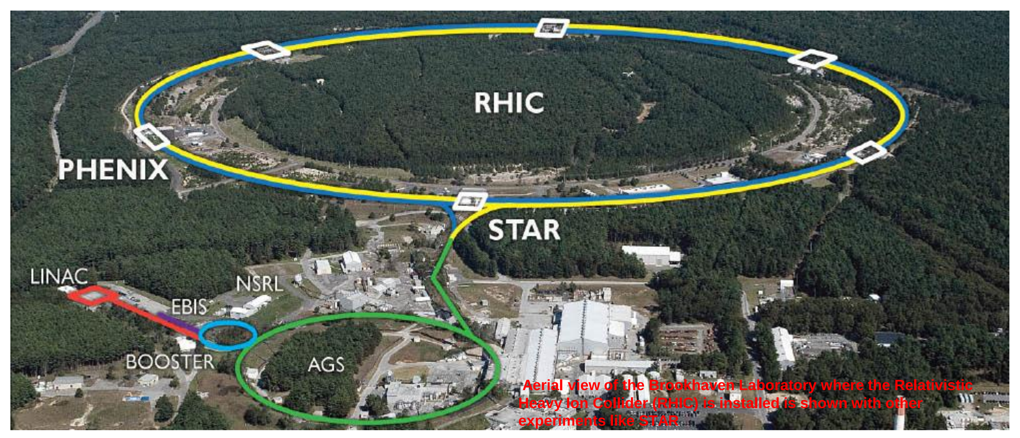
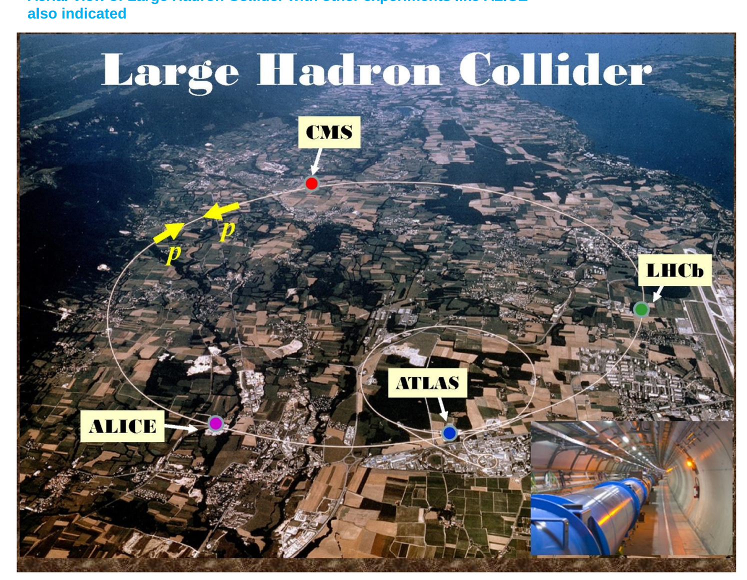
INTRODUCTION

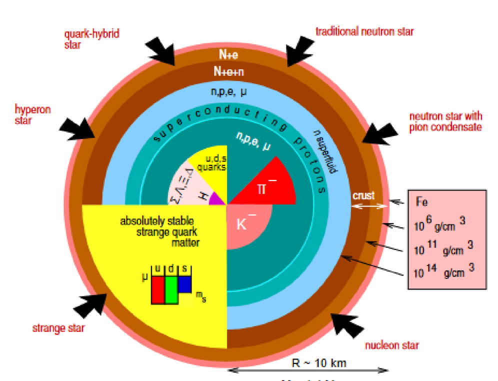
A thermal model in physics can be defined as a model that uses statistical mechanics, thermodynamics and statistical methods to describe the behaviour of a system with a variation in state variables.

- Thermal models can be used to describe particle multiplicity in heavy ion collisions as a function of the temperature T and baryon chemical potential μ_b.
- The freeze-out stage is at thermal equilibrium hence the application of statistical physics.

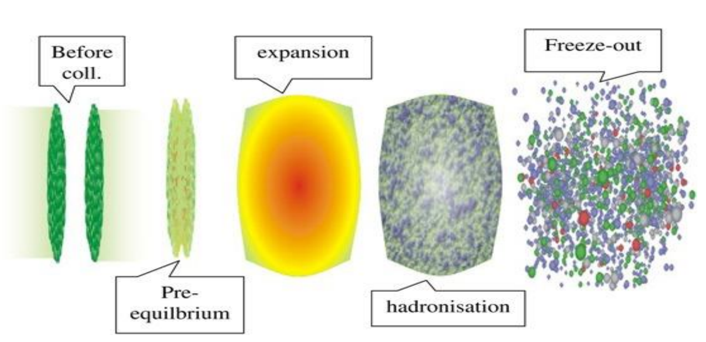


Aerial view of Large Hadron Collider with other experiments like ALICE also indicated





