Robust control of quantum systems by quantum systems

A. Rouillard and T. Konrad

School of Chemistry and Physics, University of KwaZulu-Natal

Goal

Drive a quantum system from an arbitrary initial state into a target state, stabilise it there and protect against noise.

Coherent feedback control

In this work we consider control schemes which drive a quantum system into a target state by sequential coupling to ancillary quantum systems. These ancillary systems are referred to as the controllers. A suitable coupling between system and controllers is necessary to achieve convergence to the target state.

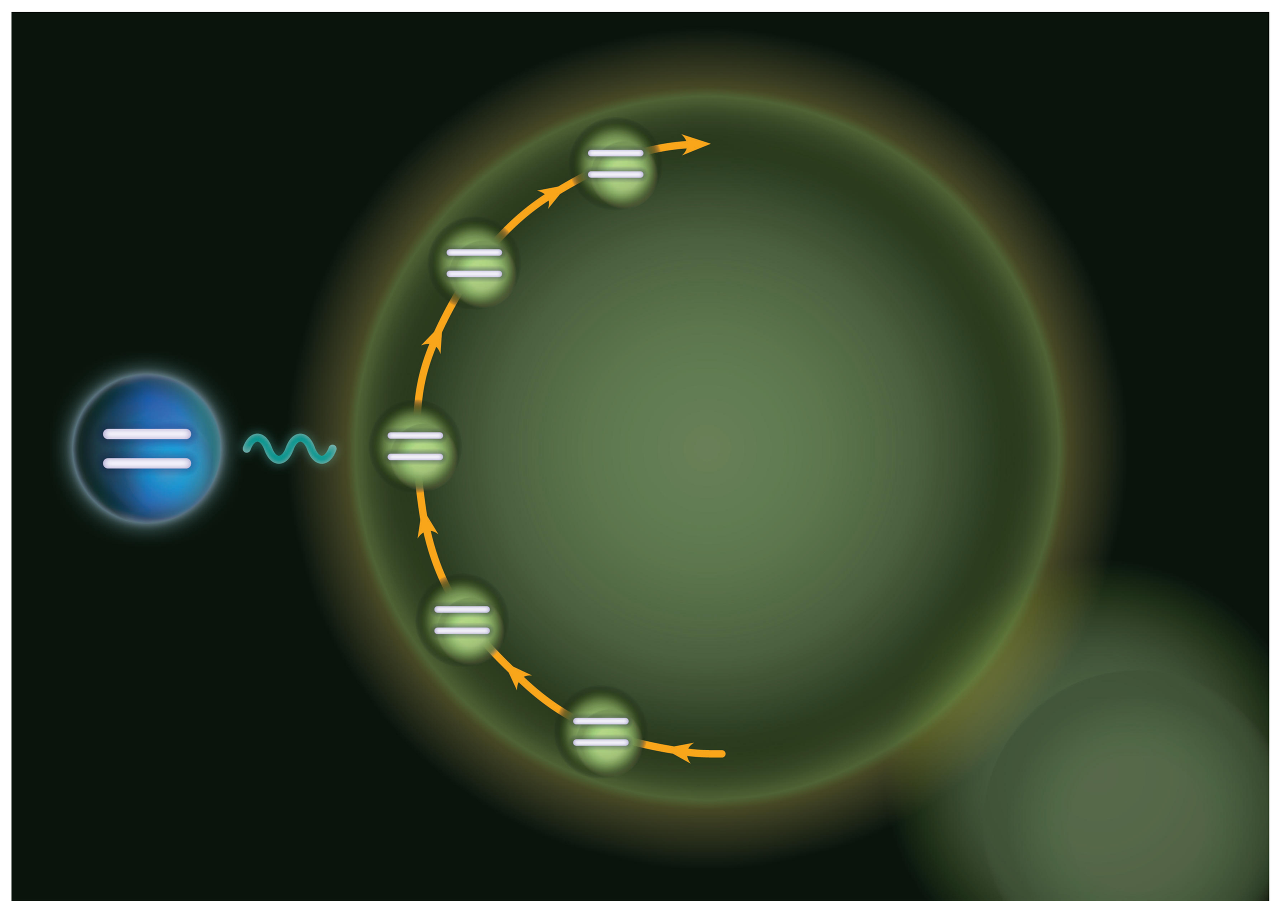


Figure 1: A quantum system (blue) is driven gradually by a sequence of interactions with other quantum systems (green) into a target state.

Conditions for convergence to the target state

In order to successfully drive a system, independent of its initial state, into a given target state $|T\rangle \in \mathcal{H}_s$, where $\mathcal{H}_s$ is the Hilbert space of the system, the following theorem can be applied.

Theorem

Let $\$(\rho) = \sum_i M_i \rho M_i^\dagger$ be a trace-preserving quantum channel with Kraus operators M_i obeying the condition

$$M_i\,|T\rangle = z_i\,|T\rangle\,,$$

$z_i \in \mathbb{C}$, with respect to some target state $|T\rangle \in \mathcal{H}_s$, and satisfying

$$\text{span}\left\{M_i^\dagger\,|T\rangle\right\}_i = \mathcal{H}_s\,.$$

Then any state ρ converges to the target state under repeated application of the quantum channel $\$$.

Any such channel $\$$ can be implemented by a unitary time evolution U which couples the system to a suitable ancilla system in initial state $|\phi\rangle$, such that

$$\$(\rho) = Tr[U(|\phi\rangle\langle\phi| \otimes \rho)U^\dagger]\,,$$

where the Kraus operator $M_i = \langle i|U|\phi\rangle$.

Example: Weak Swap

This unitary coupling $U = \exp(-i\lambda S)$, where $S(|\psi\rangle\otimes|\phi\rangle) = |\phi\rangle\otimes|\psi\rangle$ results in a weak swap between (d-level) system and controller states. The real parameter $0 < \lambda \leq \frac{\pi}{2}$ determines the strength of the swap.

The fidelity increases in an exponential fashion as a function of the number of iterations of the control scheme. The weak swap is robust against errors in implementation and is able to protect the system against noise.