Artist rendition of a neutron star merger
Source: https://www.ligo.caltech.edu/news/ligo20200106



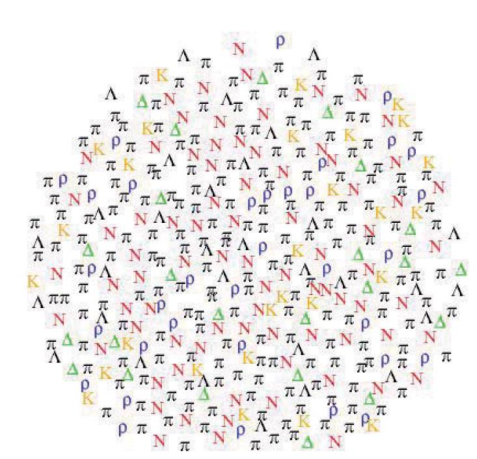
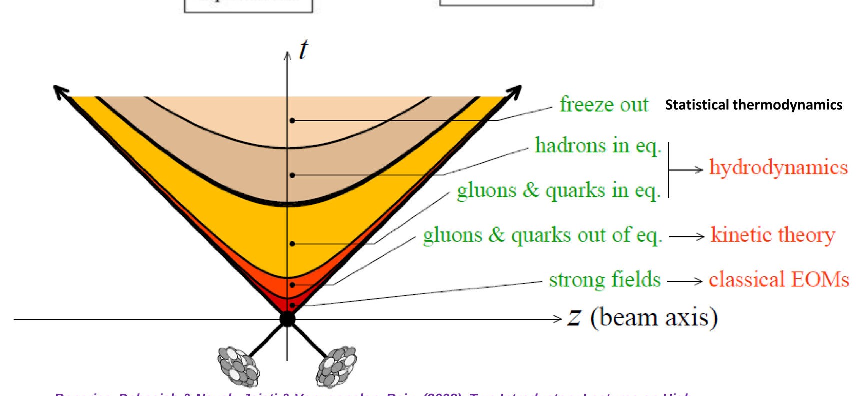


Image: Courtesy of Jean Cleymans



Banerjee, Debasish & Nayak, Jajati & Venugopalan, Raju. (2008). Two Introductory Lectures on High-Energy QCD and Heavy-Ion Collisions. Lecture Notes in Physics. 785. 10.1007/978-3-642-02286-9_3.

Maxwell-Boltzmann	Fermi-Dirac	Bose-Einstein
▪ Non-interacting and distinguishable particles e.g. ideal gas ▪ Classical systems (Quantum effects ignored)	▪ Particles that have half-integer spin ▪ Obey Pauli exclusion principle ▪ Quantum effects considered	▪ Integer-spin particles ▪ Do not obey Pauli's exclusion principle
$\bar{n}_j = \frac{1}{e^{(E-\mu)/T}}$	$\bar{n}_j = \frac{1}{1+e^{(\epsilon_j-\mu)/T}}$	$\bar{n}_j = \frac{1}{e^{(\epsilon_j-\mu)/T}-1}$
	$N = \frac{gVm^{3/2}}{2^{1/2}\pi^2\hbar^3}\int_0^\infty \frac{\sqrt{\varepsilon}\, d\varepsilon}{e^{(\varepsilon-\mu)/T}\pm 1}.$	$N = \frac{gVm^{3/2}}{2^{1/2}\pi^2\hbar^3}\int_0^\infty \frac{\sqrt{\varepsilon}\, d\varepsilon}{e^{(\varepsilon-\mu)/T}\pm 1}.$
	$\Omega = -\frac{2}{3}\frac{gVm^{3/2}}{2^{1/2}\pi^2\hbar^3}\int_0^\infty \frac{\varepsilon^{3/2}\, d\varepsilon}{e^{(\varepsilon-\mu)/T}\pm 1}.$	$\Omega = -\frac{2}{3}\frac{gVm^{3/2}}{2^{1/2}\pi^2\hbar^3}\int_0^\infty \frac{\varepsilon^{3/2}\, d\varepsilon}{e^{(\varepsilon-\mu)/T}\pm 1}.$

OBJECTIVES

- To understand the properties of nuclear matter under extreme conditions in heavy ion collisions and astrophysics
- Use thermal and statistical models to understand the chemical potential and temperature dependence of:
  - particle density
  - particle multiplicity and ratios

for particles produced in heavy-ion collisions.

METHODOLOGY &COMPUTATION

- We derived the expressions for the particle density of relativistic particles.

$$E^2 = p^2c^2 + m^2c^4.$$

- Fermi-Dirac statistics:

$$dN_\epsilon = \frac{gV}{2\pi^2(\hbar c)^3}\frac{\sqrt{\epsilon^2 + c^4m^2}\,\epsilon d\epsilon}{e^{(\epsilon-\mu)/T}+1} \qquad \frac{N}{V} = \frac{g}{2\pi^2(\hbar c)^3}\int_{mc^2}^{\infty}\frac{\sqrt{\epsilon^2 + c^4m^2}\,\epsilon d\epsilon}{e^{(\epsilon-\mu)/T}+1}.$$

- Bose-Einstein statistics:

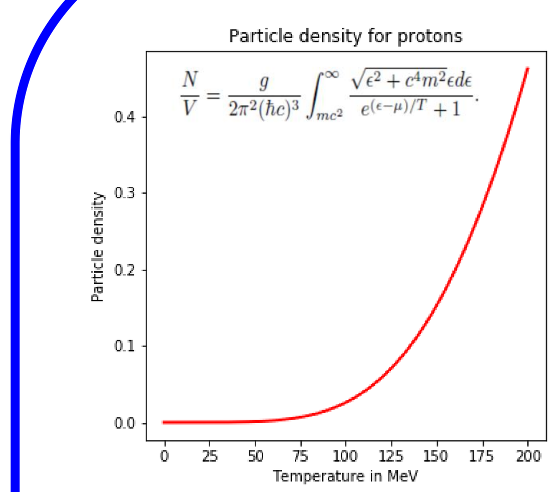
$$dN_\epsilon = \frac{gV}{2\pi^2(\hbar c)^3}\frac{\sqrt{\epsilon^2 + c^4m^2}\,\epsilon d\epsilon}{e^{(\epsilon-\mu)/T}-1} \qquad \frac{N}{V} = \frac{g}{2\pi^2(\hbar c)^3}\int_{mc^2}^{\infty}\frac{\sqrt{\epsilon^2 + c^4m^2\epsilon}}{e^{(\epsilon-\mu)/T}-1}d\epsilon$$

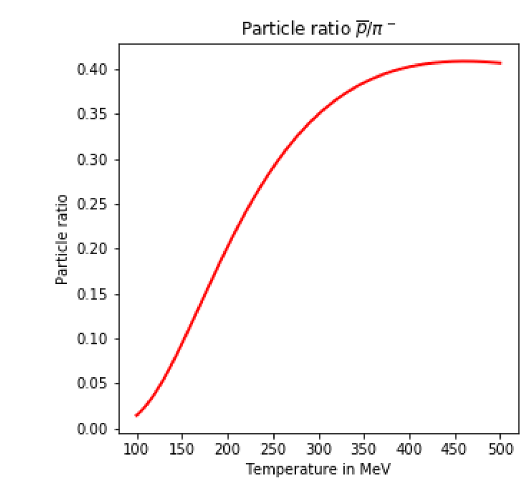
- Python used to solve the equations above
- Open source, non-proprietary software.
- Built in support for scientific computing eg. SciPy and Numpy.
- Interpreted language = no need to compile.
- Good for prototyping and quickly implementing code.
- MatPlotLib for graphs and visualization

CONCLUSIONS

- The main aim was to understand and use different statistical models to model particle yields, particle number densities and ratios.
- However, the statistical models were studied and applied to describe the last stage of Heavy Ion Collision (HIC).
- We managed to model the behaviour of quark and gluon densities and this is compatible with results in literature.
- The models derived from undergraduate statistical thermodynamics knowledge are fitting experimental results from the Relativistic Heavy-Ion Collider (RHIC). This can be easily extended to LHC experimental results.
- The models are versatile as they can be refined to better predict realistic systems.
- Given enough time one can extend our model to investigate other effects such as interactions, finite volume effects as well as studying the dependence of particle yields and ratios on beam energy, volume, equilibration factors, spin and isospin

RESULTS

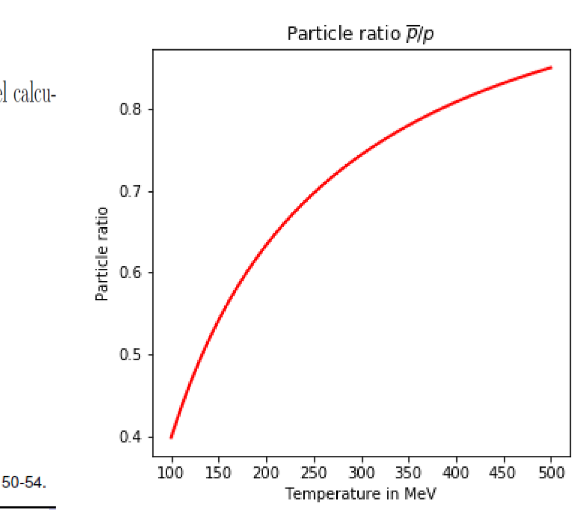




P. Braun-Munzinger *et al*. Physics Letters B 518 (2001) 41-46

Table 1: Comparison of experimental particle ratios and thermal model calculations at $T = 174$ MeV., $\mu_b = 46$ MeV at $\sqrt{s} = 130$ GeV.