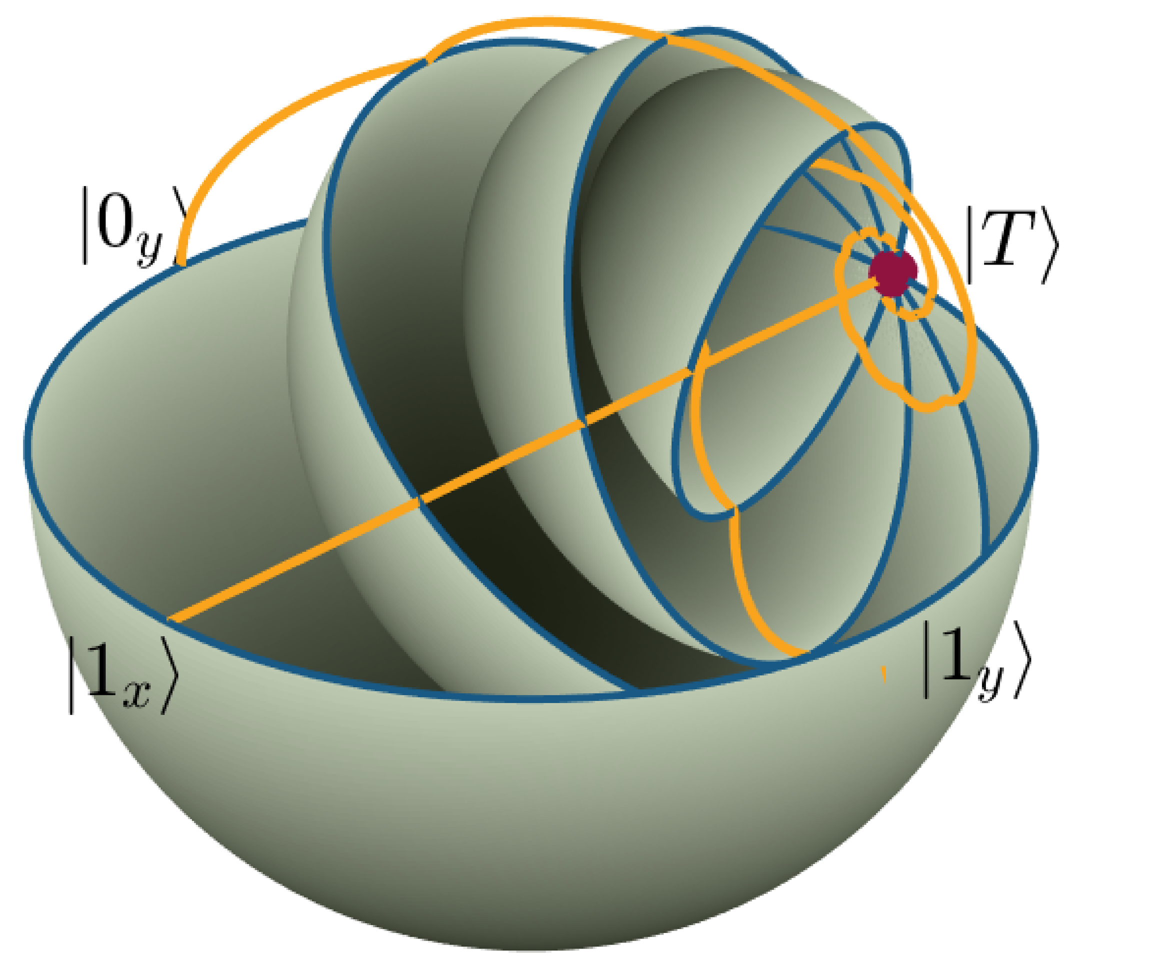


Figure 2: Weak swap between two-level systems: Evolution of the Bloch sphere of states, represented by its lower hemisphere, towards the target state (red point) after sequential weak swaps with $\lambda = \frac{\pi}{5}$. The orange lines show the evolution of three states (eigenstates of Pauli operators) during the interaction with controllers initialised in the target state $|T\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$.