Ratio	Model calculation	Experimental result
$\bar{p}/p$	0.59	0.65 ± 0.07
$\bar{p}/\pi^-$	0.09	0.08 ± 0.01
π^-/π^+	1.00	1.00 ± 0.02
K^-/K^+	1.00	0.88 ± 0.05
K^-/π^-	0.224	0.149 ± 0.02



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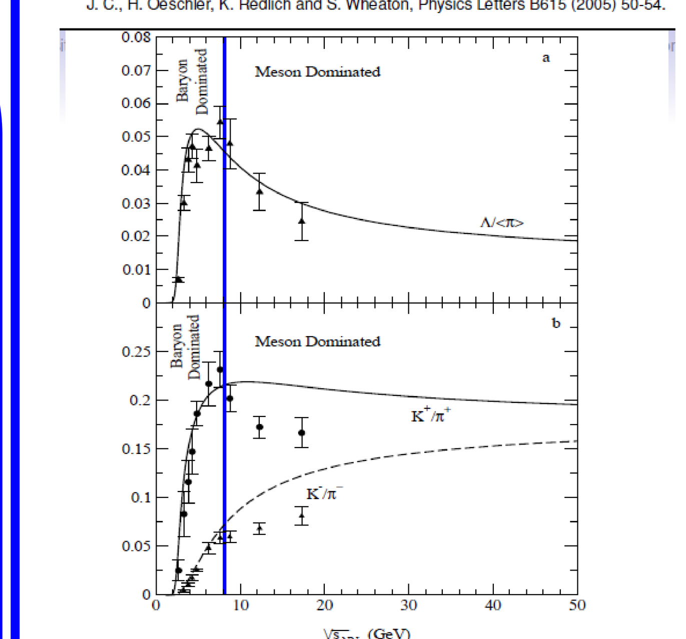

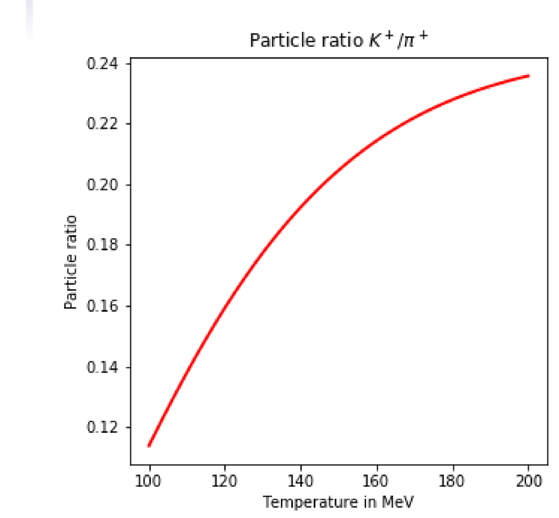

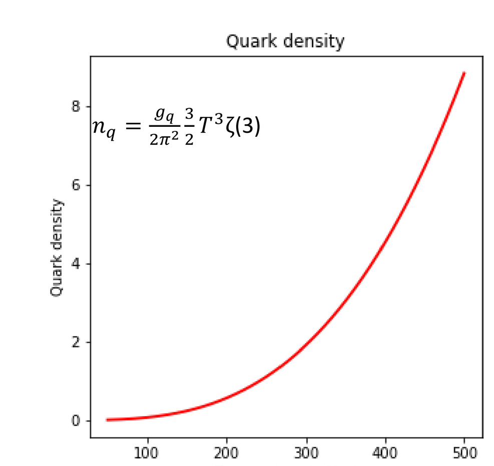

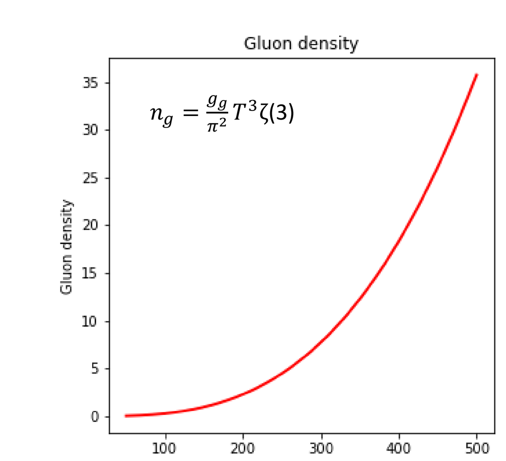

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