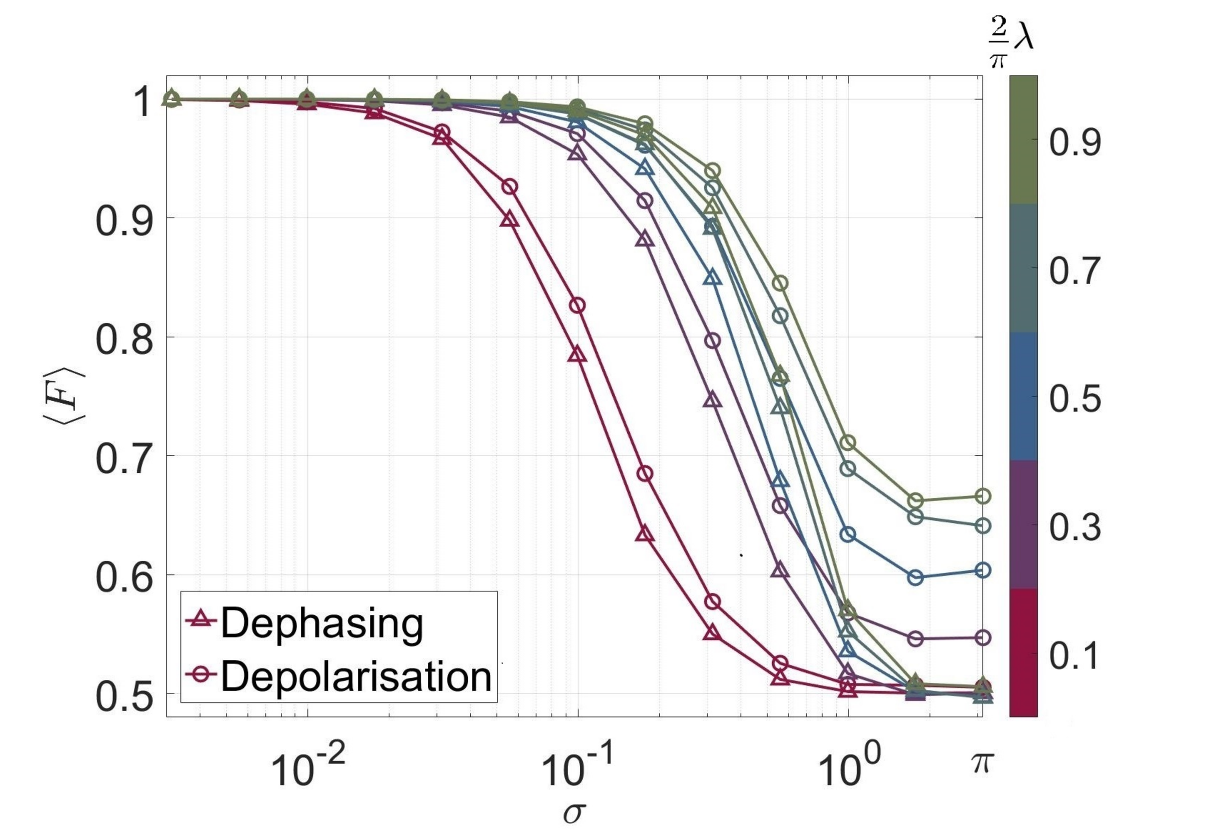
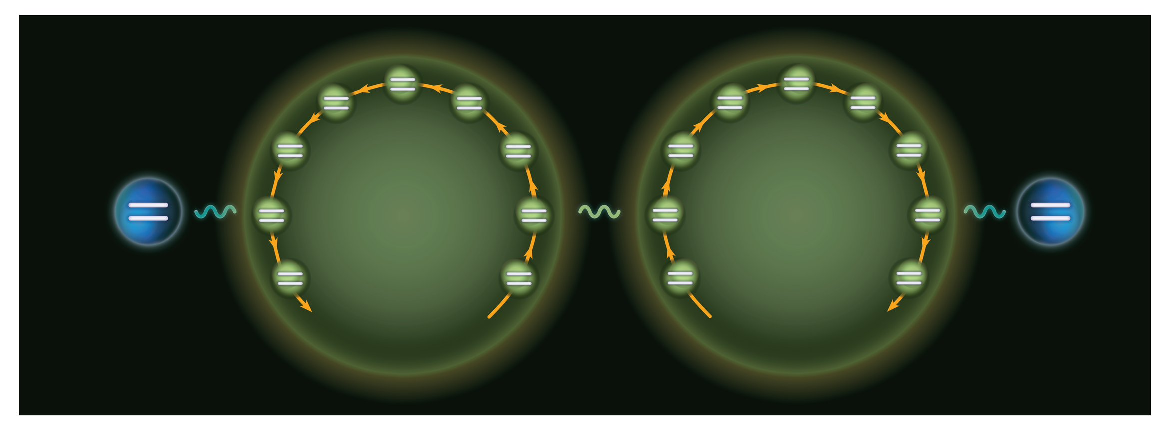


Figure 3: Quantitative assessment of the control abilities of the weak swap in the presence of dephasing noise and depolarisation noise. The plots show the averages fidelity $\langle F\rangle = \sum_n \langle T|\rho_n|T\rangle / N$ of the target fidelity over the sequence ρ_n for trajectories of length $N = 10^3$.

Potential applications

Preparation of orbital angular momentum and polarisation states of light using aditional degrees of freedom. Robust control of an atom in a cavity using photons. Preparation of entangled states without direct iteration between entangled pairs. Quantum computing, mitigating noise in state preparation and gate execution.



[1] Konrad, T., et al. arXiv:2012.01998 (2020)